Dark matter with "Second-stage annihilation"

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This talk is based on:

M. Geller, S. Iwamoto, G. Lee, Y. Shadmi, O. Telem [1802.07720].

- "Dark matter" exists:
  - galaxy rotation curve, bullet cluster, ...
- Traditional models for DM:
  thermal WIMPs with "freeze-out" [→ next slide]
  [weakly interacting massive particles]

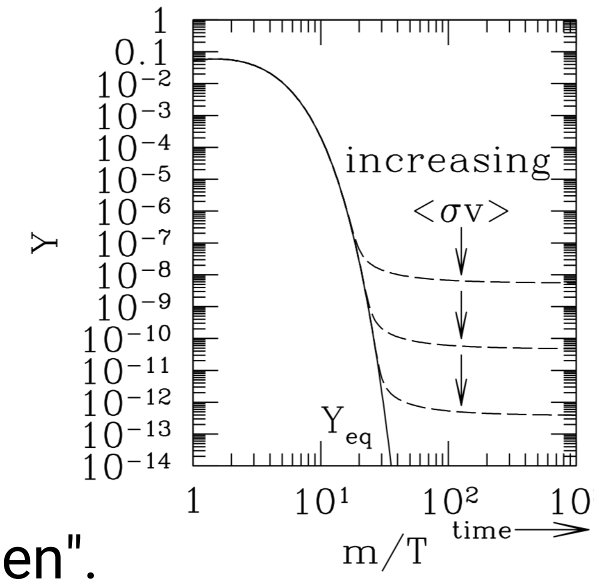
- Discrepancies in sub-galactic scale see, e.g., Tulin, Yu [1705.02358]
  - Core-cusp problem ... $\rho_{\mathrm{DM}}(r)\Big|_{r\to 0} \propto r^{-1}$ (numerical simulation)
    [low-surface-brightness galaxy]
    r^{0} (observation of LSBs, dSphs)
    [dwarf spheroidal galaxy]
  - Diversity problem ... Variety of rotation curves is observed.
  - Too-big-to-fail problem ... Big DM halos expected are not observed.

→ the effects of baryon? or: non-traditional DM?

Yet a trend to explore "exotic" DM scenarios.

- $T > m_{\rm DM}$: DM DM $\xrightleftharpoons{\text{chem.eq.}}$ SM SM ... "DM is in the thermal bath." (hosted by photons; i.e. T is "photon temperature")
  - $n_{\rm DM}$ is determined by the bath (i.e., by T).
- $T \lesssim m_{\rm DM}$: DM DM $\xrightarrow{\text{annihil.}}$ SM SM
  - DM production is reduced since $T < m_{\rm DM}$.
- $T \lesssim m_{\rm DM}/30$: DM DM ⇌̸ SM SM
  - DM annihilation is "frozen out" due to expansion.

→ DM density per comoving volume, Y, is "frozen".



$$\Omega_{\rm DM}h^2 \approx \frac{1.1\times10^9\cdot x_{\rm f}}{\sqrt{g_*}M_{\rm pl}\langle\sigma v\rangle\cdot{\rm GeV}} \approx 0.1\cdot\frac{15}{\sqrt{g_*}}\frac{x_{\rm f}}{30}\frac{3\times10^{-26}\,{\rm cm}^3/{\rm s}}{\langle\sigma v\rangle} \quad \text{with } x_{\rm f}=m_{\rm DM}/T_{\rm fo}.$$

$$\Longrightarrow \quad \langle\sigma v\rangle_{\rm annihil.} \sim 3\times10^{-26}\,{\rm cm}^3/{\rm s} \simeq \frac{\alpha_{\rm EM}^2}{(150\,{\rm GeV})^2}$$

"stable thermal WIMP as the cold DM"

Figure from Gelmini and Gondolo, 1009.3690

- $T > m_{DM}$: DM DM $\overset{\text{chem.eq.}}{\rightleftharpoons}$ SM SM ∴ "DM is in the thermal bath." (hosted by photons, i.e. T is "photon temperature")
  - n_{DM} is determined by the bath (i.e., by T).

thermal

- $T \lesssim m_{DM}$: DM DM $\xrightarrow{\text{annihil.}}$ SM SM
  - DM production is reduced since $T < m_{DM}$.

weakly-interacting

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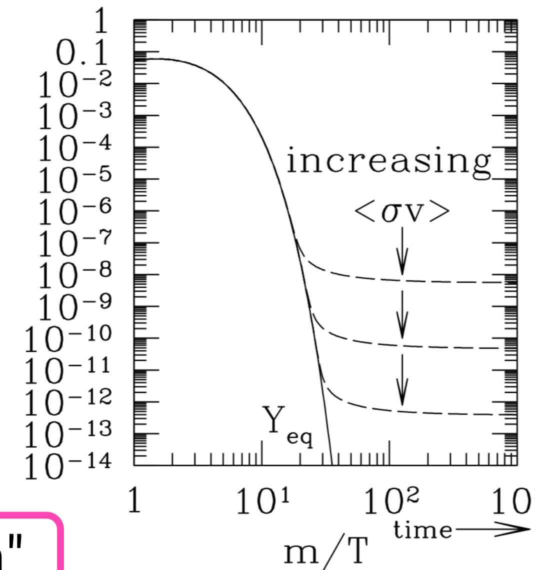
stable



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massive / cold

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see, e.g., Tulin, Yu [1705.02358]

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Yet a trend to explore "exotic" DM scenarios.

■ Thermal WIMP relic abundance

$$\Omega_{\rm DM}h^2 \approx \frac{1.1\times 10^9 \cdot x_{\rm f}}{\sqrt{g_*}M_{\rm pl}\langle\sigma v\rangle \cdot {\rm GeV}} \approx 0.1 \cdot \frac{15}{\sqrt{g_*}}\frac{x_{\rm f}}{30}\frac{3\times 10^{-26}\,{\rm cm^3/s}}{\langle\sigma v\rangle} \quad \text{with } x_{\rm f} = m_{\rm DM}/T_{\rm fo}.$$

■ Unitarity bound on partial-wave cross section

$$\sigma = \sum_J \sigma_J, \quad \sigma_J(2\to 2) \le \frac{(2J+1)\pi}{p_{\rm i}^2}$$

- For $v_{\rm rel} \ll 1$, $\langle\sigma v_{\rm rel}\rangle \sim \sigma_{J=0} v_{\rm rel}$

$$\le \frac{\pi}{(m v_{\rm rel}/2)^2} v_{\rm rel} \sim \frac{4\pi}{m^2\sqrt{6/x_{\rm f}}}$$

$\therefore \Omega_{\rm DM}h^2 \le 1 \iff m_{\rm DM} \le 340\,{\rm TeV}$ (for majorana-fermion DM).

* For $\Omega_{\rm DM}h^2 = 0.12$, the bound is ~100 TeV.

Models to break this upper bound?

Figure from Gelmini and Gondolo, 1009.3690

Our DM particle: "hidden quark" X

[a particle with SU(N)-charge]

- extra gauge symmetry
- confining at $T=\Lambda_D$

[called "hidden color" or "dark QCD".]

- $T > m_{DM}$: DM DM $\xrightleftharpoons{\text{chem.eq.}}$ SM SM
- $T \lesssim m_{DM}$: DM DM $\xrightarrow{\text{annihil.}}$ SM SM
- $T \lesssim m_{DM}/30$: DM DM $\not\rightleftharpoons$ SM SM

$\sigma_{\text{annihil.}} \sim m_X^{-2}$

$T \sim \Lambda_D$: phase transition

- $T \lesssim \Lambda_D$: [DM] [DM] $\xrightarrow{\text{annihil.}}$ SM SM

hadronized

"second stage annihilation"

$\sigma_{\text{annihil.}} \sim \Lambda_D^{-2}$

- $T \sim \Lambda_D/30$: SSA is frozen out. → DM "frozen".

$$\Rightarrow \frac{\Omega_{\rm DM}h^2}{0.1} \approx \frac{3\times 10^{-26}\,{\rm cm^3/s}}{(30 m_X \Lambda_{\rm D}^3)^{-1/2}} \simeq \sqrt{\frac{m_X\Lambda_{\rm D}^3}{(10\,{\rm TeV})^4}} \iff \Omega_{\rm DM} \sim \sqrt{\frac{\Lambda_D}{m_X}}\left(\frac{m_X}{30\,{\rm TeV}}\right)^2$$

Assuming Λ_D > MeV not to ruin BBN,

$$10\,\text{TeV} \lesssim m_X \lesssim 1\,\text{PeV}.$$

$$\begin{cases} \sigma \sim \Lambda_{\rm D}^{-2} \\ v_{\rm rel}\big|_{\rm fo} \sim \sqrt{T_{\rm fo}/m_X} \\ \qquad\quad \sim \sqrt{\Lambda_{\rm D}/30 m_X} \end{cases}$$

1. Introduction ✓

2. Dark QCD models and Our toy model

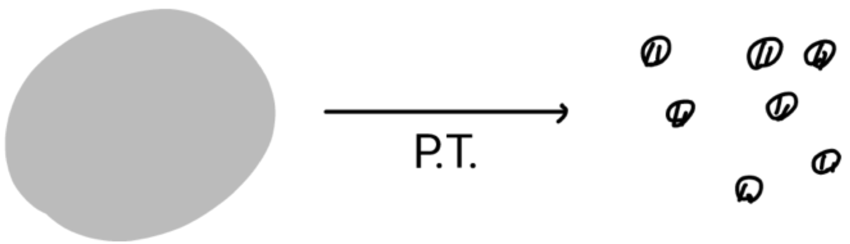
3. Annihilation cross section: $\sigma_{\text{annihil.}} \overset{?}{\sim} \Lambda_D^{-2}$
   - "Rearrangement" is the dominant process.

4. Conclusion

Dark QCD: hidden confining SU(N) gauge symmetry

- If no "dark quarks" are added:
  [particles charged under dark QCD]

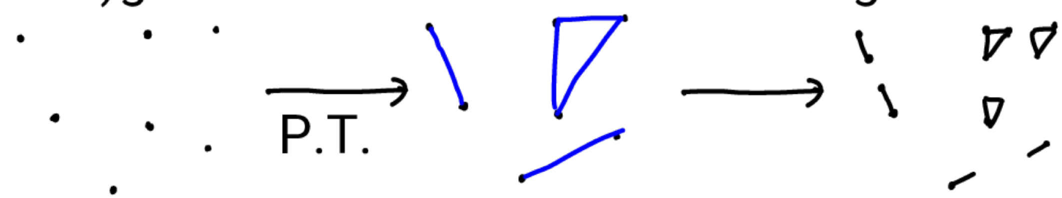
dark gluons (radiation) glueballs $\cdots \begin{cases} m_{gb} \sim 7\Lambda_D, \\ R_{gb} \sim \Lambda_D \end{cases}$ (ground state 0^{++})



$T \gg \Lambda_D$ $T < \Lambda_D$

This is via "dark-QCD string formation":

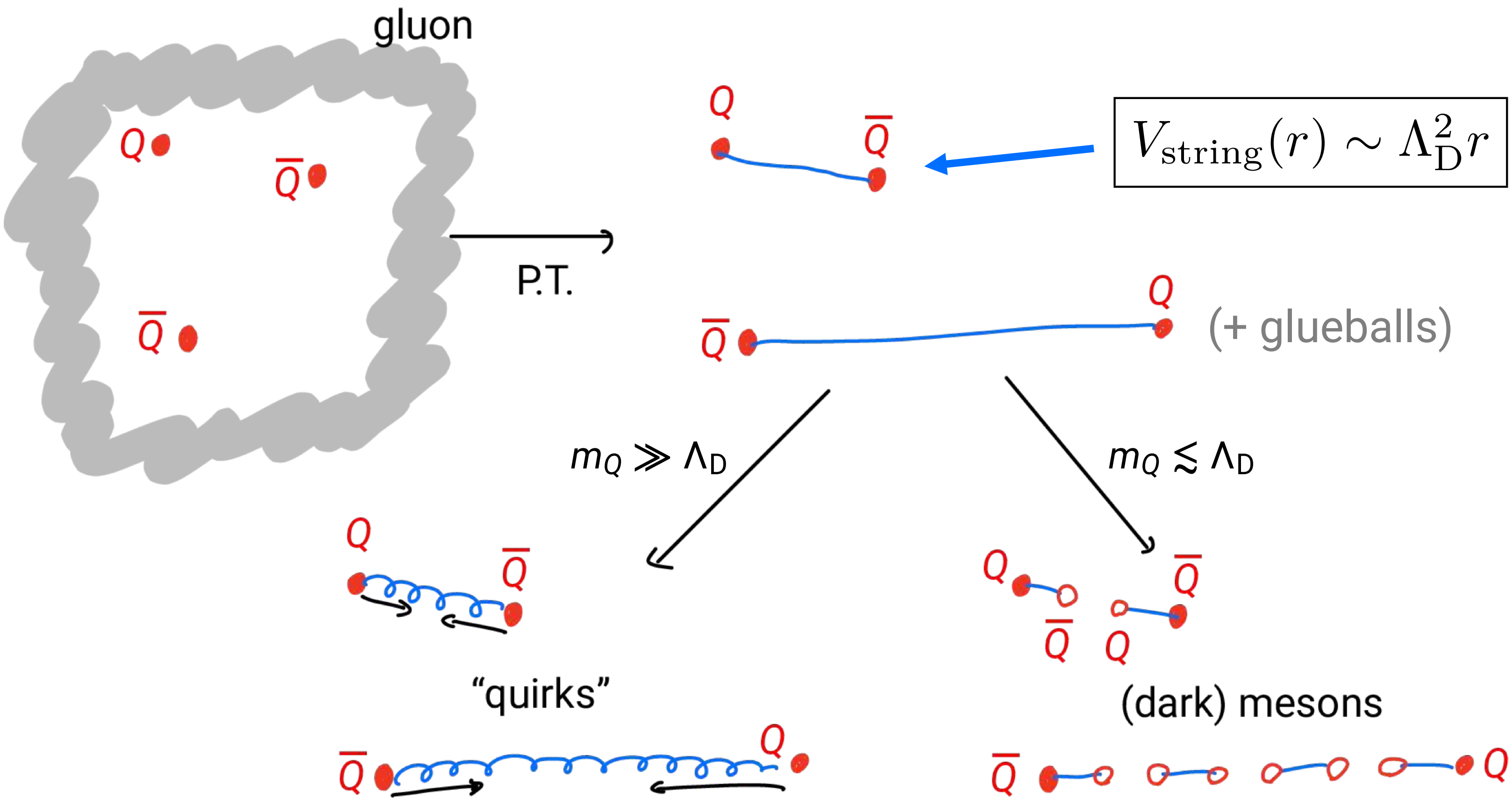
(dark)gluons glueballs



see Forestell, Morrissey, Sigurdson [1710.06447] * Note the subtlety on "temperature".

Dark QCD: hidden confining SU(N) gauge symmetry

- With one quark Q with SU(N)-fundamental representation:

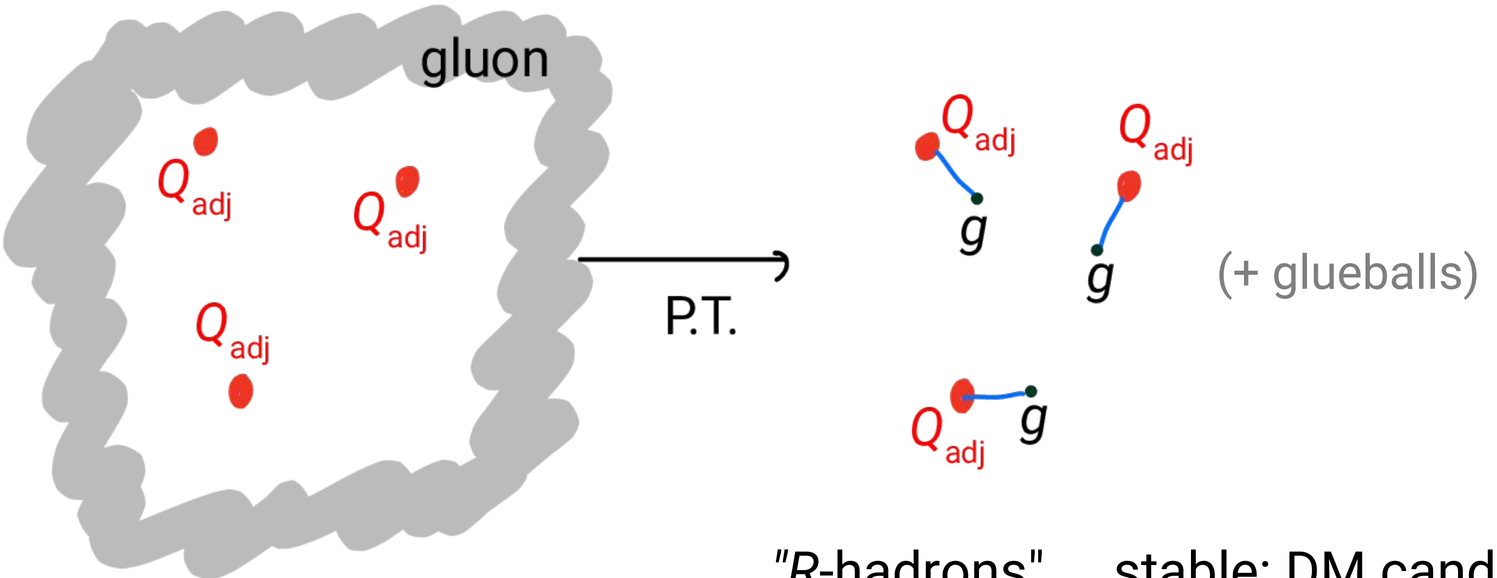


(Their fate depends on model-details.)

see
Kang, Luty [0805.4642], Kribs, Roy, Terning, Zurek [0909.2034] for quirk models,
Kang, Luty, Nasri [hep-ph/0611322], Appelquist, Brower, Buchoff, et al. [1503.04203] for meson models.

Dark QCD: hidden confining SU(N) gauge symmetry

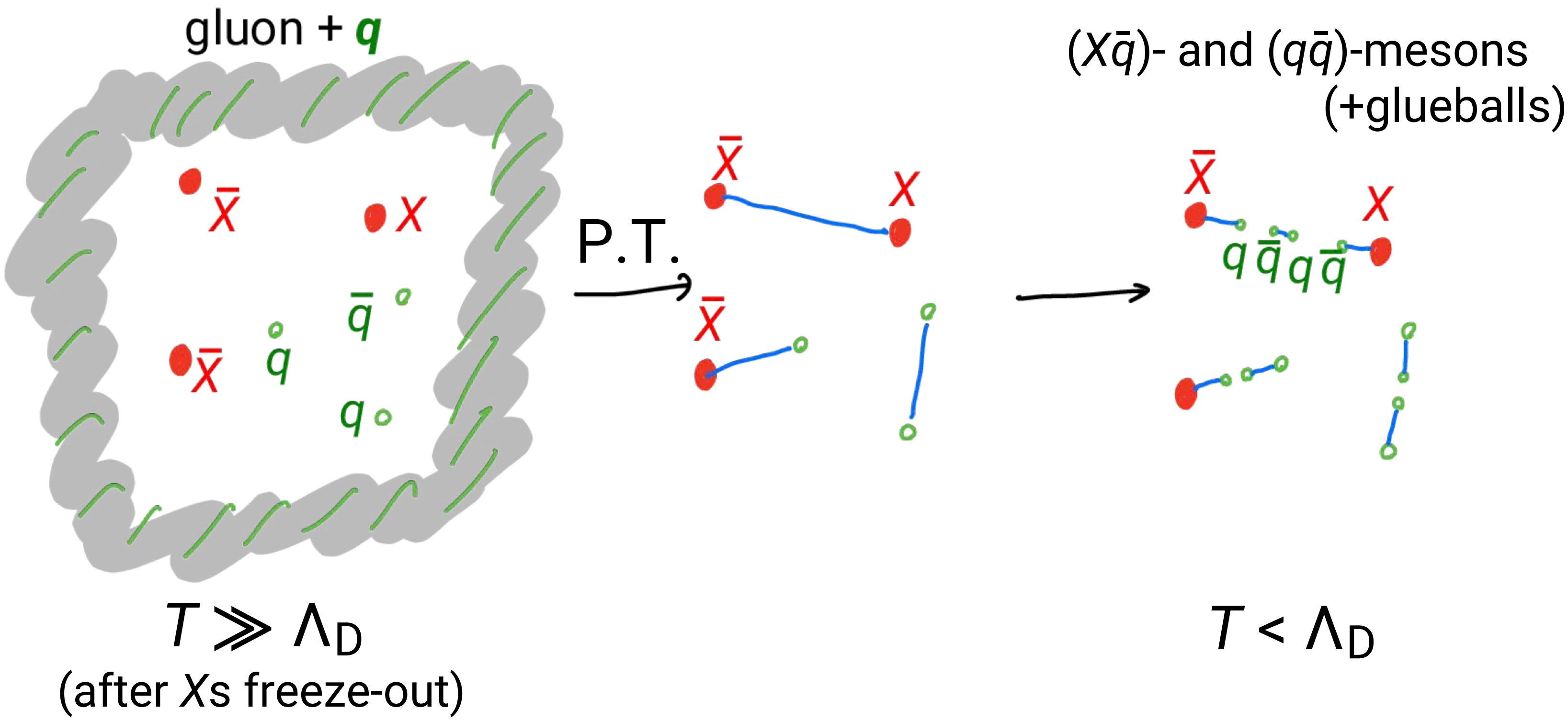
- With one quark Q_{adj} with SU(N)-adjoint representation:



"*R*-hadrons" ... stable: DM candidate.

see Arvanitaki, Dimopoulos, Dubovsky, Graham, et al. [0812.2075], Feng, Shadmi [1102.0282]

Our Model: two fundamental quarks X and q: $m_X \gg \Lambda_D \gtrsim m_q$



$H_X = (X\bar{q})$-meson

Our Model: two fundamental quarks X and q: $m_X \gg \Lambda_D \gtrsim m_q$

- At phase transition,

$$\underset{\text{"matter"}}{X, \bar{X},} \quad \underset{\text{"radiation"}}{q, \quad g} \longrightarrow \underbrace{H_X, \bar{H}_X,}_{\text{mass} \sim m_X} \quad \underbrace{(q\bar{q}), \quad G}_{\text{mass} \sim \Lambda_D}$$

[glueball]

- H_X : stable → DM candidate **[our focus]**
- $(q\bar{q})$, G : decays to the lightest one $((q\bar{q})_{{}^1S_0}$ if $m_q \ll \Lambda_D)$, whose fate depends on model-details.
- Baryonic DM may be produced:

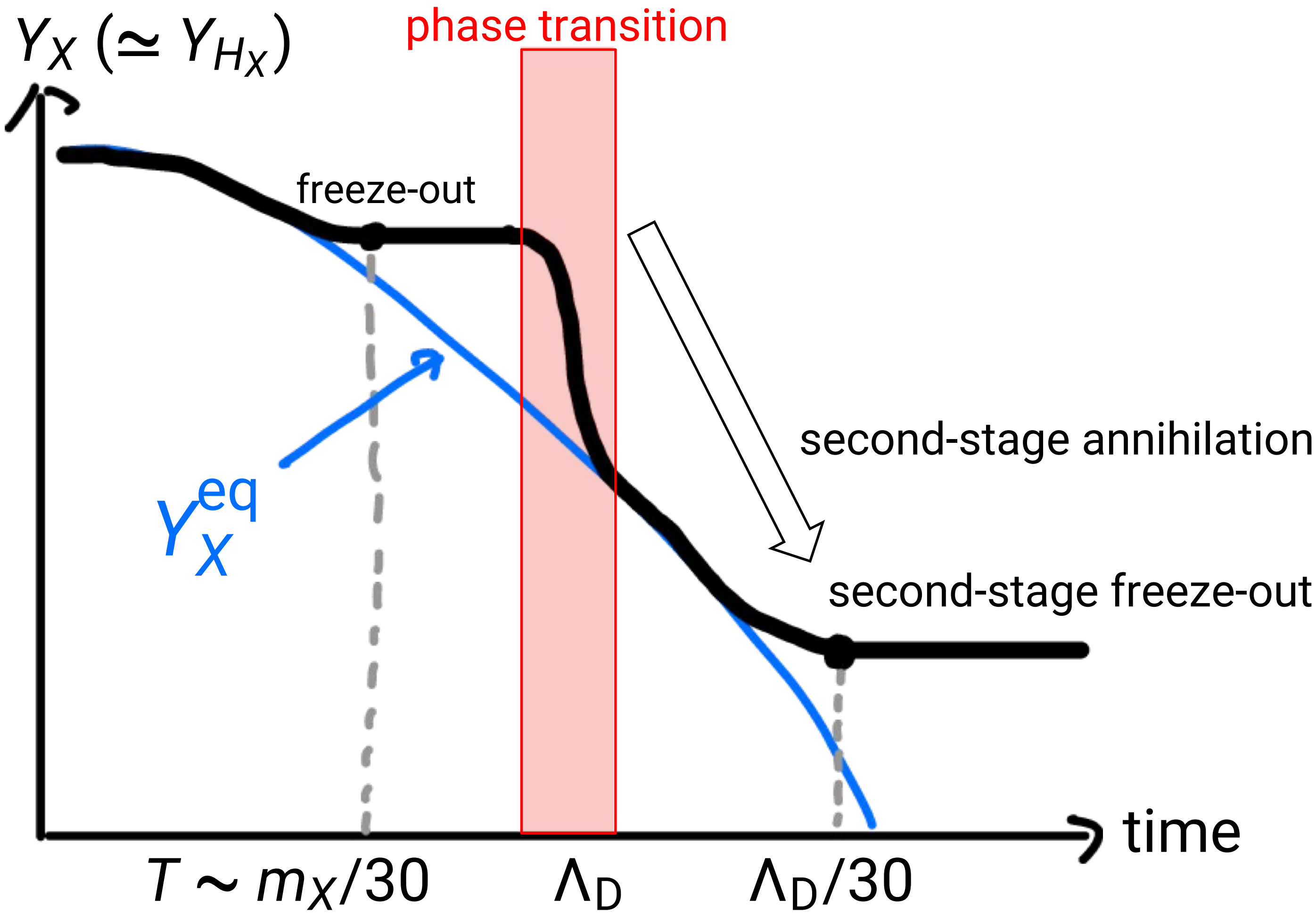
$$Xq^{N-1} \longrightarrow X^2 q^{N-2} \longrightarrow \cdots \longrightarrow X^N$$

[size $\sim \Lambda_D^{-1}$; interacting] [size $\sim m_X^{-1} \Rightarrow$ relic ··· another DM component]

beyond this work

see Arvanitaki, Dimopoulos, Dubovsky, Graham, et al. [0812.2075], Feng, Shadmi [1102.0282]

$H_X = (X\bar{q})$-meson



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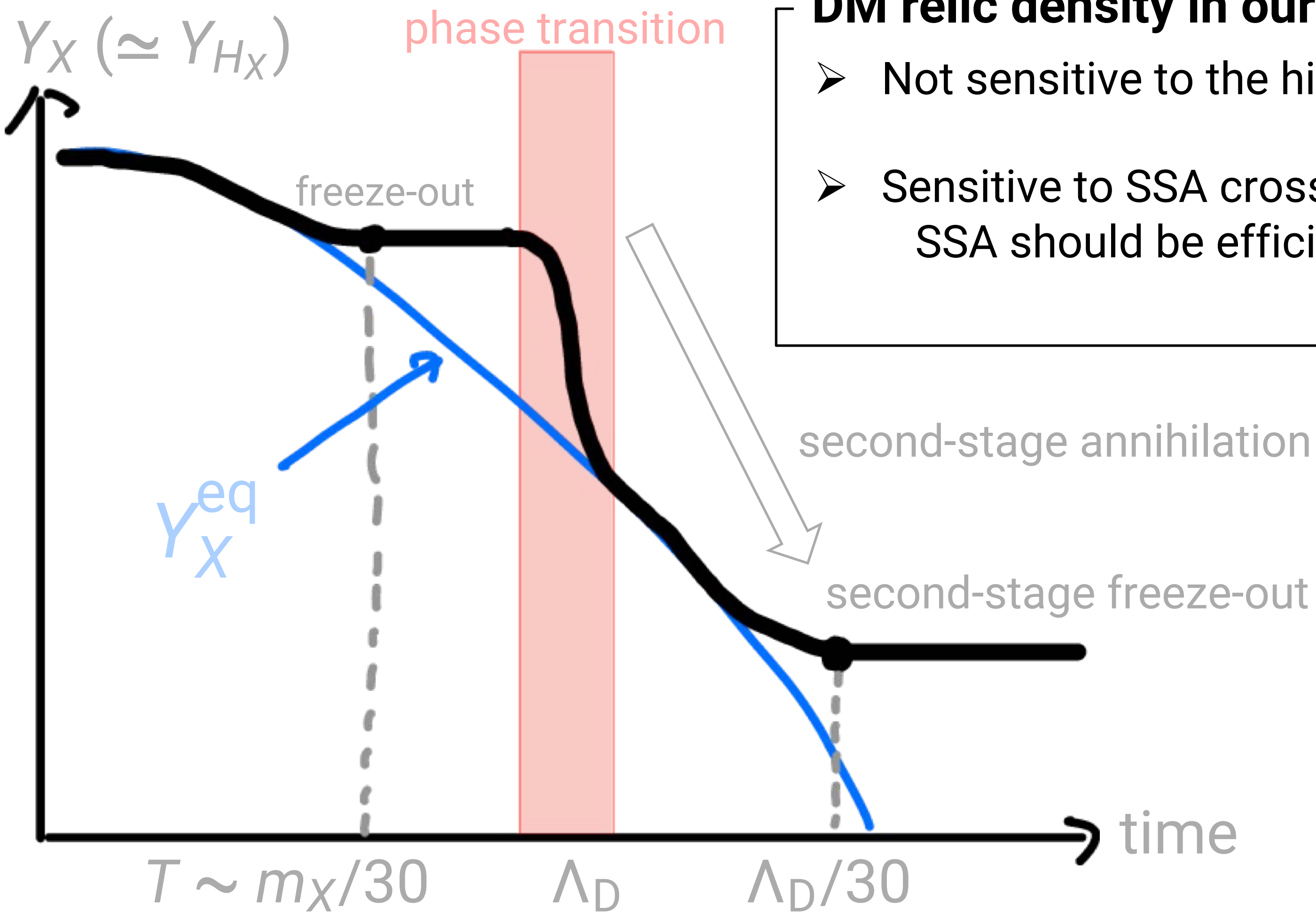


DM relic density in our scenario:

- Not sensitive to the history before SSA.
- Sensitive to SSA cross section: SSA should be efficient and $\sigma_{SSA} \sim \Lambda_D^{-2}$.

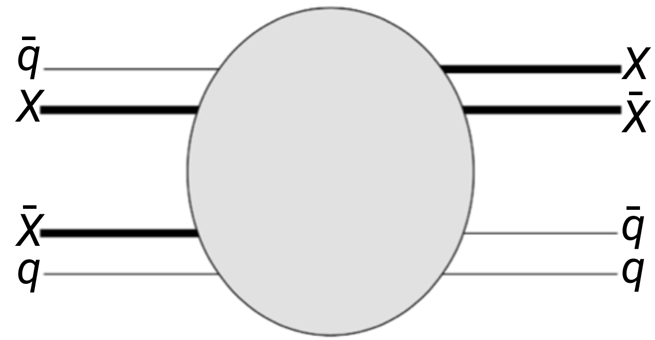
What are the processes for SSA?

Processes for Second-stage annihilation

$H_X = (X\bar{q})$-meson

1) Rearrangement

$$H_X + \bar{H}_X \longrightarrow (X\bar{X}) + (q\bar{q})$$

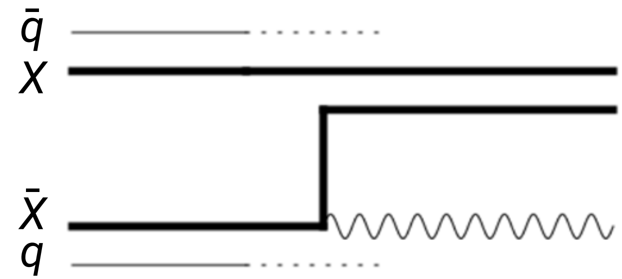


2) Radiative processes such as

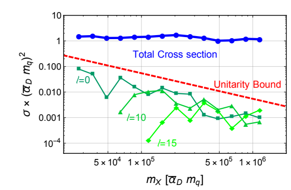
$$X + \bar{X} \longrightarrow (X\bar{X}) + \varphi$$

with "q"s as spectators

some light particles (model dependent)



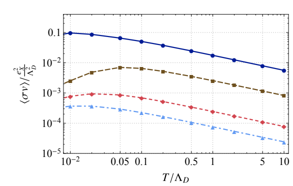
Our work suggests:



$\sigma_{\rm rearrangement} \sim 1/\Lambda_D^2$ → **responsible for SSA**

[SSA scenario works!]

$\sigma_{\rm radiative} \sim 1/\sqrt{m_X^3 \Lambda_D}$: subdominant



1) Rearrangement

$$H_X + \bar{H}_X \longrightarrow (X\bar{X}) + (q\bar{q})$$

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Remark

Both should be followed by $(X\bar{X})$-decays:

$$(X\bar{X}) \rightarrow G \text{ and/or } (q\bar{q})$$

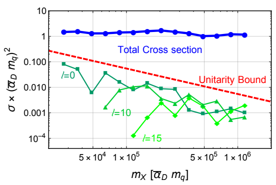
(otherwise X-number conservation).

So DM relic density is sensitive also to

$$\Gamma_{(X\bar{X})} \text{ and } 2m_{H_X} - m_{(X\bar{X})}$$

(but beyond the scope of this work).

Our work suggests:



$$\sigma_{\text{rearrangement}} \sim 1/\Lambda_D^2$$ → **responsible for SSA**

[SSA scenario works!]

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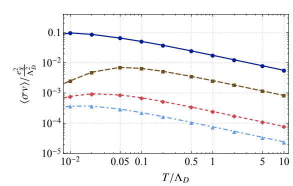

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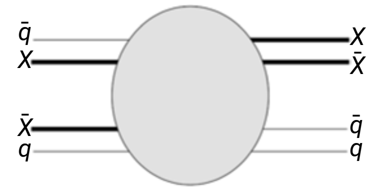
$\sigma_{\text{rearrangement}}$ $H_X + \bar{H}_X \longrightarrow (X\bar{X}) + (q\bar{q})$



- Lattice QCD
- perturbative non-relativistic QCD (pNRQCD)

too much...

- If $m_q \gtrsim \Lambda_{\rm D}$, we can use semi-classical analysis.

... calculate semi-classically and extrapolate;

"Quantum-mechanical scattering under Cornell potentials".

potentials for quarkoniums:

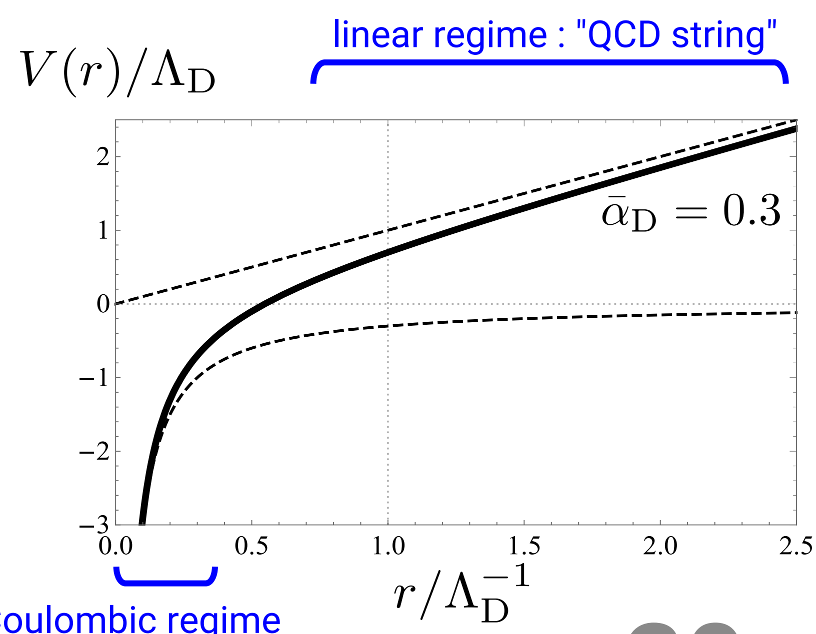
$$V(r) = -\frac{\bar{\alpha}_{\rm D}}{r} + \Lambda_{\rm D}^2 r$$

$[\bar{\alpha}_{\rm D} = C\alpha_{\rm D}; \quad C = (C_1 + C_2 - C_{12})/2]$

dark-QCD coupling ($\alpha_{\rm D}$)

quadratic Casimir of constituent 1, 2, and their bound state.

($C = (N^2 - 1)/(2N)$ for our model)



σ rearrangement $H_X + \bar{H}_X \longrightarrow (X\bar{X}) + (q\bar{q})$

$$V(r) = -\frac{\bar{\alpha}_{\rm D}}{r} + \Lambda_{\rm D}^2 r$$

We can ignore since size ≲ $1/\Lambda_{\rm D}$.

= equivalent to hydrogen-antihydrogen rearrangement. ⇦

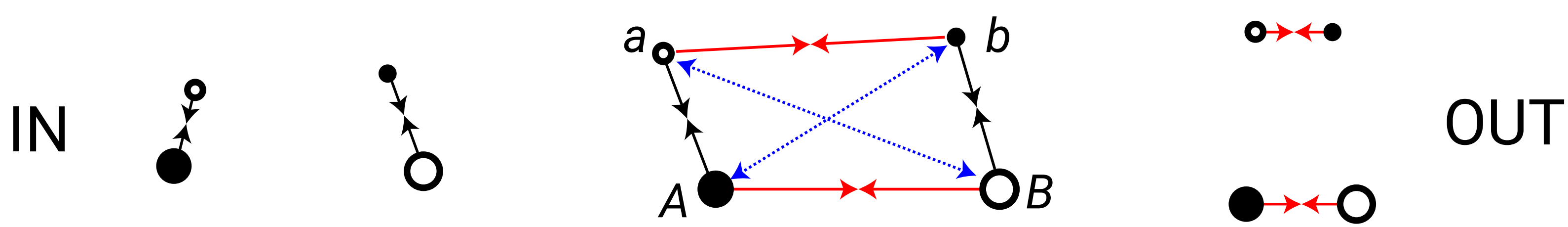
Froelich, Jonsell, Saens, Zygelman, Dalgarno
PRL **84** (2000) 4577, PRA **64** (2001) 052712, JPB **37** (2004) 1195.

$$\mathcal{H} = \mathcal{H}_{\rm free} + \mathcal{H}_{\rm int}$$

$$\mathcal{H}_{\rm free} = (\text{kinetic energies})$$

$$\mathcal{H}_{\rm int} = V_{Aa} + V_{Bb} + V_{AB} + V_{ab} + V_{Ab} + V_{aB}$$

$$= \underbrace{-\frac{\bar{\alpha}_{\rm D}}{\|\boldsymbol{r}_{Aa}\|} - \frac{\bar{\alpha}_{\rm D}}{\|\boldsymbol{r}_{Bb}\|}}_{\text{active for IN state}} \underbrace{- \frac{\bar{\alpha}_{\rm D}}{\|\boldsymbol{r}_{AB}\|} - \frac{\bar{\alpha}_{\rm D}}{\|\boldsymbol{r}_{ab}\|}}_{\text{active for OUT state}} + \frac{\bar{\alpha}_{\rm D}}{\|\boldsymbol{r}_{Ab}\|} + \frac{\bar{\alpha}_{\rm D}}{\|\boldsymbol{r}_{aB}\|}$$



$H_X + \bar{H}_X$ | interacting | $(X\bar{X}) + (q\bar{q})$

$$H = H_0 + V;$$ $V \sim 0$ for separated particles

$$H\,|\Psi_\alpha^\pm\rangle = E_\alpha\,|\Psi_\alpha^\pm\rangle$$ IN/OUT states are (subset of) *H*-eigenstates.

$$H_0\,|\Phi_\alpha\rangle = E_\alpha\,|\Phi_\alpha\rangle$$ "free states"; orthonormal to each other.

* Schrödinger repr.

$$\frac{\partial}{\partial t}\,|\psi;t\rangle = -\mathrm{i}H\,|\psi;t\rangle$$

$$|\psi\rangle := |\psi;0\rangle$$

Definitions of IN states $|\Psi_\alpha^+\rangle$ and OUT states $|\Psi_\alpha^-\rangle$:

$$\int \mathrm{d}\alpha\, g(\alpha)\mathrm{e}^{-\mathrm{i}E_\alpha t}\,|\Psi_\alpha^\pm\rangle \xrightarrow{t\to\mp\infty} \int \mathrm{d}\alpha\, g(\alpha)\mathrm{e}^{-\mathrm{i}E_\alpha t}\,|\Phi_\alpha\rangle$$

$$\Rightarrow\quad |\Psi_\alpha^\pm\rangle = |\Phi_\alpha\rangle + \left(E_\alpha - H_0 \pm \mathrm{i}\epsilon\right)^{-1} V\,|\Psi_\alpha^\pm\rangle$$ Lippmann-Schwinger eq.

Definition of *S*-matrix

$$|\Psi_\alpha^+\rangle =: \int \mathrm{d}\beta\, S_{\beta\alpha}\,|\Psi_\alpha^-\rangle$$

$$\left(S_{\beta\alpha} = \langle\Psi_\beta^-|\Psi_\alpha^+\rangle\right)$$

$$\Rightarrow\quad S_{\beta\alpha} = \delta(\beta-\alpha) - 2\pi\mathrm{i}\delta(E_\beta - E_\alpha)\,\langle\Phi_\beta|V|\Psi_\alpha^+\rangle$$

$$=: \delta(\beta-\alpha) + \delta^{(4)}(p_\beta - p_\alpha)\mathcal{M}_{\beta\alpha}$$ (defining *M*-matrix)

$$\Rightarrow\quad \sigma_{AB\to12} = \frac{1}{2E_A 2E_B v_{\mathrm{rel}}}\int \frac{\mathrm{d}\Omega}{4\pi}\frac{2p}{8\pi E}\left[(4\pi)^4 E_A E_B E_1 E_2\,|\mathcal{M}|^2\right]$$

$$= (2\pi)^2\left(\frac{p_\beta}{p_\alpha}\right)\mu_\alpha\mu_\beta\,|\mathcal{M}|^2\,\mathrm{d}\Omega$$

[momenta and reduced masses of initial/final system]

$$H = H_0 + V;$$ $V \sim 0$ for separated particles

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Definitions of IN states $|\Psi^{+}_{\alpha}\rangle$ and OUT states $|\Psi^{-}_{\alpha}\rangle$:

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$$= \mathrm{e}^{-\mathrm{i}Ht}\,|\Psi^{\pm}_{\alpha}\rangle$$

$t = -\infty$ | $t = 0$ | $t = +\infty$

$$\mathrm{e}^{-\mathrm{i}H(-\infty)}\,|\Psi^{+}_{\alpha}\rangle \xrightarrow{H} |\Psi^{+}_{\alpha}\rangle$$

$$\mathrm{e}^{-\mathrm{i}H(-\infty)}\,|\Psi^{+}_{\alpha}\rangle = \mathrm{e}^{-\mathrm{i}H_0(-\infty)}\,|\Phi_\alpha\rangle$$

$$\mathrm{e}^{-\mathrm{i}H_0(-\infty)}\,|\Phi_\alpha\rangle \xleftarrow{H_0} |\Phi_\alpha\rangle \xrightarrow{H_0} \mathrm{e}^{-\mathrm{i}H\infty}\,|\Phi_\alpha\rangle$$

$$\mathrm{e}^{-\mathrm{i}H\infty}\,|\Phi_\alpha\rangle = \mathrm{e}^{-\mathrm{i}H\infty}\,|\Psi^{-}_{\alpha}\rangle$$

$$|\Psi^{-}_{\alpha}\rangle \xleftarrow{H} \mathrm{e}^{-\mathrm{i}H\infty}\,|\Psi^{-}_{\alpha}\rangle$$

$$|\Psi^{+}_{\alpha}\rangle = \Omega_+\,|\Phi_\alpha\rangle\,;$$

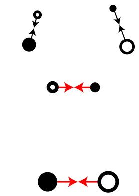
$$\Omega_+ := \lim_{t\to-\infty} \mathrm{e}^{\mathrm{i}Ht}\mathrm{e}^{-\mathrm{i}H_0 t}$$

[Møller operator]

$$S_{\beta\alpha} = \langle\Psi^{-}_{\beta}|\Psi^{+}_{\alpha}\rangle$$ [time independent!]

$$\mathcal{H}\,|\Psi_\alpha^\pm\rangle = E_\alpha\,|\Psi_\alpha^\pm\rangle$$
$$\mathcal{H}_{\text{in}}\,|\Phi_\alpha^{\text{in}}\rangle = E_\alpha\,|\Phi_\alpha^{\text{in}}\rangle$$
$$\mathcal{H}_{\text{out}}\,|\Phi_\alpha^{\text{out}}\rangle = E_\alpha\,|\Phi_\alpha^{\text{out}}\rangle$$

$$\mathcal{H}_{\text{in}} = \mathcal{H}_{\text{free}} + V_{Aa} + V_{Bb}$$
$$\mathcal{H}_{\text{out}} = \mathcal{H}_{\text{free}} + V_{AB} + V_{ab}$$



$t = -\infty$ | $t = 0$ | $t = +\infty$

$$e^{-\mathrm{i}\mathcal{H}(-\infty)}\,|\Psi_\alpha^+\rangle \xrightarrow{\ \mathcal{H}\ } |\Psi_\alpha^+\rangle$$

$$\|$$

$$e^{-\mathrm{i}\mathcal{H}_{\text{in}}(-\infty)}\,|\Phi_\alpha^{\text{in}}\rangle \xleftarrow{\ \mathcal{H}_{\text{in}}\ } |\Phi_\alpha^{\text{in}}\rangle$$

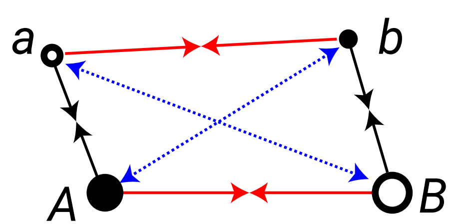
$$|\Phi_\alpha^{\text{out}}\rangle \xrightarrow{\ \mathcal{H}_{\text{out}}\ } e^{-\mathrm{i}\mathcal{H}_{\text{out}}\infty}\,|\Phi_\alpha^{\text{out}}\rangle$$

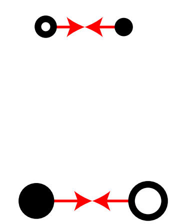
$$\|$$

$$|\Psi_\alpha^-\rangle \xleftarrow{\ \mathcal{H}\ } e^{-\mathrm{i}\mathcal{H}\infty}\,|\Psi_\alpha^-\rangle$$

$$\Omega_\pm^A := \lim_{t\to\mp\infty} \exp(\mathrm{i}Ht)\exp(-\mathrm{i}H_A t)$$

[channel Møller operator]





$$\mathcal{H}_{\rm in} = \mathcal{H}_{\rm free} + V_{Aa} + V_{Bb}$$
$$\mathcal{H}_{\rm out} = \mathcal{H}_{\rm free} + V_{AB} + V_{ab}$$

$|\Psi_\alpha^+\rangle = \Omega_+^{\rm in}|\Phi_\alpha^{\rm in}\rangle$ ... "For any state $|\Phi_\alpha^{\rm in}\rangle$ that approaches to at $t = -\infty$,

there exists a state $|\Psi_\alpha^+\rangle$ that satisfies

$$e^{-iHt}|\Psi_\alpha^+\rangle \xrightarrow{t\to-\infty} e^{-iH_{\rm in}t}|\Phi_\alpha^{\rm in}\rangle$$

(and it is given as such).

$$\Omega_\pm^A := \lim_{t\to\mp\infty} \exp(iHt)\exp(-iH_At)$$

[channel Møller operator]

$$S_{\beta\alpha} = \langle\Psi_\beta^-|\Psi_\alpha^+\rangle = \left\langle \Phi_\beta^{\rm out} \middle| \Omega_-^{\rm out\dagger}\Omega_+^{\rm in} \middle| \Phi_\alpha^{\rm in}\right\rangle$$

$$\mathcal{M} = 2\pi \left\langle \Phi_\beta^{\rm out} \middle| (\mathcal{H} - \mathcal{H}_{\rm out}) \middle| \Psi_\alpha^+ \right\rangle$$

Remark

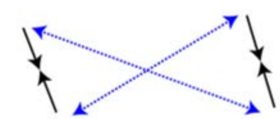
Orthogonality & completeness

[Faddeev (1965), Hepp (1969)]

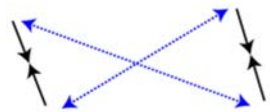
e.g.) $\mathscr{R}_\pm^A \perp \mathscr{B}$,

$\mathscr{R}_\pm^A \perp \mathscr{R}_\pm^{A'}$ if $A \neq A'$;

$$\mathscr{H} = \mathscr{B} \oplus \sum_A \mathscr{R}_+^A = \mathscr{B} \oplus \sum_A \mathscr{R}_-^A$$

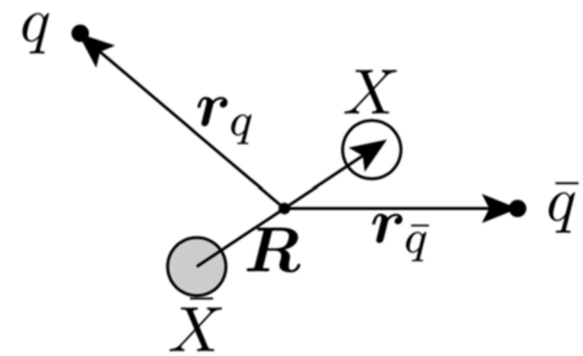


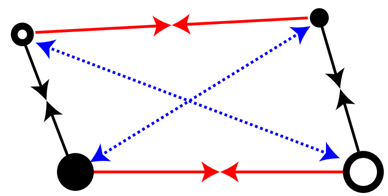
cf.) $\mathcal{M} \propto \langle\Phi_\beta|V|\Psi_\alpha^+\rangle$ for single channel.



$$= -\frac{\bar{\alpha}_D}{\|\boldsymbol{r}_{\bar{q}} - \boldsymbol{R}/2\|} - \frac{\bar{\alpha}_D}{\|\boldsymbol{r}_q + \boldsymbol{R}/2\|} + \frac{\bar{\alpha}_D}{\|\boldsymbol{r}_q - \boldsymbol{R}/2\|} + \frac{\bar{\alpha}_D}{\|\boldsymbol{r}_{\bar{q}} + \boldsymbol{R}/2\|}$$

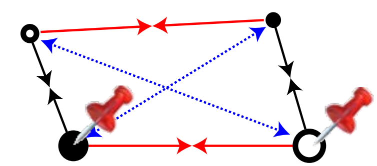
$$\mathcal{M} = 2\pi \left\langle \Phi_\beta^{\text{out}} \,\middle|\, (\mathcal{H} - \mathcal{H}_{\text{out}}) \,\middle|\, \Psi_\alpha^+ \right\rangle$$





(already factorized)

Born-Oppenheimer approximation



$$\Psi(\boldsymbol{r}, \boldsymbol{R}) = \phi(\boldsymbol{R})\psi(\boldsymbol{r}; \boldsymbol{R})$$

where $(K_q + K_{\bar{q}} + V)\psi(\boldsymbol{r}; \boldsymbol{R}) = E'(\boldsymbol{R})\psi(\boldsymbol{r}; \boldsymbol{R})$

$$(K_X + K_{\bar{X}})\phi(\boldsymbol{R}) = (E - E'(\boldsymbol{R}))\phi(\boldsymbol{R})$$

factorized form $\Psi(\boldsymbol{R}, \boldsymbol{r}_q, \boldsymbol{r}_{\bar{q}}) = \psi^{X\bar{X}}(\boldsymbol{R})\, \psi^{q\bar{q}}(R; \boldsymbol{r}_q, \boldsymbol{r}_{\bar{q}})$

[motion of heavy quarks]

[light quarks around the heavies]

$$\left(\mathcal{H}_{\rm tot} - \mathcal{H}_{X\bar{X}}\right)\psi_{\rm in}^{q\bar{q}} = V_{\rm BO}(\boldsymbol{R})\psi_{\rm in}^{q\bar{q}}$$

[with fixed ***R***]

→ gives $V_{\rm BO}$ & $\psi_{\rm in}^{q\bar{q}}$ as func. of $\boldsymbol{R}$

$$\left[-\frac{1}{2m_X}\nabla_R^2 + \underbrace{V_{X\bar{X}}(R) + V_{\rm BO}(\boldsymbol{R})}_{=:\, V_{\rm in}(\boldsymbol{R})}\right]\psi_{\rm in}^{X\bar{X}} = E\psi_{\rm in}^{X\bar{X}}$$

factorized form $\Psi(\boldsymbol{R}, \boldsymbol{r}_q, \boldsymbol{r}_{\bar{q}}) = \psi^{X\bar{X}}(\boldsymbol{R})\,\psi^{q\bar{q}}(R; \boldsymbol{r}_q, \boldsymbol{r}_{\bar{q}})$

[motion of heavy quarks]

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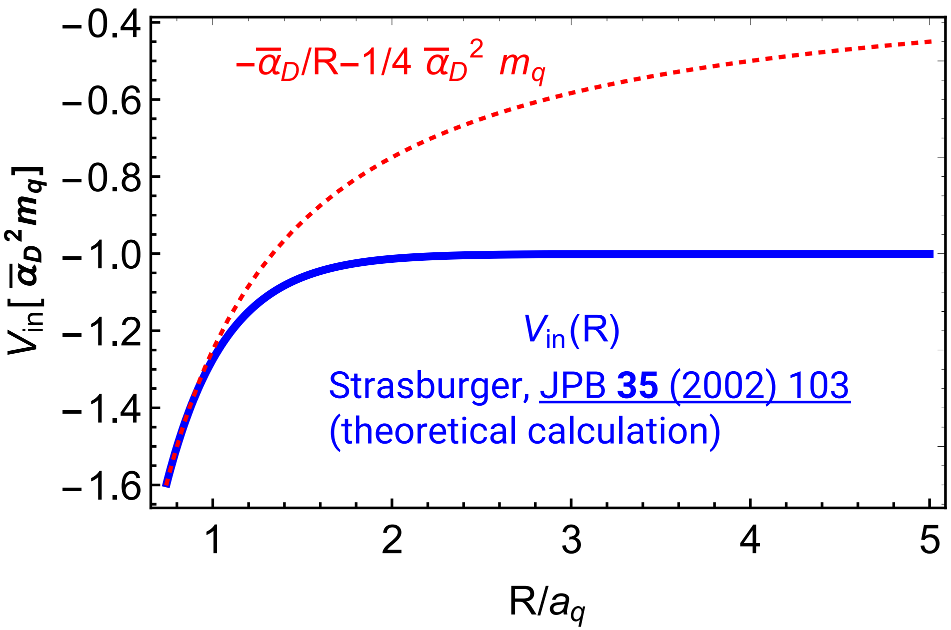
$$\begin{aligned} V_{\rm in} &\sim \frac{1}{2m_q}(\nabla_r)^2 - \frac{\bar{\alpha}_{\rm D}}{r} \\ &= \frac{1}{2m_q}\frac{(\nabla_x)^2}{a_q^2} - \frac{\bar{\alpha}_{\rm D}}{x a_q} \\ &= \frac{1}{a_q^2 m_q}\left[\frac{(\nabla_x)^2}{2} - \frac{1}{x}\right] \end{aligned}$$

$V_{\rm in}$ depends only on "Bohr radius" $a_q = (\bar{\alpha}_{\rm D} m_q)^{-1}$

→ we can use the result for H-H̄ system.



$$\begin{cases} R \gg 1:\ V_{\rm BO} \sim -\bar{\alpha}_{\rm D}^2 m_q = 2\times(H_X \text{ binding energy}) \\ R \ll 1:\ V_{\rm BO} \sim -\bar{\alpha}_{\rm D}^2 m_q/4 = (q\text{-}\bar{q} \text{ binding energy}) \end{cases}$$

(c.f. Bohr model $E_n = \mu\alpha^2/2n^2$)

flatness = screening by *qs*

(**N.B.** we are still under the assumption $(1/a_q) \sim \Lambda_{\rm D}$.)

E_b = binding energy of H_X

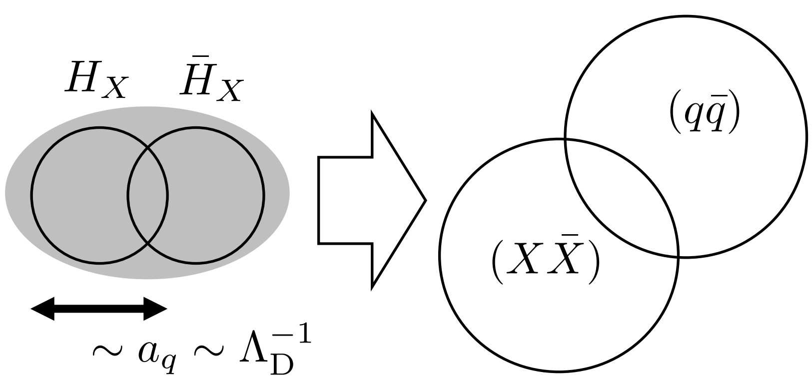
$$\mathcal{M} = 2\pi \left\langle \Phi_\beta^\mathrm{out} \middle| (\mathcal{H} - \mathcal{H}_\mathrm{out}) \middle| \Psi_\alpha^+ \right\rangle$$
$$= \int \mathrm{d}^3\boldsymbol{R}\, \psi_\mathrm{out}^{X\bar{X}}(\boldsymbol{R})^*\, T(\boldsymbol{R})\, \psi_\mathrm{in}^{X\bar{X}}(\boldsymbol{R})$$

$$T(\boldsymbol{R}) := \int \mathrm{d}^3\boldsymbol{r}_q \mathrm{d}^3\boldsymbol{r}_{\bar{q}}\, \psi_\mathrm{out}^{q\bar{q}*} (\mathcal{H} - \mathcal{H}_\mathrm{out})\, \psi_\mathrm{in}^{q\bar{q}}$$

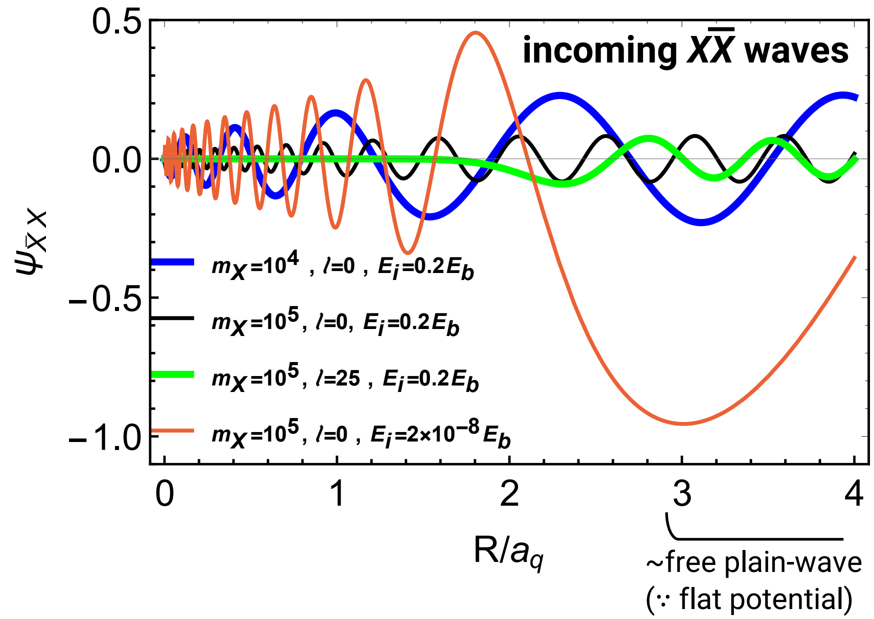
Incoming wavefunctions: BO approx.

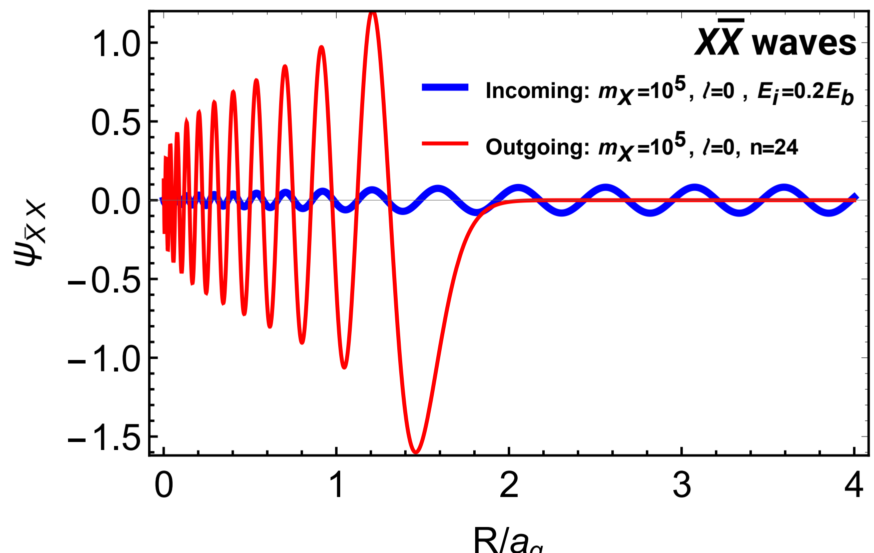
$$\left[-\frac{1}{2m_X}\nabla_R^2 + V_\mathrm{in}(\boldsymbol{R})\right] \psi_\mathrm{in}^{X\bar{X}} = E\psi_\mathrm{in}^{X\bar{X}}$$
$$(\mathcal{H}_\mathrm{tot} - \mathcal{H}_{X\bar{X}})\, \psi_\mathrm{in}^{q\bar{q}} = V_\mathrm{BO}(\boldsymbol{R})\psi_\mathrm{in}^{q\bar{q}}$$

Outgoing wavefunctions: free waves



$a_q = (\bar{\alpha}_\mathrm{D} m_q)^{-1}$ (Bohr rad.)





E_b = binding energy of H_X

$$\mathcal{M} = 2\pi \left\langle \Phi_\beta^{\rm out} \middle| (\mathcal{H} - \mathcal{H}_{\rm out}) \middle| \Psi_\alpha^+ \right\rangle = \int \mathrm{d}^3\boldsymbol{R}\, \psi_{\rm out}^{X\bar{X}}(\boldsymbol{R})^*\, T(\boldsymbol{R})\, \psi_{\rm in}^{X\bar{X}}(\boldsymbol{R})$$

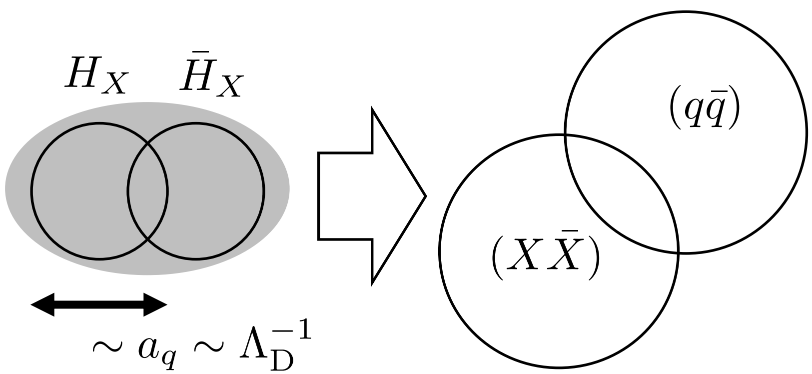
$$T(\boldsymbol{R}) := \int \mathrm{d}^3\boldsymbol{r}_q \mathrm{d}^3\boldsymbol{r}_{\bar{q}}\, \psi_{\rm out}^{q\bar{q}*}\, (\mathcal{H} - \mathcal{H}_{\rm out})\, \psi_{\rm in}^{q\bar{q}}$$

Incoming wavefunctions: BO approx.

$$\left[-\frac{1}{2m_X}\nabla_R^2 + V_{\rm in}(\boldsymbol{R})\right] \psi_{\rm in}^{X\bar{X}} = E\psi_{\rm in}^{X\bar{X}}$$

$$(\mathcal{H}_{\rm tot} - \mathcal{H}_{X\bar{X}})\, \psi_{\rm in}^{q\bar{q}} = V_{\rm BO}(\boldsymbol{R})\psi_{\rm in}^{q\bar{q}}$$

Outgoing wavefunctions: free waves



$a_q = (\bar{\alpha}_{\rm D} m_q)^{-1}$ (Bohr rad.)

We neglect the angular dependence of T

$$T(\boldsymbol{R}) = T(R, \Omega_{\boldsymbol{R}}) \approx T(R)$$

so that we use H-H̄ results

Jonsell, Saens, Froelich, Zygelman, Dalgarno JPB **37** (2004) 1195.

$$T(R) = \begin{cases} (\text{const.}) \times \left[E_f + \frac{1}{4}\bar{\alpha}_D^2 m_q - V_{\rm BO}(R)\right] & \text{for } R > 0.74 a_q \\ 0 & \text{for } R \leq 0.74 a_q \end{cases}$$

BO is no longer valid since qs are attracted by each other.

Strasburger, JPB **35** (2002) 103

Remark

This corresponds to "dipole approx."

$$\exp\left[\mathrm{i}\, \boldsymbol{k}_{\rm f} \cdot (\boldsymbol{r}_q + \boldsymbol{r}_{\bar{q}})\right] \approx 1\,,$$

which we can validate for

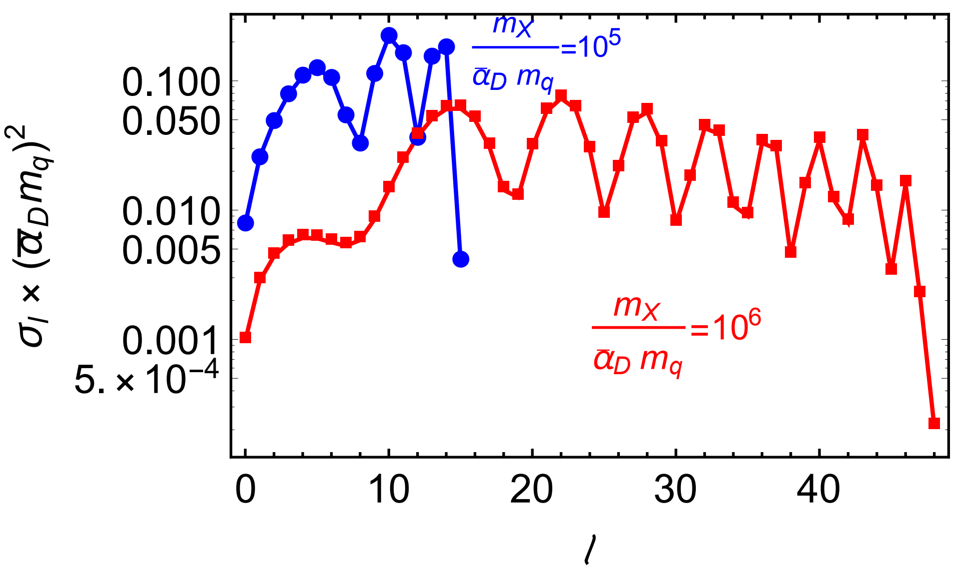
$\boldsymbol{k}_{\rm f} \lesssim \Lambda_{\rm D}$ [shallower $(X\bar{X})$]

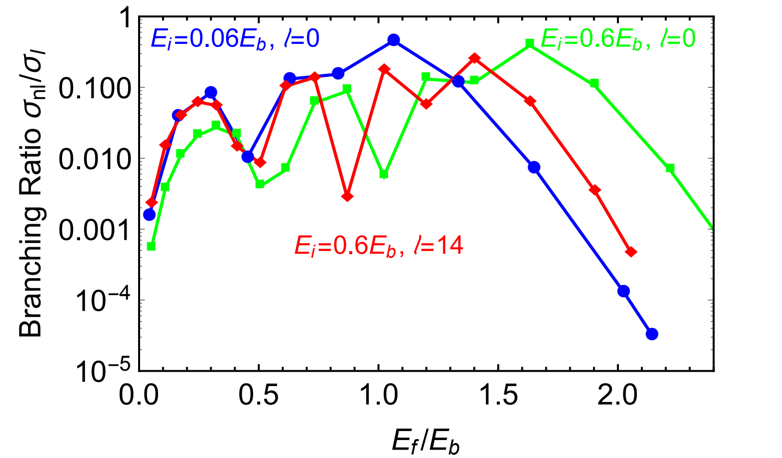
$\boldsymbol{r}_q \lesssim \Lambda_{\rm D}^{-1}$ [deeper/ground $(q\bar{q})$].

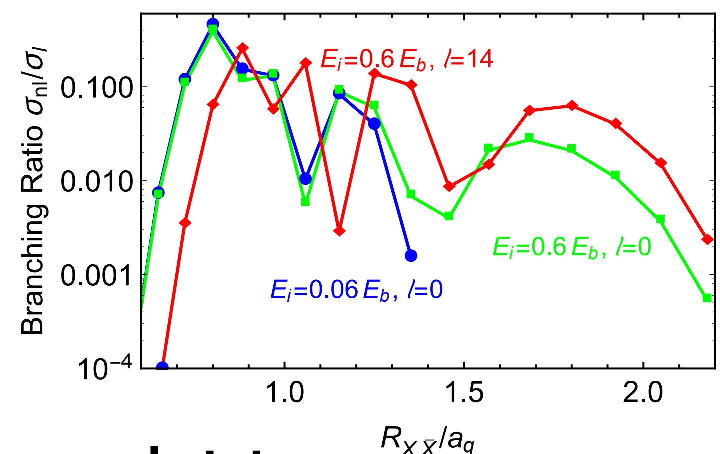
$$\mathcal{M}_{nlm}(E_{\rm in}) = \int \mathrm{d}^3\boldsymbol{R}\, \left[\psi_{\rm out}^{X\bar{X}}\right]_{nlm}^* T(R) \left[\psi_{\rm in}^{X\bar{X}}\right]_{E_{\rm in}, l}$$

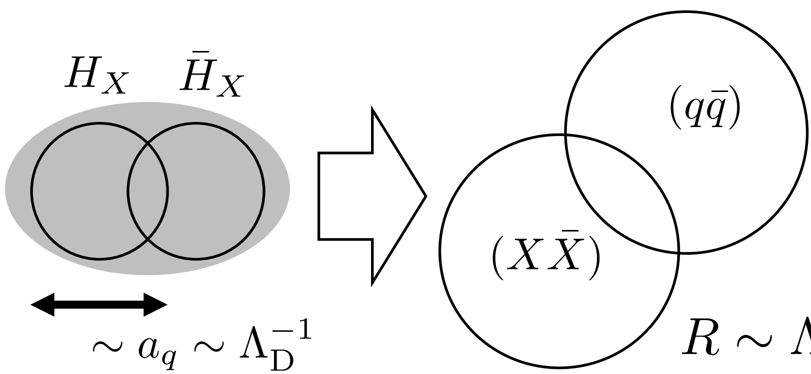
Then we got results; several interesting properties.

$E_{\rm b}$ = binding energy of H_X









$R \sim \Lambda_{\rm D}^{-1}$

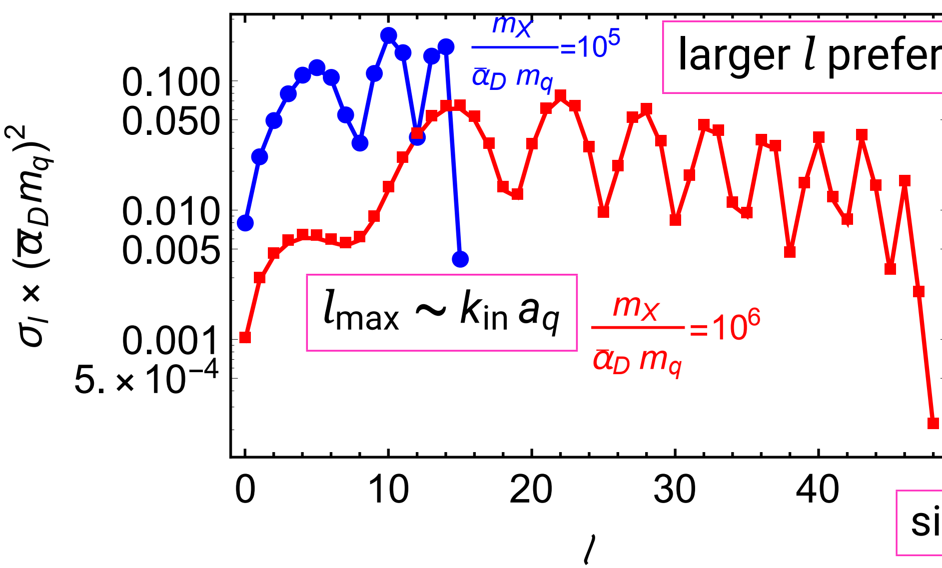
... deeper/ground states

... highly-excited states

$a_q = (\bar{\alpha}_{\rm D} m_q)^{-1}$ (Bohr rad.)

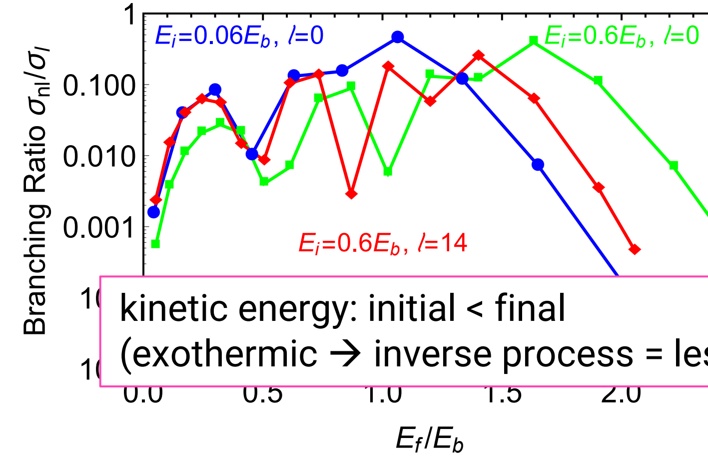
Coulomb potential expectation: $\frac{\langle r \rangle}{a} \simeq \frac{3n^2 - l^2}{2}$

E_b = binding energy of H_X



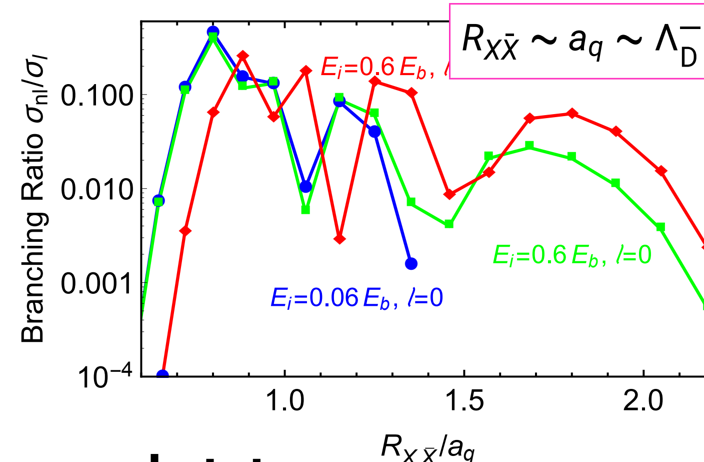
larger l preferred

$l_{\max} \sim k_{\rm in}\, a_q$

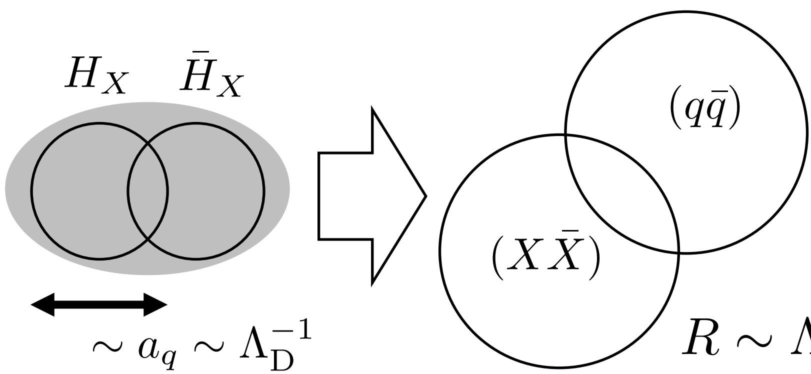


kinetic energy: initial < final
(exothermic → inverse process = less)

similar in $T \sim E_{\rm in} = 0.06$−0.6



$R_{X\bar{X}} \sim a_q \sim \Lambda_{\rm D}^{-1}$



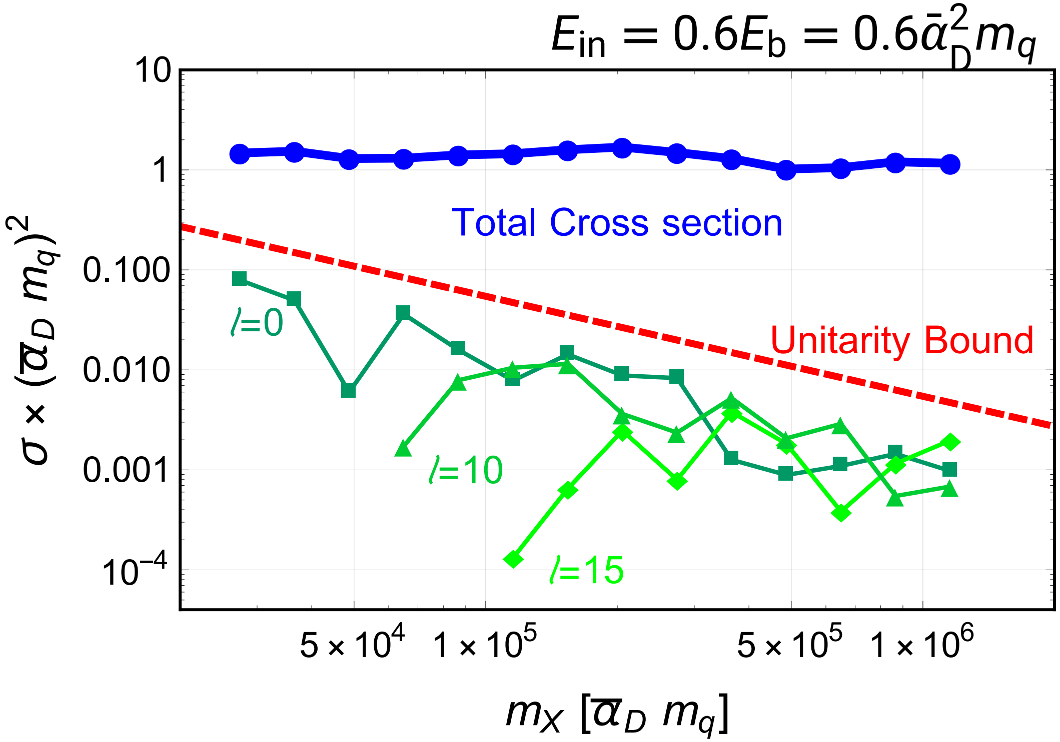
$R \sim \Lambda_{\rm D}^{-1}$ **... deeper/ground states**

$R \sim \Lambda_{\rm D}^{-1}$ **... highly-excited states**

$a_q = (\bar{\alpha}_{\rm D} m_q)^{-1}$ (Bohr rad.)

Coulomb potential expectation: $\frac{\langle r \rangle}{a} \simeq \frac{3n^2 - l^2}{2}$

Then we got results; cross section is geometric and SSA works well!



$= 4\pi / k_{\rm in}^2$

$$\sigma_{\rm rearr} = \frac{1\text{–}2}{(\bar{\alpha}_{\rm D} m_q)^2} = (1\text{–}2)a_q^2 \approx \Lambda_{\rm D}^{-2}$$

"geometric" cross section

$\dfrac{\sigma_l}{2l+1}$ are shown.

… $\sigma_l \simeq \dfrac{4\pi(2l+1)}{k_{\rm in}}$

close to the unitarity bound.

$$\left(\sigma = \sum_J \sigma_J, \quad \sigma_J(2\to 2) \le \frac{(2J+1)\pi}{p_{\rm i}^2}\right)$$

$$\sigma = \sum_{l=0}^{l_{\rm max}} \sigma_l \sim \frac{4\pi}{k_{\rm in}^2}\sum_l (2l+1) \sim \frac{4\pi}{k_{\rm in}^2}(l_{\rm max})^2 \sim 4\pi a_q^2$$

for $T \sim \mathcal{O}(0.01 - 0.1)\Lambda_{\rm D}$

Note: for $T \to 0$, $l_{\rm max} \to 0$ and only s-wave interaction is available; $\sigma \sim a_q/k_{\rm in}$.

✓ Introduction

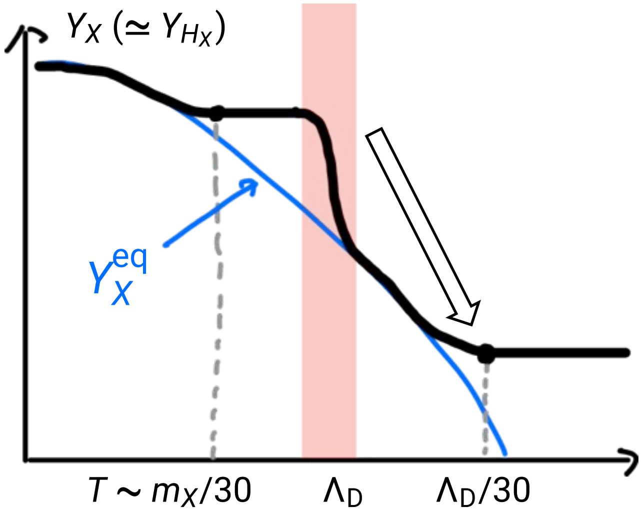
✓ Dark QCD models and Our toy model

✓ Annihilation cross section: $\sigma_{\text{annihil.}} \overset{?}{\sim} \Lambda_D^{-2}$

- "Rearrangement" is the dominant process.

4. Conclusion

Michael Geller, SI, Gabriel Lee, Yael Shadmi, Ofri Telem [1802.07720]

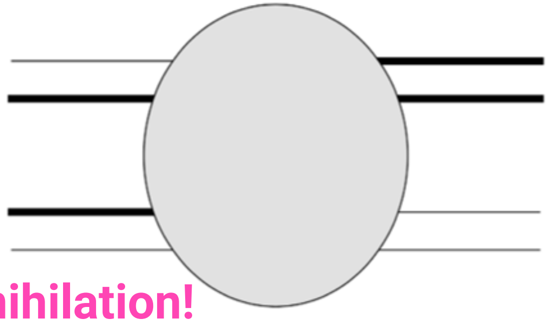


Rearrangement

$$H_X + \bar{H}_X \longrightarrow (X\bar{X}) + (q\bar{q})$$

$$\sigma_{\text{rearr}} \simeq (a_q)^2 \sim \Lambda_D^{-2}$$

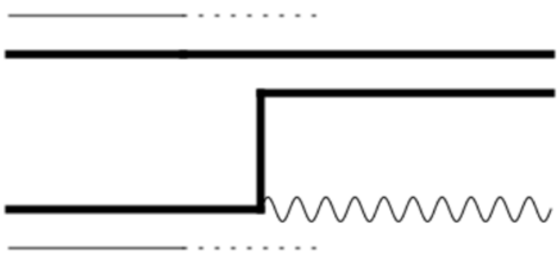
→ Second stage annihilation!

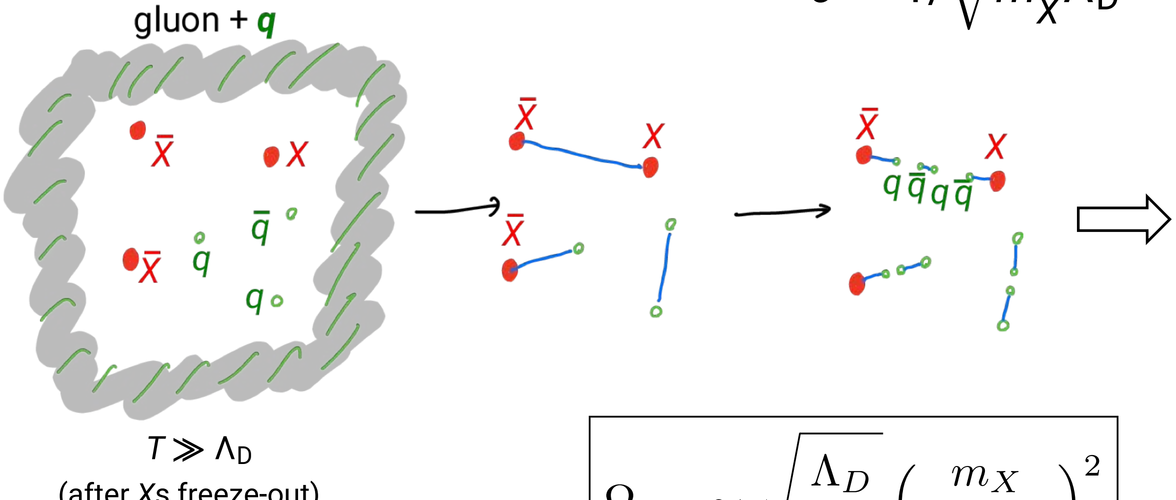


Radiative process

e.g., $X + \bar{X} \longrightarrow (X\bar{X}) + \varphi$

$$\sigma \sim 1/\sqrt{m_X^3 \Lambda_D}$$





$$H_X + \bar{H}_X \longrightarrow (X\bar{X}) + (q\bar{q})$$

↓ decay

$$\Omega_{\rm DM} \sim \sqrt{\frac{\Lambda_D}{m_X}} \left(\frac{m_X}{30\ {\rm TeV}}\right)^2$$

... PeV DM?

1. Introduction

2. Dark QCD models and Our toy model

3. Annihilation cross section: $\sigma_{\text{annihil.}} \overset{?}{\sim} \Lambda_D^{-2}$
   - "Rearrangement" is the dominant process.

4. Conclusion

E1. radiative process?

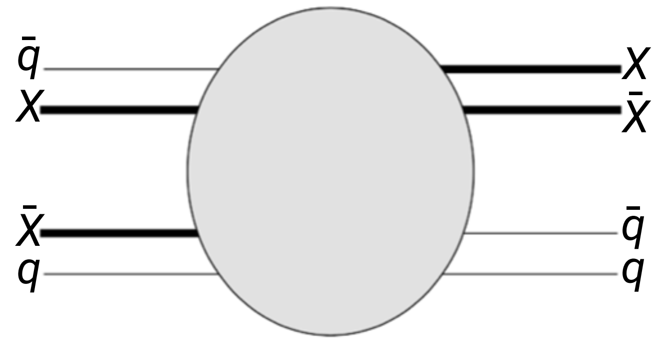
E2. states and wavefunctions

Extra 1: radiative process?

$H_X = (X\bar{q})$-meson

1) Rearrangement

$$H_X + \bar{H}_X \longrightarrow (X\bar{X}) + (q\bar{q})$$

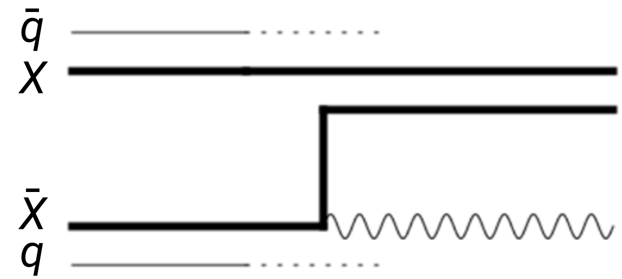


2) Radiative processes such as

$$X + \bar{X} \longrightarrow (X\bar{X}) + \varphi$$

with "q"s as spectators

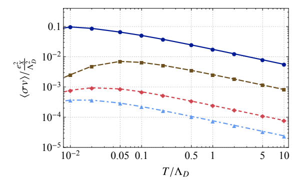
some light particles (model dependent)



Our work suggests:

$\sigma_{\text{rearrangement}} \sim 1/\Lambda_D^2$ → **responsible for SSA**

$\sigma_{\text{radiative}} \sim 1/\sqrt{m_X^3 \Lambda_D}$: subdominant



φ: some light particles (model dependent)

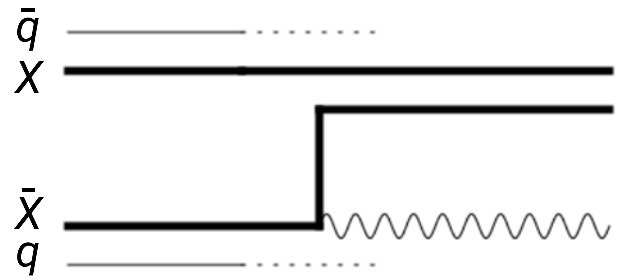
We consider scalar X with a U(1) gauge interaction:

$$\mathcal{L} \ni -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \sum_{i=1,2}\left(|\mathrm{D}_\mu X_i|^2 - m_i^2|X_i|^2\right);$$

$$F_{\mu\nu} = \partial_\mu\phi_\nu - \partial_\nu\phi_\mu, \qquad D_\mu X_i = (\partial_\mu - \mathrm{i}c_i g\phi_\mu)X_i,$$

Then, for example,

$$X + \bar{X} \longrightarrow (X\bar{X}) + \varphi$$



which is seen as

$$\left|\mathcal{U}_{\boldsymbol{K},\boldsymbol{k}}\right\rangle \to \left|\mathcal{B}_{\boldsymbol{P},lm,n}\right\rangle \otimes \left|\varphi_{\boldsymbol{P}_\varphi}\right\rangle$$

scattering state bound state

under

the Cornell potential

$$V(\boldsymbol{r}) = \begin{cases} -\frac{\alpha}{\|\boldsymbol{r}\|} + \Lambda^2\|\boldsymbol{r}\| & \text{for } \|\boldsymbol{r}\| < r_c, \\ -\frac{\alpha}{r_c} + \Lambda^2 r_c & \text{for } \|\boldsymbol{r}\| > r_c. \end{cases}$$

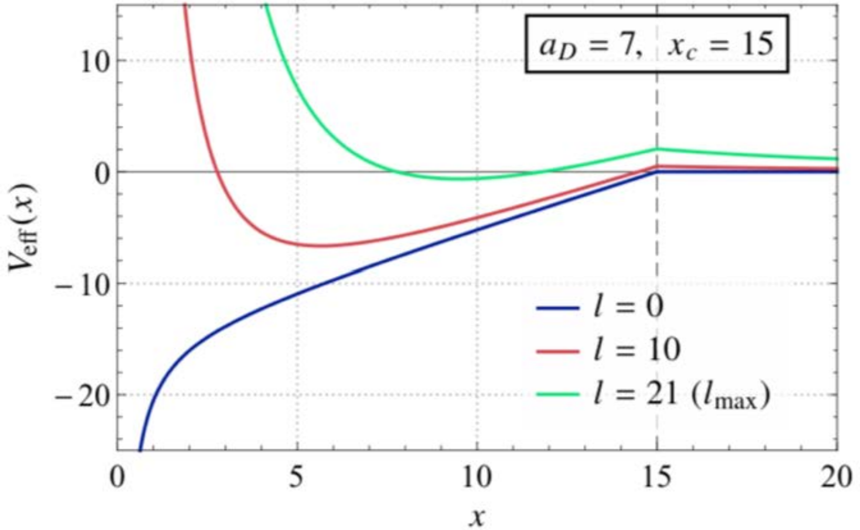
We introduce a cut-off at r_c.

$$\mathcal{M}^j_{\boldsymbol{k}\to lm,n} = -4e_X\sqrt{m_X}\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\,p^j\,\tilde{\psi}^*_{lm,n}(\boldsymbol{p})\left[\tilde{\phi}_{\boldsymbol{k}}\left(\boldsymbol{p}+\frac{\boldsymbol{P}_\varphi}{2}\right)+\tilde{\phi}_{\boldsymbol{k}}\left(\boldsymbol{p}-\frac{\boldsymbol{P}_\varphi}{2}\right)\right]$$

■ Cornell potential rescaling

$$V(r) = -\frac{\alpha}{r} + \Lambda^2 r = \left(-\frac{a}{x} + x\right) E_0$$

$$\begin{cases} \epsilon := \dfrac{E}{E_0}, \quad E_0 := \left(\dfrac{\Lambda}{2\mu}\right)^{1/3} \Lambda, \qquad a := \left(\dfrac{\Lambda}{2\mu}\right)^{-2/3} \alpha, \\ x := \dfrac{r}{r_0}, \quad r_0 := \left(\dfrac{\Lambda}{2\mu}\right)^{1/3} \dfrac{1}{\Lambda}, \end{cases}$$



■ Schrödinger equation (for both scattering and bound states)

$$\left(-\nabla_{\boldsymbol{x}}^2 - \frac{a}{x'} + x'\right) \psi(\boldsymbol{x} r_0) = \epsilon \psi(\boldsymbol{x} r_0)$$

$$\Rightarrow \psi_{lm,n}(\boldsymbol{r}) = \frac{1}{r_0^{3/2}} \frac{\chi_{ln}(x)}{x} Y_{lm}(\theta, \phi), \qquad \phi_{\boldsymbol{k}}(\boldsymbol{r}) = \sum_{l=0}^{\infty} (2l+1) \frac{\chi_{kl}(x)}{x} P_l(\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{x}})$$

$$\Rightarrow \left[-\frac{\partial^2}{\partial x^2} + \underbrace{\frac{l(l+1)}{x^2} - \frac{a}{x} + x}_{V_{\rm eff}(x)}\right] \chi(x) = \epsilon\, \chi(x)$$

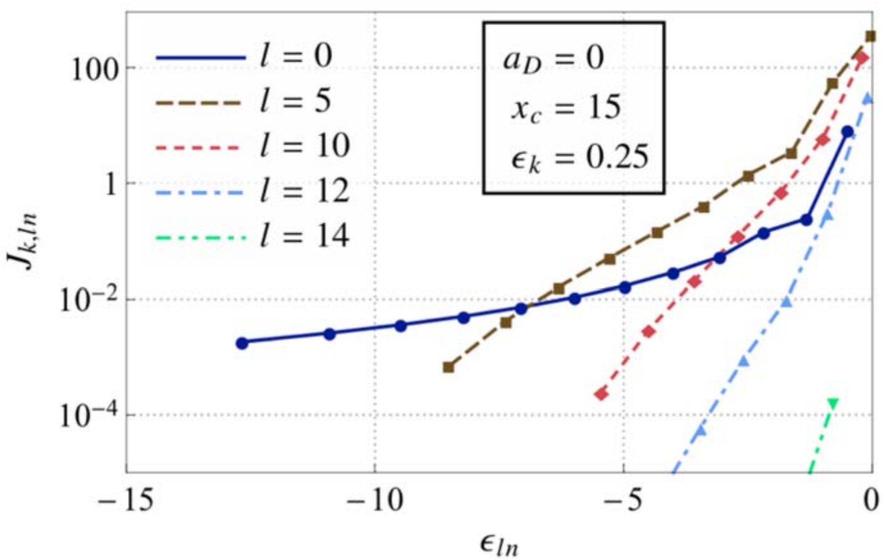
- After a long long calculation, we reached

$$\mathcal{M}^j_{\boldsymbol{k}\to lm,n} = -4e_X\sqrt{m_X}\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\,p^j\,\tilde{\psi}^*_{lm,n}(\boldsymbol{p})\left[\tilde{\phi}_{\boldsymbol{k}}\left(\boldsymbol{p}+\frac{\boldsymbol{P}_\varphi}{2}\right)+\tilde{\phi}_{\boldsymbol{k}}\left(\boldsymbol{p}-\frac{\boldsymbol{P}_\varphi}{2}\right)\right]$$

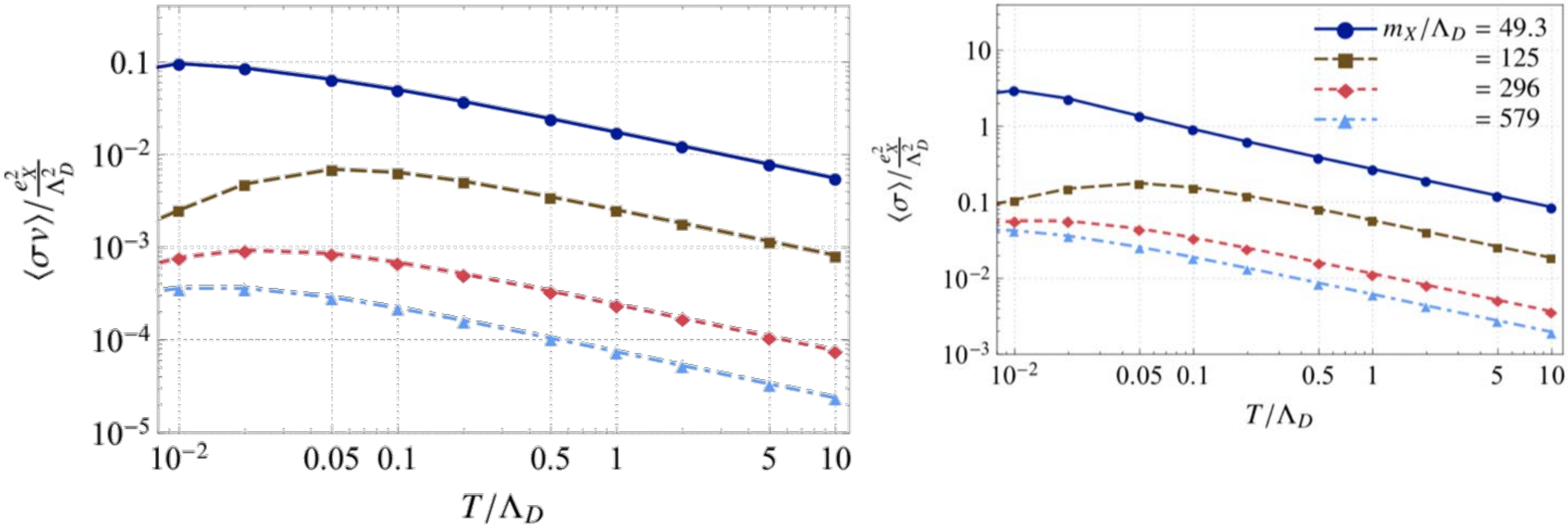
$$\Rightarrow\quad v_{\rm rel}\,\sigma_{k\hat{\boldsymbol{z}}\to ln} = \sum_{m=-l}^{l} v_{\rm rel}\,\sigma_{k\hat{\boldsymbol{z}}\to lmn} = \frac{2e_X^2}{m_X^2}\left(\frac{\Lambda_D}{m_X}\right)^{2/3} J_{k,ln}$$

$$J_{k,ln} = (\epsilon_k-\epsilon_{ln})^3\left[(l+1)\,|I_{k,l+1\to ln}|^2 + l\,|I_{k,l-1\to ln}|^2\right]$$

$$I_{k,l\pm1\to ln} = \int dx\,x\,\chi^*_{ln}(x)\,\chi_{k,l\pm1}(x)$$



and...



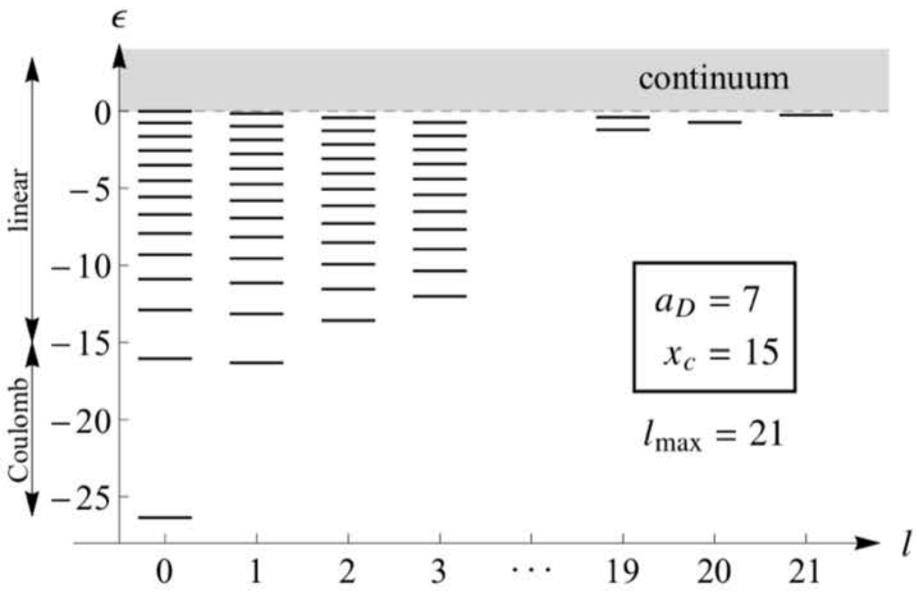
For $T \gtrsim 0.1\Lambda_D$,

$\langle v_{rel}\,\sigma\rangle \propto m_X^{-2}$

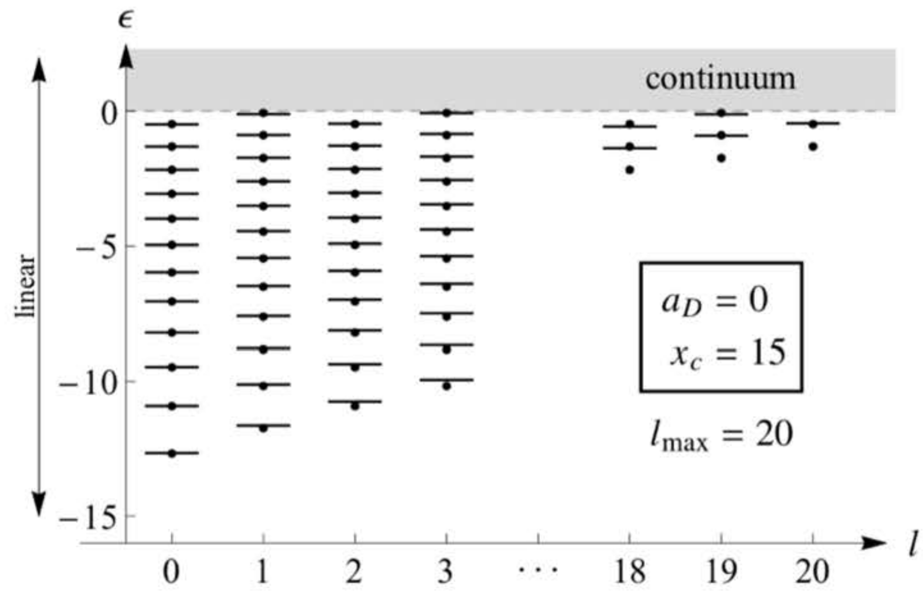
$\langle\sigma\rangle \propto (m_X^3\Lambda_D)^{-1/2}$

.... subdominant compared to the rearrangement.

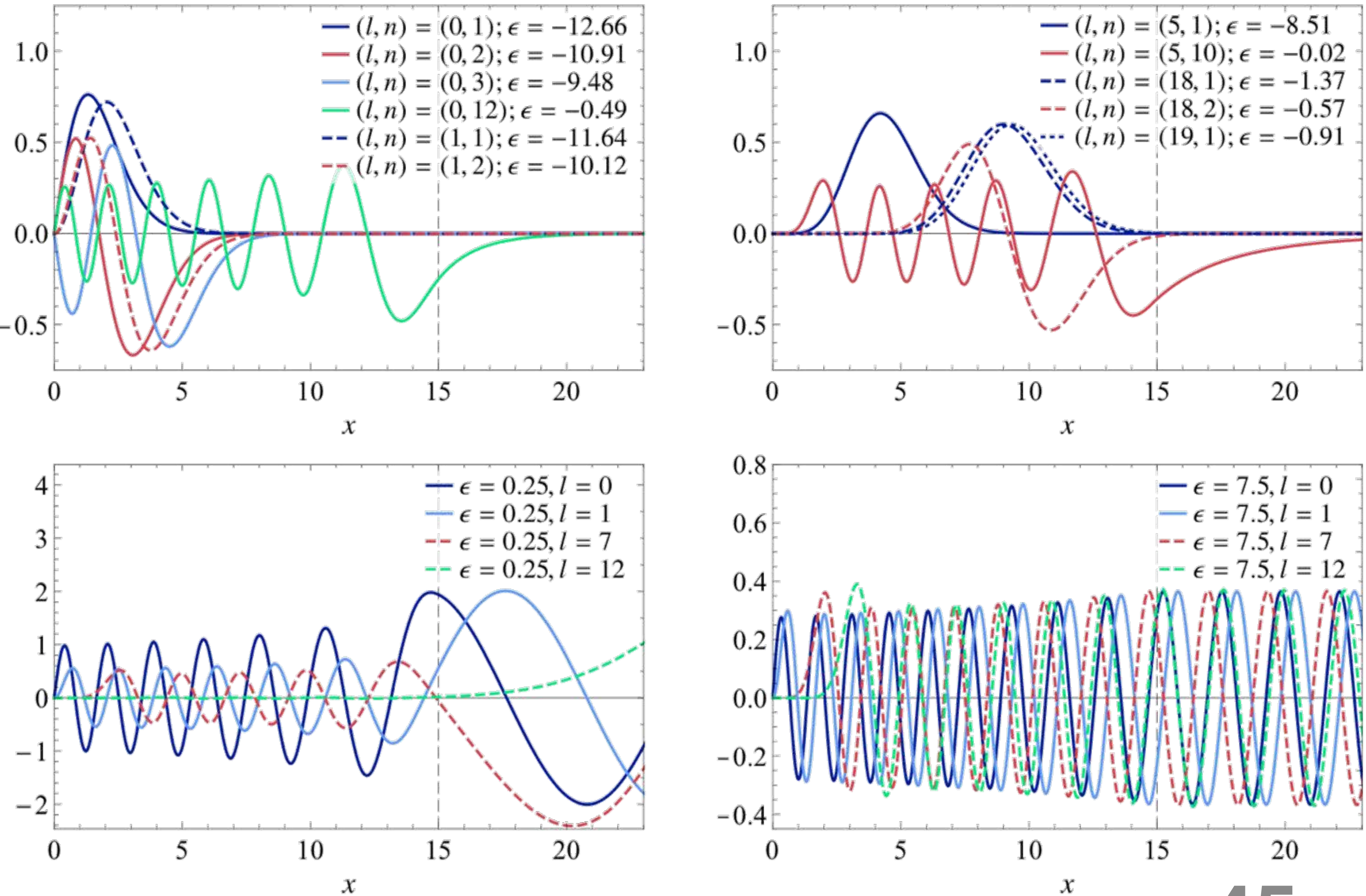
Extra 2: states and wavefunctions



(a) $a_D = 7$



(b) $a_D = 0$ (linear)


(l, n) = (0, 1); ϵ = −12.66
(l, n) = (0, 2); ϵ = −10.91
(l, n) = (0, 3); ϵ = −9.48
(l, n) = (0, 12); ϵ = −0.49
(l, n) = (1, 1); ϵ = −11.64
(l, n) = (1, 2); ϵ = −10.12
(l, n) = (5, 1); ϵ = −8.51
(l, n) = (5, 10); ϵ = −0.02
(l, n) = (18, 1); ϵ = −1.37
(l, n) = (18, 2); ϵ = −0.57
(l, n) = (19, 1); ϵ = −0.91
ϵ = 0.25, l = 0
ϵ = 0.25, l = 1
ϵ = 0.25, l = 7
ϵ = 0.25, l = 12
ϵ = 7.5, l = 0
ϵ = 7.5, l = 1
ϵ = 7.5, l = 7
ϵ = 7.5, l = 12
x