# MSSM in the light of $(g-2)_\mu$ and DM

Sho IWAMOTO（岩本　祥）

4 May 2018
KAIST−KU−PNU Joint Workshop @ KAIST

Based on

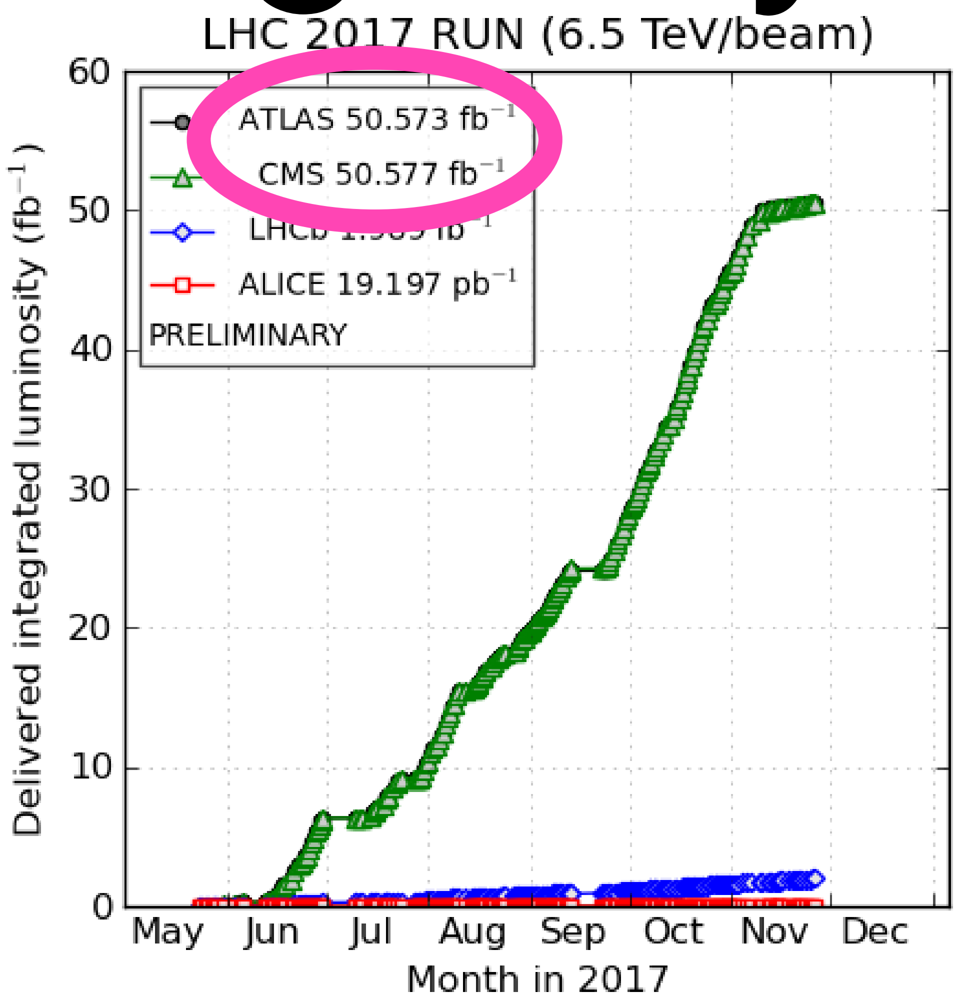
- Endo, Hamaguchi, Iwamoto, Yanagi [1704.05287]
- Endo, Hamaguchi, Iwamoto, Yoshinaga [1303.4256]

and a few ongoing projects.

N.B. briefly overlapped with my plenary talk @ Gwangju (18−19 Oct. 2016)

# LHC

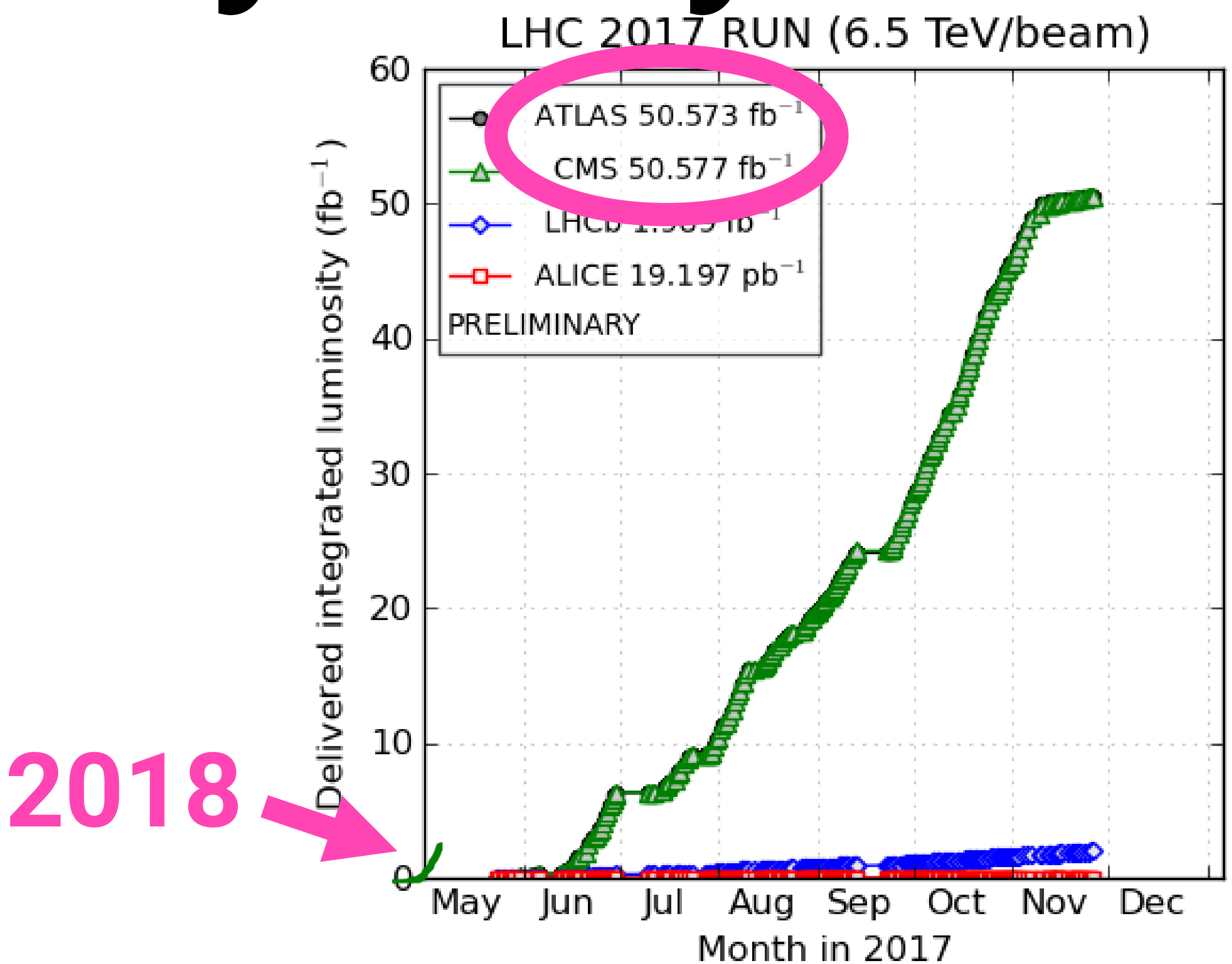
# running very well.




( 2018-03-13 14:40 including fill 6417; scripts by C. Barschel )

*Data in these plots are preliminary and may be updated by the experiments at any point in time.*
*These plots are intended for expert usage and* ***must not be used in any publication or scientific analysis****.*

# very very well...




( 2018-03-13 14:40 including fill 6417; scripts by C. Barschel )

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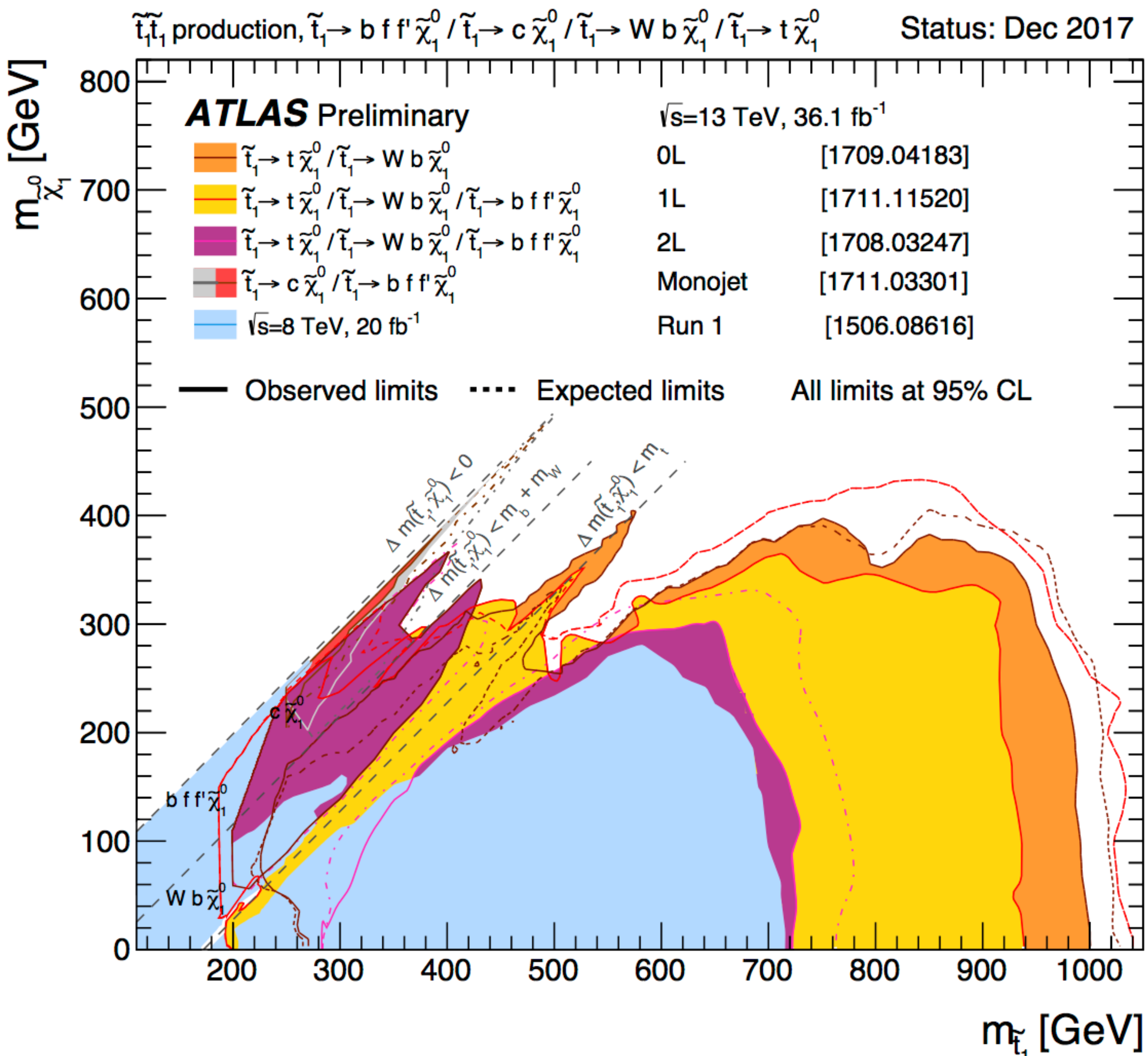
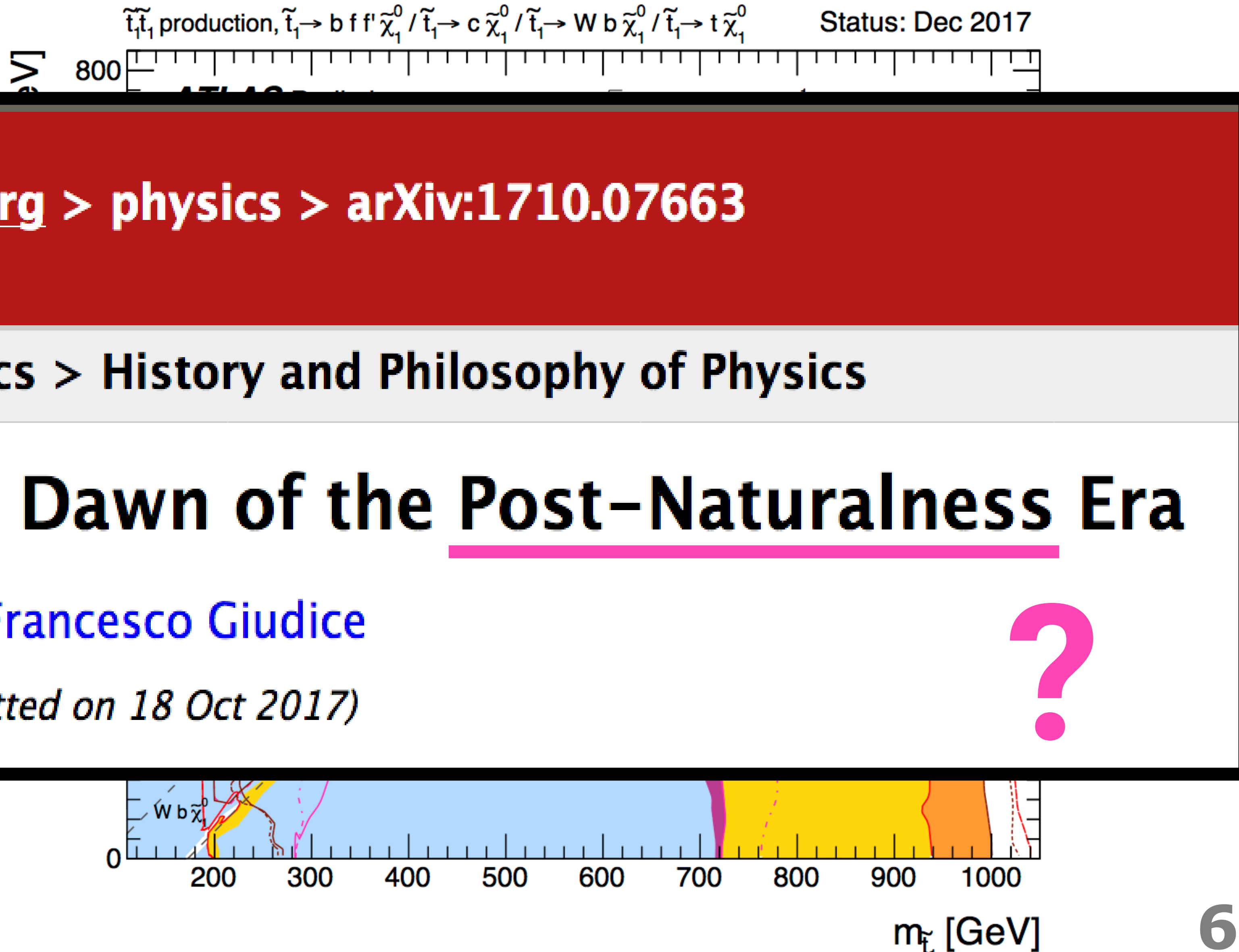
# too well...



$\tilde{t}_1\tilde{t}_1$ production, $\tilde{t}_1 \to b f f' \tilde{\chi}_1^0$ / $\tilde{t}_1 \to c \tilde{\chi}_1^0$ / $\tilde{t}_1 \to W b \tilde{\chi}_1^0$ / $\tilde{t}_1 \to t \tilde{\chi}_1^0$
Status: Dec 2017
ATLAS Preliminary
$\sqrt{s}$=13 TeV, 36.1 fb$^{-1}$
$\tilde{t}_1 \to t \tilde{\chi}_1^0$ / $\tilde{t}_1 \to W b \tilde{\chi}_1^0$ 0L [1709.04183]
$\tilde{t}_1 \to t \tilde{\chi}_1^0$ / $\tilde{t}_1 \to W b \tilde{\chi}_1^0$ / $\tilde{t}_1 \to b f f' \tilde{\chi}_1^0$ 1L [1711.11520]
$\tilde{t}_1 \to t \tilde{\chi}_1^0$ / $\tilde{t}_1 \to W b \tilde{\chi}_1^0$ / $\tilde{t}_1 \to b f f' \tilde{\chi}_1^0$ 2L [1708.03247]
$\tilde{t}_1 \to c \tilde{\chi}_1^0$ / $\tilde{t}_1 \to b f f' \tilde{\chi}_1^0$ Monojet [1711.03301]
$\sqrt{s}$=8 TeV, 20 fb$^{-1}$ Run 1 [1506.08616]
Observed limits
Expected limits
All limits at 95% CL
$\Delta m(\tilde{t}_1, \tilde{\chi}_1^0) < 0$
$\Delta m(\tilde{t}_1, \tilde{\chi}_1^0) < m_b + m_W$
$\Delta m(\tilde{t}_1, \tilde{\chi}_1^0) < m_t$
$c \tilde{\chi}_1^0$
$b f f' \tilde{\chi}_1^0$
$W b \tilde{\chi}_1^0$
$m_{\tilde{\chi}_1^0}$ [GeV]
$m_{\tilde{t}_1}$ [GeV]
0
100
200
300
400
500
600
700
800
900
1000

# too well...




arXiv.org > physics > arXiv:1710.07663

Physics > History and Philosophy of Physics

## The Dawn of the Post-Naturalness Era

Gian Francesco Giudice

*(Submitted on 18 Oct 2017)*

?


Some blog post proposes...

- Astrophysics: DM, baryogenesis, inflation,
  →Kohei's and Fang's talk
- Precision physics: neutrino mass, Higgs sector, flavor anomalies(?)
  →Seung's talk
  →LHC + Belle II
- Formal theoretical developments

- What can we expect at **the energy frontier**?

  ... How can we motivate students in ATLAS/CMS?

## ■ Motivation for $O(100)$GeV new physics

- Dark matter "WIMP miracle"

  ... simplest scenario predicts

$$\langle \sigma v \rangle_{\mathrm{DM\,DM}\to\mathrm{any}} \sim 3\times10^{-26}\,\mathrm{cm}^3/\mathrm{s} = \frac{\alpha_{\mathrm{em}}^2}{(150\,\mathrm{GeV})^2}.$$

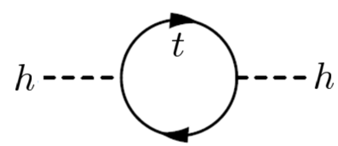
(DM as a thermal relic, freezing out by pair-annihilation)

- Naturalness

$$m_h^2 \sim m_{\mathrm{bare}}^2 + \Delta m_h^2, \qquad \Delta m_h^2(\mathrm{SM}) \sim -\frac{3|\lambda|^2}{8\pi^2}\Lambda_{\mathrm{cutoff}}^2 + \mathrm{finite}.$$

⇓

$$(100\,\mathrm{GeV})^2 \sim \Lambda_{\mathrm{cutoff}}^2 - \Lambda_{\mathrm{cutoff}}^2$$



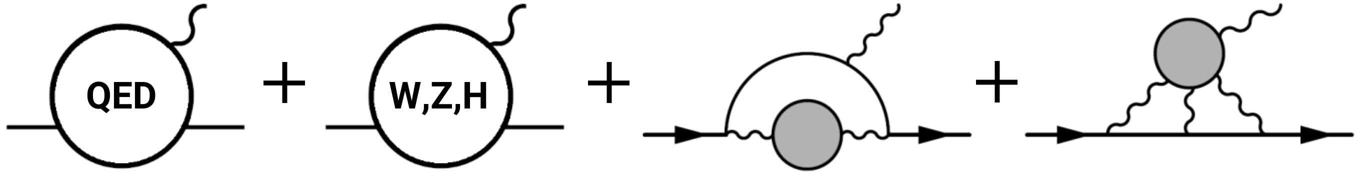

- $(g-2)_\mu$ anomaly [$g$−2 = anomalous magnetic moment]

  →Deok-Ki's talk

→Deok-Ki's talk

(5-loop) (2⁺-loop) (dispersion rel.) (low-energy EFT)

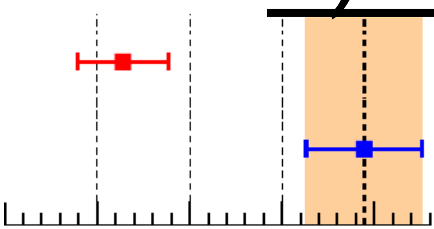
$a_\mu^{\rm SM} \simeq$




had. vac. polarization had. light-by-light

$$a_\mu(\text{QED}) = (11\,658\,471.886 \pm 0.003) \times 10^{-10},$$
$$a_\mu(\text{EW}) = (15.36 \pm 0.11) \times 10^{-10},$$
$$a_\mu(\text{HVP-LO}) = (689.46 \pm 3.25) \times 10^{-10},$$
$$a_\mu(\text{HVP-HO}) = (-8.70 \pm 0.07) \times 10^{-10},$$
$$a_\mu(\text{HLbL}) = (10.34 \pm 2.88) \times 10^{-10}.$$

+) $a_\mu(\text{NP})$? … $10 \times 10^{-10} \approx \dfrac{\alpha_{\rm em}}{4\pi}\left(\dfrac{m_\mu}{200\,\text{GeV}}\right)^2$



$$a_\mu^{\rm SM} = (11\,659\,178.3 \pm 4.3) \times 10^{-10}$$
$$a_\mu^{\rm SM} = (11\,659\,209.2 \pm 6.3) \times 10^{-10}$$ (BNL '04+CODATA '14)

→ ±1.6 @ Fermilab in 1–2 year!

SM combination according to Jegerlehner [1804.07409].
QED: Aoyama, Hayakawa, Kinoshita, Nio [1205.5370] (cf. [1712.06060]).
EW: Gnendiger, Stöckinger, Stöckinger-Kim [1306.5546].
QCD: Jegerlehner [1711.06089] [1705.00263].

See also:
QED: Laporta [1704.06996], Marquard et al. [1708.07138].
HVP-LO: Keshavarzi, Nomura, Teubner [1802.02995]
HVP-HO: Kurz, Liu, Marquard, Steinhauser [1403.6400],
HLbL: Jegerlehner, Nyffeler [0902.3360],
Colangelo, Hoferichter, Nyffeler, Passera, Stoffer [1403.7512]

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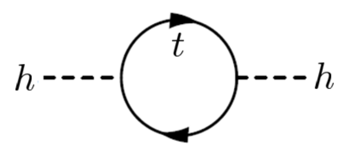
$$\Downarrow$$

$$(100\,\mathrm{GeV})^2 \sim \Lambda_{\mathrm{cutoff}}^2 - \Lambda_{\mathrm{cutoff}}^2$$




## ➢ $(g-2)_\mu$ anomaly [$g$−2 = anomalous magnetic moment]

$$\Delta a_\mu = 10\times 10^{-10} \approx \frac{\alpha_{\mathrm{em}}}{4\pi}\left(\frac{m_\mu}{200\,\mathrm{GeV}}\right)^2 \quad + \text{Fermilab}$$

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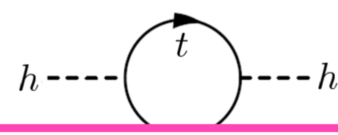


**we are in muon *g*−2 era!!**

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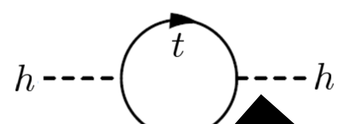


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➢ $(g-2)_\mu$ anomaly [$g-2$ = anomalous magnetic moment]

$$\Delta a_\mu = 10\times 10^{-10} \approx \frac{\alpha_{\mathrm{em}}}{4\pi}\left(\frac{m_\mu}{200\,\mathrm{GeV}}\right)^2 + $$

**years [2016–2020]**

→Deok-Ki's talk

## ■ Muon $g$−2 anomaly: What is the origin?

- Just a statistical fluctuation.
- Just an issue in the experiment.
- $O(100)$ GeV particles with $O(0.1)$ couplings
  - KK gravition, MSSM
- keV−MeV particles with tiny couplings.
  - dark photon (extra U(1) gauge boson)

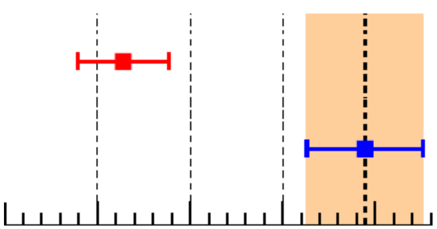
$$10\times10^{-10}\approx\frac{\alpha_{\rm em}}{4\pi}\left(\frac{m_\mu}{m_{\rm new}}\right)^2$$

~200GeV

$$10\times10^{-10}\approx\frac{(\varepsilon^2/4\pi)}{4\pi}\left(\frac{m_\mu}{m_{\rm new}}\right)^2$$

keV−MeV

$a_\mu$(NP)? … $10\times10^{-10}\approx\frac{\alpha_{\rm em}}{4\pi}\left(\frac{m_\mu}{200\,{\rm GeV}}\right)^2$



$$a_\mu^{\rm SM}=(11\,659\,178.3\pm4.3)\times10^{-10}$$

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*we assume it is "actual".*

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$$10 \times 10^{-10} \approx \frac{\alpha_{\text{em}}}{4\pi}\left(\frac{m_\mu}{m_{\text{new}}}\right)^2$$

$m_{\text{new}}$ ~200GeV

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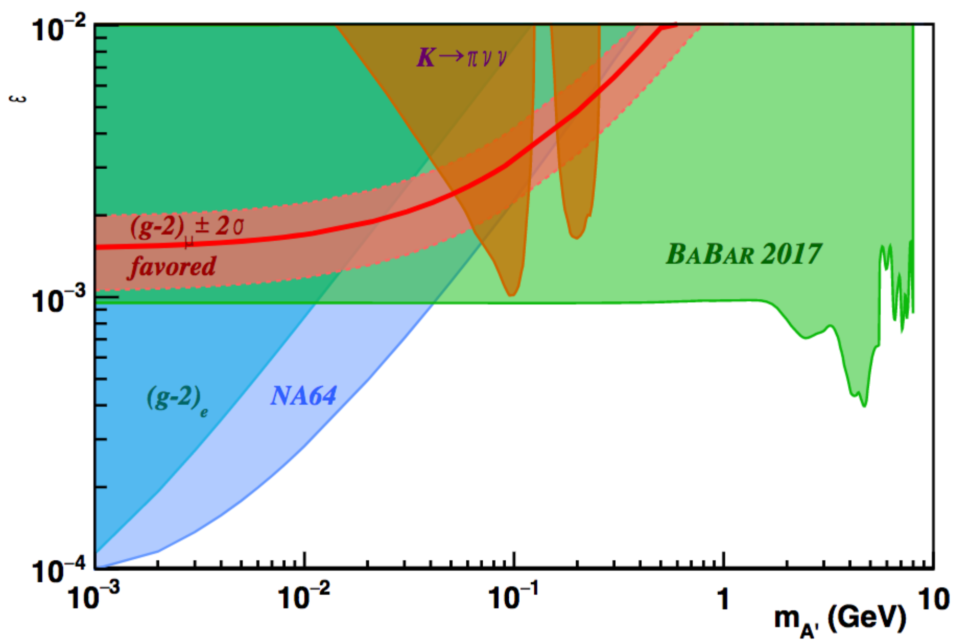
$m_{\text{new}}$ keV−MeV




$e^+e^- \to \gamma A',\ A' \to$ invisible

BaBar [1702.03327]

## Muon $g$−2 anomaly: What is the origin?

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- ~~Just an issue i…~~ …periment.

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- $O(100)$ GeV pa… …ith $O(0.1)$ couplings
  - MSSM

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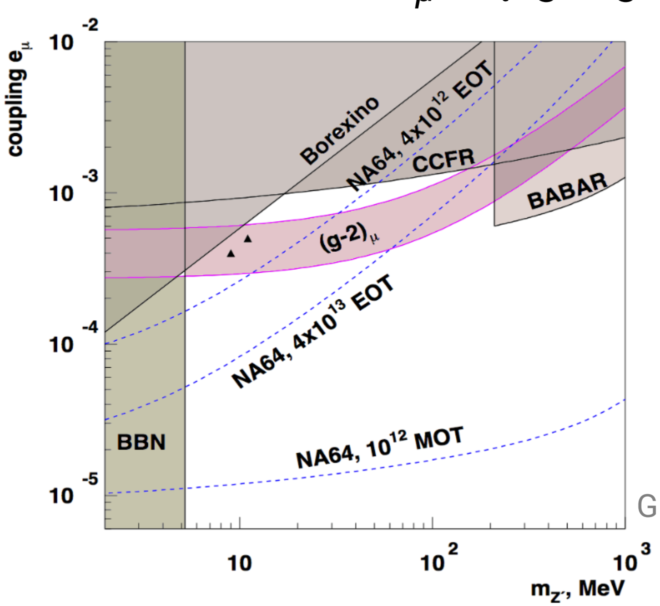
$m_{\rm new}$ ~200GeV

- keV−MeV particles with tiny couplings.
  - ~~dark photon (extra U(1) gauge boson)~~
  - extra $L_\mu\!-\!L_\tau$ gauge boson

$$10\times10^{-10}\approx\frac{(\varepsilon^2/4\pi)}{4\pi}\left(\frac{m_\mu}{m_{\rm new}}\right)^2$$

$m_{\rm new}$ keV−MeV

Gninenko, Krasnikov [ph/0102222], Baek, Deshpande, He, Ko [ph/0104141]




$$L_{Z'}=e_\mu Z'_\nu[\bar\mu\gamma^\nu\mu-\bar\tau\gamma^\nu\tau+\bar\nu_\mu\gamma^\nu\nu_\mu-\bar\nu_\tau\gamma^\nu\nu_\tau]$$

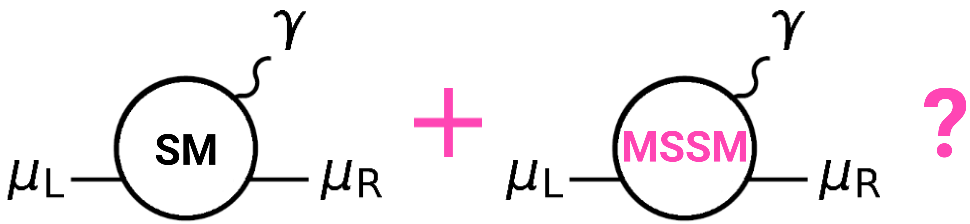
Gninenko, Krasnikov [1801.10448]

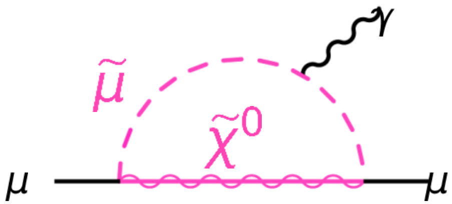
1. Introduction

2. Four scenarios of MSSM as the solution of $(g-2)_\mu$ anomaly
   - Overview
   - Collider physics
   - Dark Matter

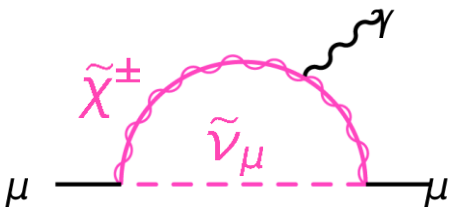
3. Discussion for each scenario
   - "Chargino" scenario: multi-lepton signature is promising.
   - "Pure-bino" scenario: di-lepton, but production not sufficient.
   - "BHR" or "BHL": multi-tau, combined with direct detections.

$$a_\mu \equiv \frac{(g-2)_\mu}{2} =$$




$$a_\mu^{\mathrm{SUSY}}\left(\tilde{\chi}^0, \tilde{\mu}\right) \approx \frac{g_Y^2}{(4\pi)^2}\frac{m_\mu^2}{m_{\mathrm{soft}}^2}\,\mathrm{sgn}(\mu)\tan\beta + \cdots,$$




$$a_\mu^{\mathrm{SUSY}}\left(\tilde{\chi}^\pm, \tilde{\nu}_\mu\right) \approx \frac{g_2^2}{(4\pi)^2}\frac{m_\mu^2}{m_{\mathrm{soft}}^2}\,\mathrm{sgn}(\mu)\tan\beta.$$

- lighter SUSY-particles
- larger $\tan\beta$

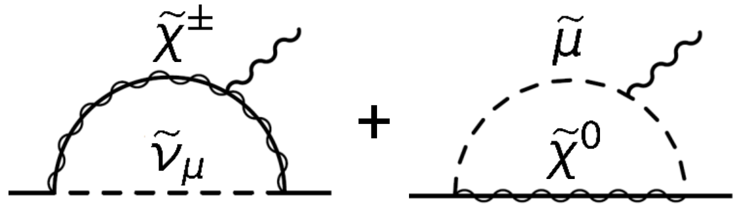
⇨ larger $a_\mu^{\mathrm{SUSY}}$

$W \ni \mu H_\mathrm{u} H_\mathrm{d}$ (higgsino mass term), $\tan\beta = \langle H_\mathrm{u}\rangle / \langle H_\mathrm{d}\rangle$,
$m_{\mathrm{soft}}$ : SUSY-particle mass-scale, $g_i$ : gauge couplings.

Lopez, Nanopoulos, Wang [ph/9308336]
Chattopadhyay, Nath [ph/9507386]
Moroi [ph/9512396]

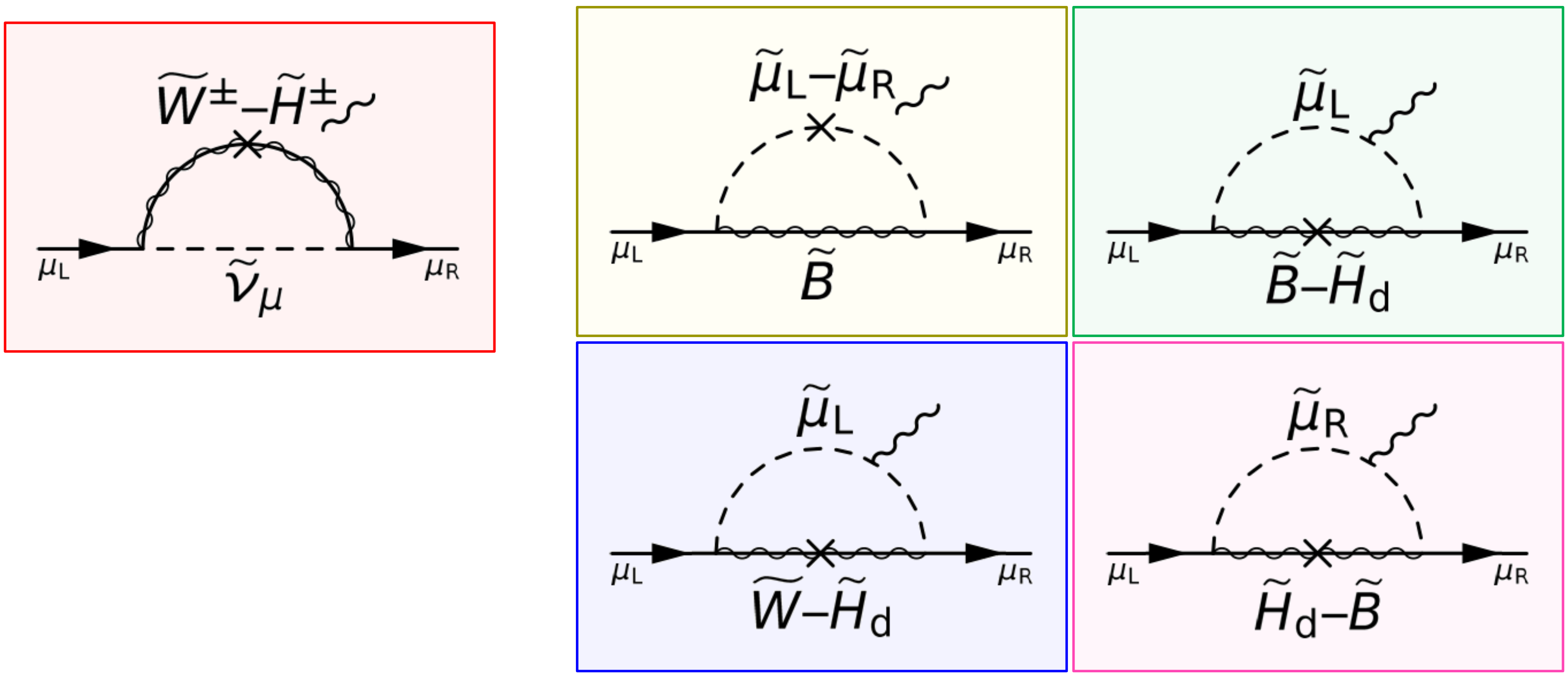
**mass eigenstates**

$a_\mu^{\mathrm{SUSY}} \simeq$

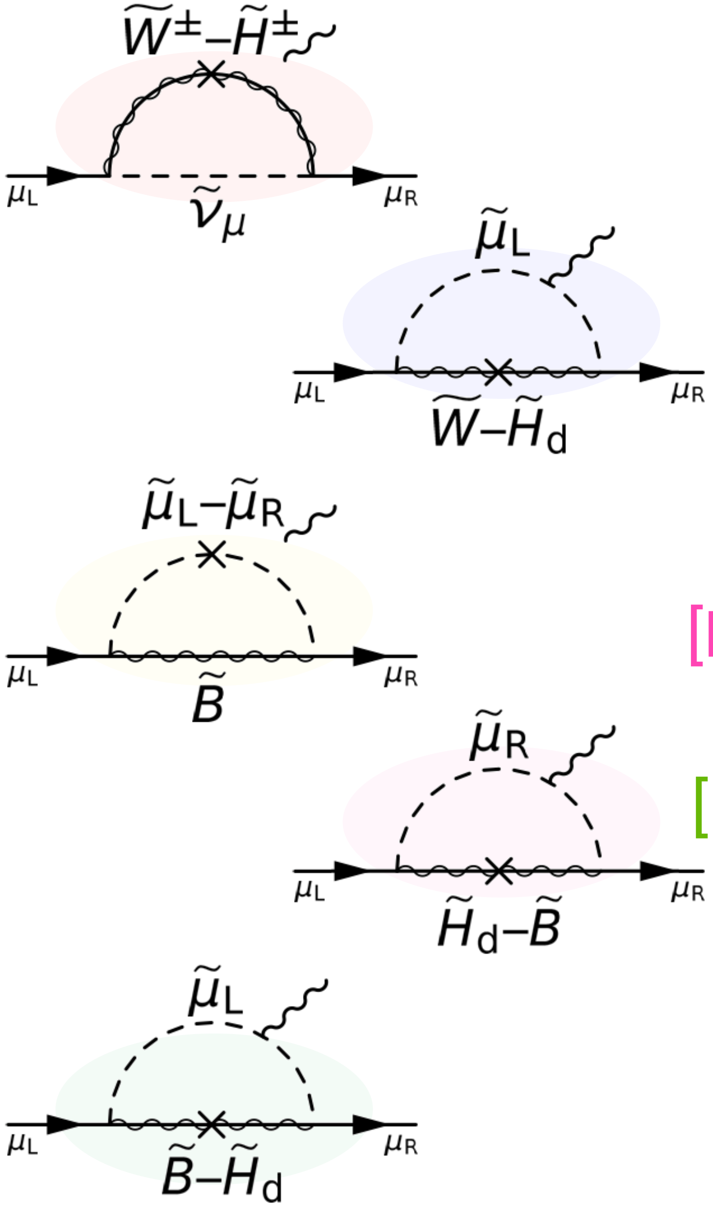



$$\tilde{\chi}^0_{1-4} = \tilde{B} \oplus \tilde{W}^0 \oplus \tilde{H}_\mathrm{d} \oplus \tilde{H}_\mathrm{u}, \quad \tilde{\chi}^\pm_{1,2} = \tilde{W}^\pm \oplus \tilde{H}^\pm.$$

**gauge eigenstates**




**(“mass insertion” technique)**




[C] $$\frac{g_2^2 m_\mu^2}{8\pi^2}\frac{M_2\mu\tan\beta}{m_{\tilde{\nu}_\mu}^4}\cdot F_a\left(\frac{M_2}{m_{\tilde{\nu}_\mu}},\frac{\mu}{m_{\tilde{\nu}_\mu}}\right)$$

[C'] $$-\frac{g_2^2 m_\mu^2}{16\pi^2}\frac{M_2\mu\tan\beta}{m_{\tilde{\mu}_L}^4}\cdot F_b\left(\frac{M_2}{m_{\tilde{\mu}_L}},\frac{\mu}{m_{\tilde{\mu}_L}}\right)$$

[B] $$\frac{g_Y^2 m_\mu^2}{8\pi^2}\frac{\mu\tan\beta}{M_1^3}\cdot F_b\left(\frac{m_{\tilde{\mu}_L}}{M_1},\frac{m_{\tilde{\mu}_R}}{M_1}\right)$$

[BHR] $$-\frac{g_Y^2 m_\mu^2}{8\pi^2}\frac{M_1\mu\tan\beta}{m_{\tilde{\mu}_R}^4}\cdot F_b\left(\frac{M_1}{m_{\tilde{\mu}_R}},\frac{\mu}{m_{\tilde{\mu}_R}}\right)$$

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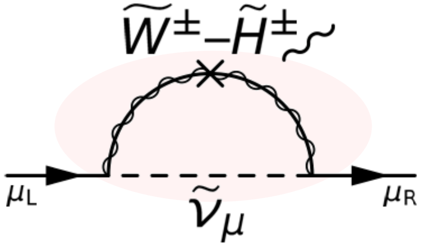
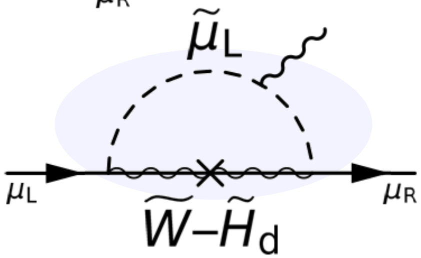
$F_a, F_b$ are loop functions and positive.

$$F_a(x,y)=\frac{1}{2}\frac{C_1(x^2)-C_1(y^2)}{x^2-y^2},\qquad F_b(x,y)=-\frac{1}{2}\frac{N_2(x^2)-N_2(y^2)}{x^2-y^2};$$

$$C_1(x)=\frac{3-4x+x^2+2\log x}{(1-x)^3},\qquad N_2(x)=\frac{1-x^2+2x\log x}{(1-x)^3}.$$




[C] $$\frac{g_2^2 m_\mu^2}{8\pi^2}\frac{M_2\mu\tan\beta}{m_{\widetilde{\nu}_\mu}^4}\cdot F_a\left(\frac{M_2}{m_{\widetilde{\nu}_\mu}},\frac{\mu}{m_{\widetilde{\nu}_\mu}}\right)$$

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- "Chargino contribution"
- $\propto g_2^2$ (not $g_Y^2$) → tends to be the dominant contribution.
- SU(2) pair → [C'] $\simeq -0.5$[C] → $\mu > 0$ to be positive.
- Higgsino, Wino, and $\widetilde{\mu}_L$ must be $O(100)$GeV.

$F_a, F_b$ are loop functions and positive.

$$F_a(x,y) = \frac{1}{2}\frac{C_1(x^2)-C_1(y^2)}{x^2-y^2}, \qquad F_b(x,y) = -\frac{1}{2}\frac{N_2(x^2)-N_2(y^2)}{x^2-y^2};$$

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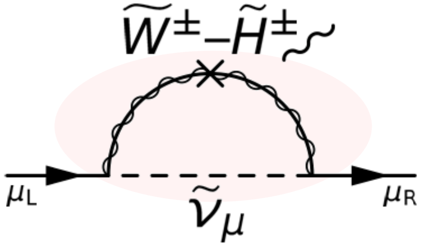


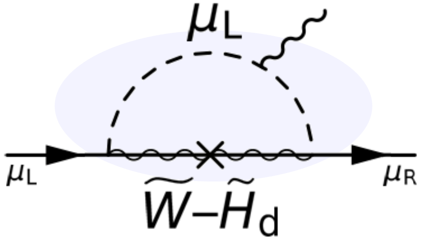


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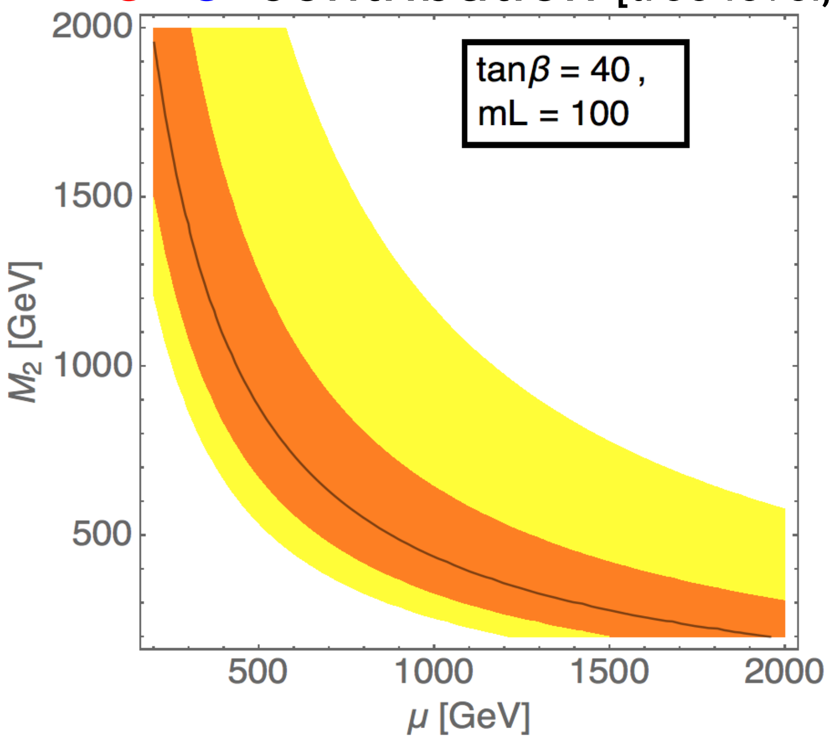
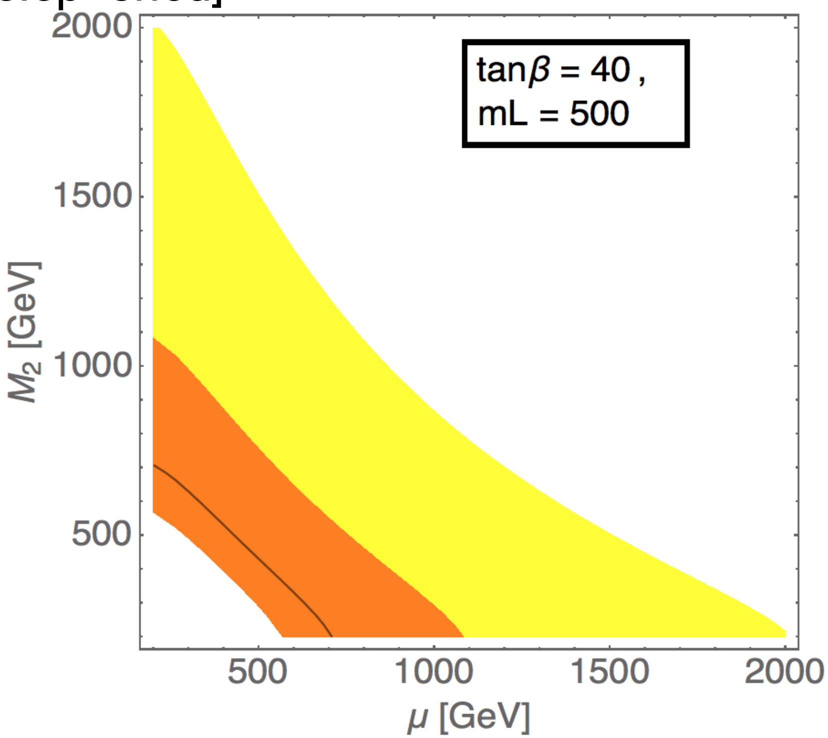
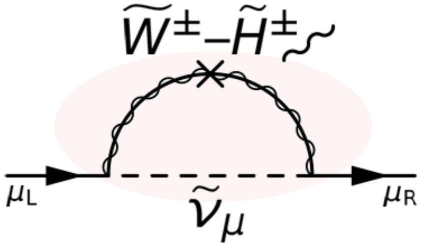
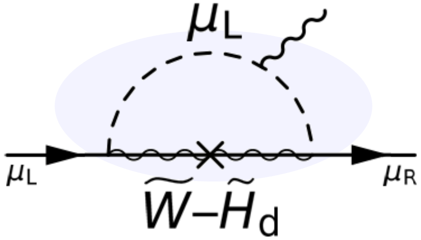
## C+C'-contribution [tree-level; slep=sneu]



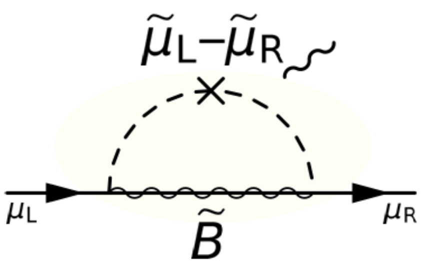





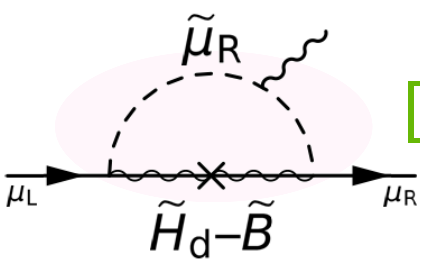

[C] $$\frac{g_2^2 m_\mu^2}{8\pi^2}\frac{M_2\mu\tan\beta}{m_{\tilde{\nu}_\mu}^4}\cdot F_a\left(\frac{M_2}{m_{\tilde{\nu}_\mu}},\frac{\mu}{m_{\tilde{\nu}_\mu}}\right)$$



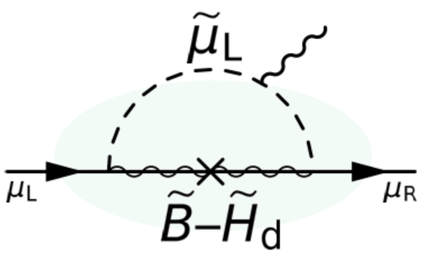

[C'] $$-\frac{g_2^2 m_\mu^2}{16\pi^2}\frac{M_2\mu\tan\beta}{m_{\tilde{\mu}_L}^4}\cdot F_b\left(\frac{M_2}{m_{\tilde{\mu}_L}},\frac{\mu}{m_{\tilde{\mu}_L}}\right)$$




[B] $$\frac{g_Y^2 m_\mu^2}{8\pi^2}\frac{\mu\tan\beta}{M_1^3}\cdot F_b\left(\frac{m_{\tilde{\mu}_L}}{M_1},\frac{m_{\tilde{\mu}_R}}{M_1}\right)$$




[BHR] $$-\frac{g_Y^2 m_\mu^2}{8\pi^2}\frac{M_1\mu\tan\beta}{m_{\tilde{\mu}_R}^4}\cdot F_b\left(\frac{M_1}{m_{\tilde{\mu}_R}},\frac{\mu}{m_{\tilde{\mu}_R}}\right)$$




[BHL] $$\frac{g_Y^2 m_\mu^2}{16\pi^2}\frac{M_1\mu\tan\beta}{m_{\tilde{\mu}_L}^4}\cdot F_b\left(\frac{M_1}{m_{\tilde{\mu}_L}},\frac{\mu}{m_{\tilde{\mu}_L}}\right)$$

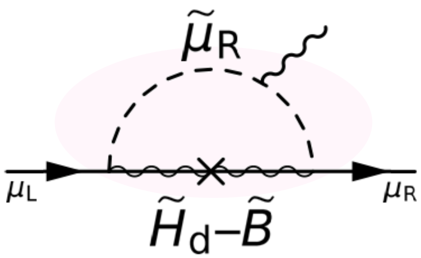
$F_a$, $F_b$ are loop functions and positive.

$$F_a(x,y)=\frac{1}{2}\frac{C_1(x^2)-C_1(y^2)}{x^2-y^2},\qquad F_b(x,y)=-\frac{1}{2}\frac{N_2(x^2)-N_2(y^2)}{x^2-y^2};$$

$$C_1(x)=\frac{3-4x+x^2+2\log x}{(1-x)^3},\qquad N_2(x)=\frac{1-x^2+2x\log x}{(1-x)^3}.$$

- "BHR contribution" (Bino, Higgsino, $\tilde{\mu}_R$ must be $O(100)$GeV)
- If $\mu$-parameter < 0, this is the only viable contribution.
  (Higgsino-mass parameter)
- $\propto g_Y^2$

[BHR] $$-\frac{g_Y^2 m_\mu^2}{8\pi^2}\frac{M_1 \mu \tan\beta}{m_{\tilde{\mu}_R}^4}\cdot F_b\left(\frac{M_1}{m_{\tilde{\mu}_R}},\frac{\mu}{m_{\tilde{\mu}_R}}\right)$$

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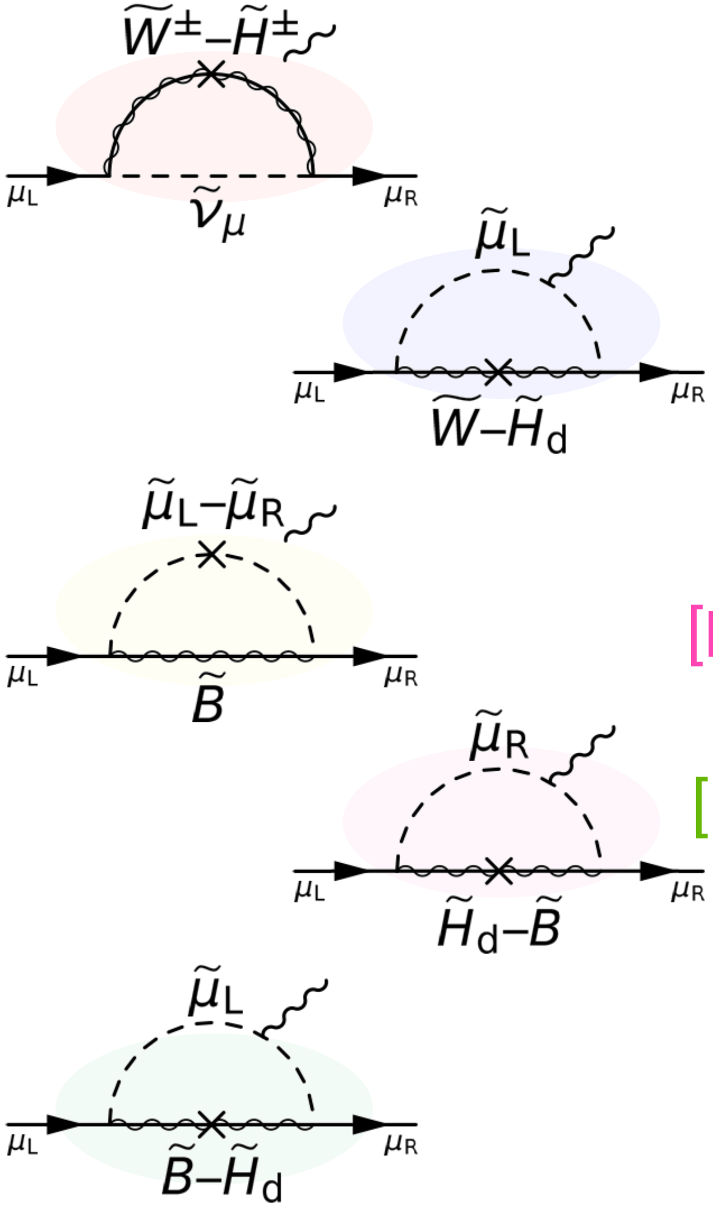


[C]
$$\frac{g_2^2 m_\mu^2}{8\pi^2}\frac{M_2\mu\tan\beta}{m_{\tilde{\nu}_\mu}^4}\cdot F_a\left(\frac{M_2}{m_{\tilde{\nu}_\mu}},\frac{\mu}{m_{\tilde{\nu}_\mu}}\right)$$

[C']
$$-\frac{g_2^2 m_\mu^2}{16\pi^2}\frac{M_2\mu\tan\beta}{m_{\tilde{\mu}_L}^4}\cdot F_b\left(\frac{M_2}{m_{\tilde{\mu}_L}},\frac{\mu}{m_{\tilde{\mu}_L}}\right)$$

[B]
$$\frac{g_Y^2 m_\mu^2}{8\pi^2}\frac{\mu\tan\beta}{M_1^3}\cdot F_b\left(\frac{m_{\tilde{\mu}_L}}{M_1},\frac{m_{\tilde{\mu}_R}}{M_1}\right)$$

[BHR]
$$-\frac{g_Y^2 m_\mu^2}{8\pi^2}\frac{M_1\mu\tan\beta}{m_{\tilde{\mu}_R}^4}\cdot F_b\left(\frac{M_1}{m_{\tilde{\mu}_R}},\frac{\mu}{m_{\tilde{\mu}_R}}\right)$$

[BHL]
$$\frac{g_Y^2 m_\mu^2}{16\pi^2}\frac{M_1\mu\tan\beta}{m_{\tilde{\mu}_L}^4}\cdot F_b\left(\frac{M_1}{m_{\tilde{\mu}_L}},\frac{\mu}{m_{\tilde{\mu}_L}}\right)$$

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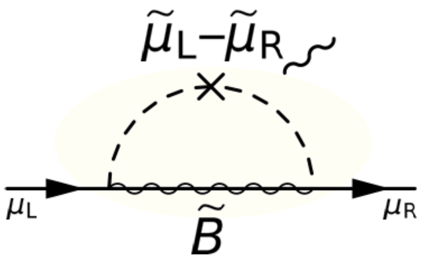
$$C_1(x)=\frac{3-4x+x^2+2\log x}{(1-x)^3},\qquad N_2(x)=\frac{1-x^2+2x\log x}{(1-x)^3}.$$

- "pure-Bino contribution": Bino and $\tilde{\mu}_\mathrm{L}, \tilde{\mu}_\mathrm{R}$ must be $O(100)$GeV.
  - Higgsino and Wino can be any heavy.
- $\propto \mu \tan\beta$ → heavier Higgsino gives larger contribution.




[B] $$\frac{g_Y^2 m_\mu^2}{8\pi^2} \frac{\mu \tan\beta}{M_1^3} \cdot F_b\left(\frac{m_{\tilde{\mu}_\mathrm{L}}}{M_1}, \frac{m_{\tilde{\mu}_\mathrm{R}}}{M_1}\right)$$

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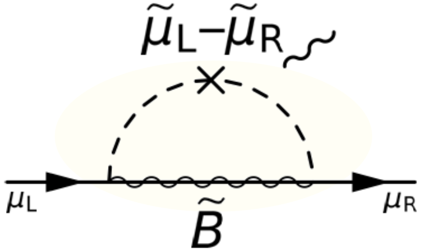
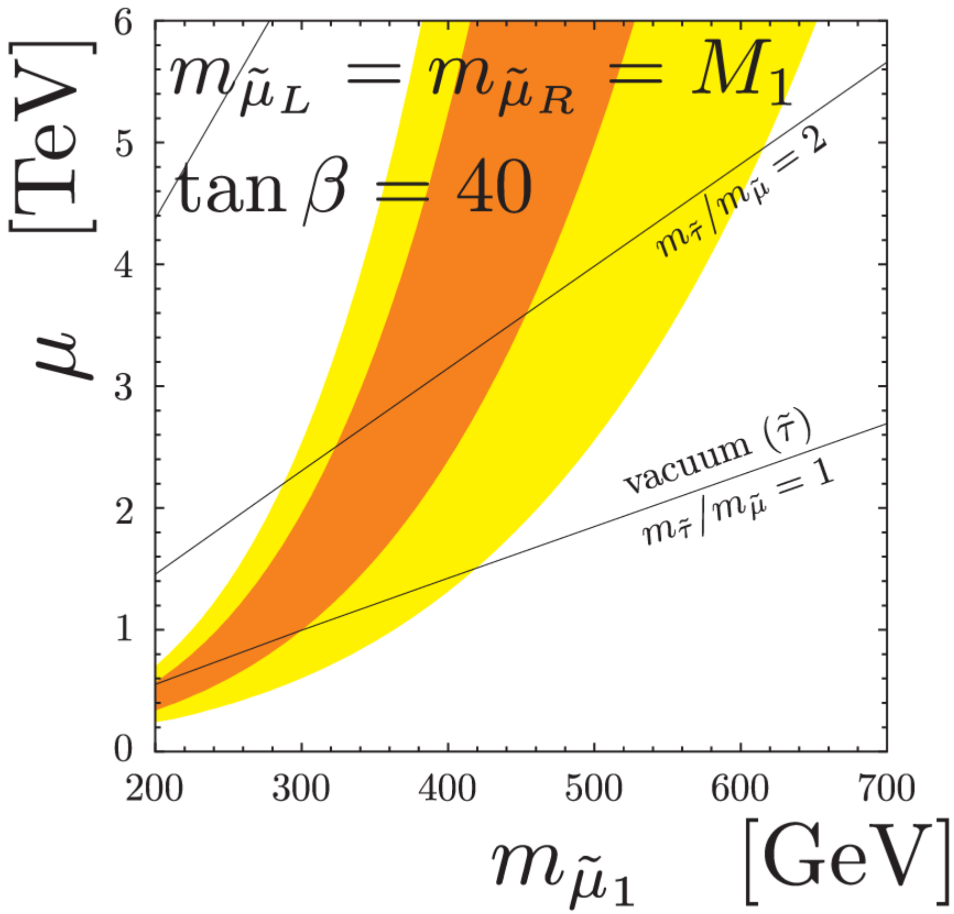
Endo, Hamaguchi, Kitahara, Yoshinaga [1309.3065]




$$\frac{g_Y^2 m_\mu^2}{8\pi^2}\frac{\mu\tan\beta}{M_1^3}\cdot F_b\left(\frac{m_{\tilde{\mu}_L}}{M_1},\frac{m_{\tilde{\mu}_R}}{M_1}\right)$$

from $M_{\tilde{\mu}}^2 = \begin{pmatrix} m(l_L)^2 & m_\mu(A_\mu^* - \mu\tan\beta) \\ m_\mu(A_\mu^* - \mu\tan\beta) & m(l_R)^2 \end{pmatrix}$




***μ* tan*β* has upper bounds:**

$$V_{\rm Higgs} \supset -\left(m_\tau\,\mu\tan\beta\cdot\tilde{\tau}_L^*\tilde{\tau}_R h + m_\mu\,\mu\tan\beta\cdot\tilde{\mu}_L^*\tilde{\mu}_R h\right)$$

$m_{\tilde{\tau}}/m_{\tilde{\mu}}$

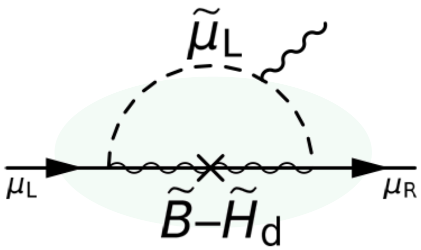
$= 1 \;\Rightarrow m_{\tilde{\mu}} \lesssim 300(420)$ GeV

$= 2 \;\Rightarrow \lesssim 440(620)$ GeV

$= \infty \;\Rightarrow \lesssim 1.4(1.9)$ TeV

- "BHL contribution" (Bino, Higgsino, $\tilde{\mu}_L$ must be $O(100)$GeV)
- nothing special.

[BHL] $$\frac{g_Y^2 m_\mu^2}{16\pi^2} \frac{M_1 \mu \tan\beta}{m_{\tilde{\mu}_L}^4} \cdot F_b\left(\frac{M_1}{m_{\tilde{\mu}_L}}, \frac{\mu}{m_{\tilde{\mu}_L}}\right)$$

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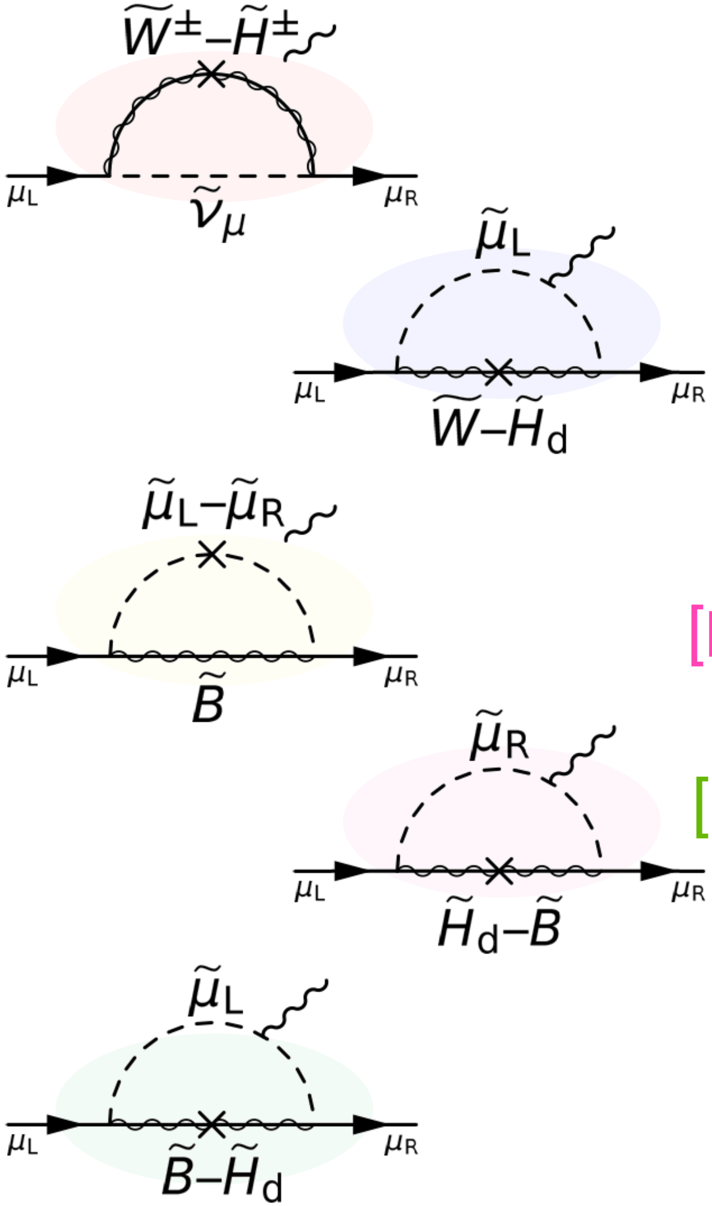
$$C_1(x) = \frac{3-4x+x^2+2\log x}{(1-x)^3}, \qquad N_2(x) = \frac{1-x^2+2x\log x}{(1-x)^3}.$$




[C] $\frac{g_2^2 m_\mu^2}{8\pi^2}\frac{M_2\mu\tan\beta}{m^4}\cdot F_a\left(\frac{M_2}{m},\frac{\mu}{m}\right)$

**tend to be large/dominant**

[C'] $-\frac{g_2^2 m_\mu^2}{16\pi^2}\frac{M_2\mu\tan\beta}{m_{\widetilde{\mu}_L}^4}\cdot F_b\left(\frac{M_2}{m_{\widetilde{\mu}_L}},\frac{\mu}{m_{\widetilde{\mu}_L}}\right)$

[B] $\frac{g_Y^2 m_\mu^2}{8\pi^2}\,\cdots\,\frac{\ }{M_1}\left(\frac{\ }{M_1},\frac{m_{\widetilde{\mu}_R}}{M_1}\right)$

**$\mu$-enhancement**

[BHR] $-\frac{g_Y^2 m_\mu^2}{8\pi^2}\frac{M_1\mu\,\cdots}{m_{\widetilde{\mu}_R}}\,\cdots\left(\frac{M_1}{m_{\widetilde{\mu}_R}},\frac{\mu}{m_{\widetilde{\mu}_R}}\right)$

**negative**

[BHL] $\frac{g_Y^2 m_\mu^2}{16\pi^2}\,\cdots\,\frac{\ }{m_{\widetilde{\mu}_L}}\left(\cdots,\frac{\mu}{m_{\widetilde{\mu}_L}}\right)$

**nothing special**

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$$C_1(x)=\frac{3-4x+x^2+2\log x}{(1-x)^3},\qquad N_2(x)=\frac{1-x^2+2x\log x}{(1-x)^3}.$$

# 1. Introduction

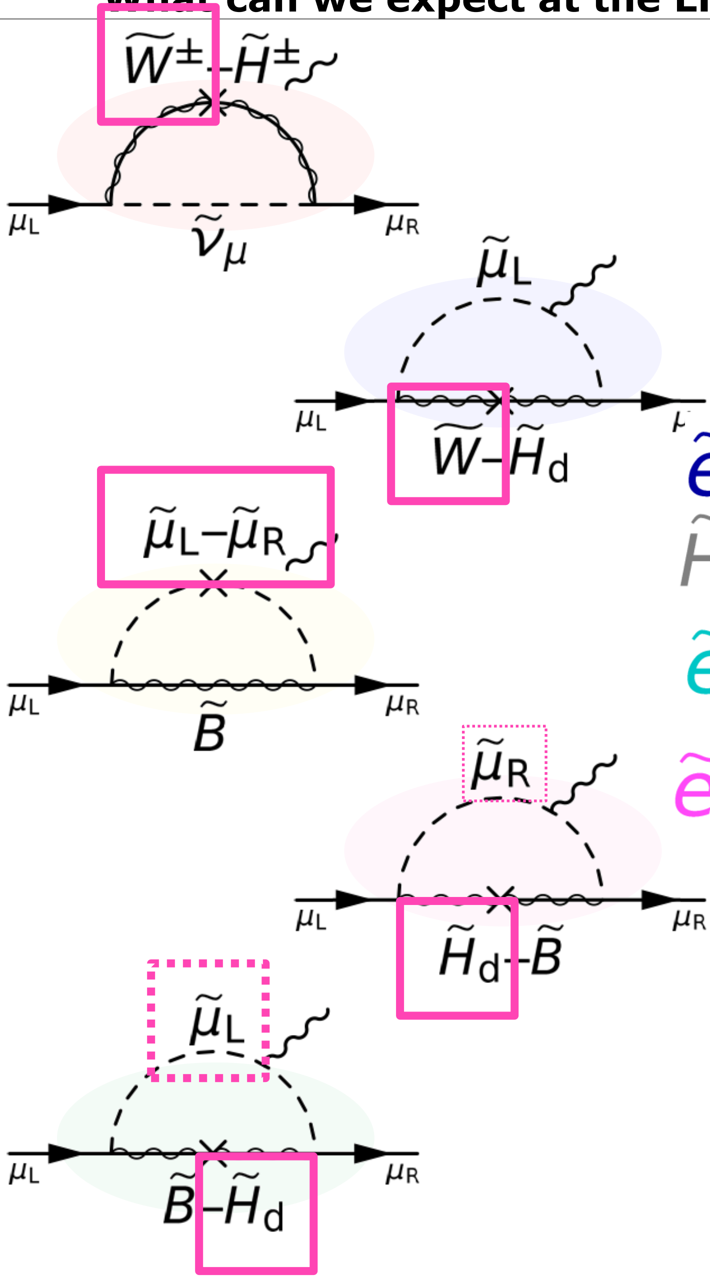
# 2. Four scenarios of MSSM as the solution of $(g-2)_\mu$ anomaly

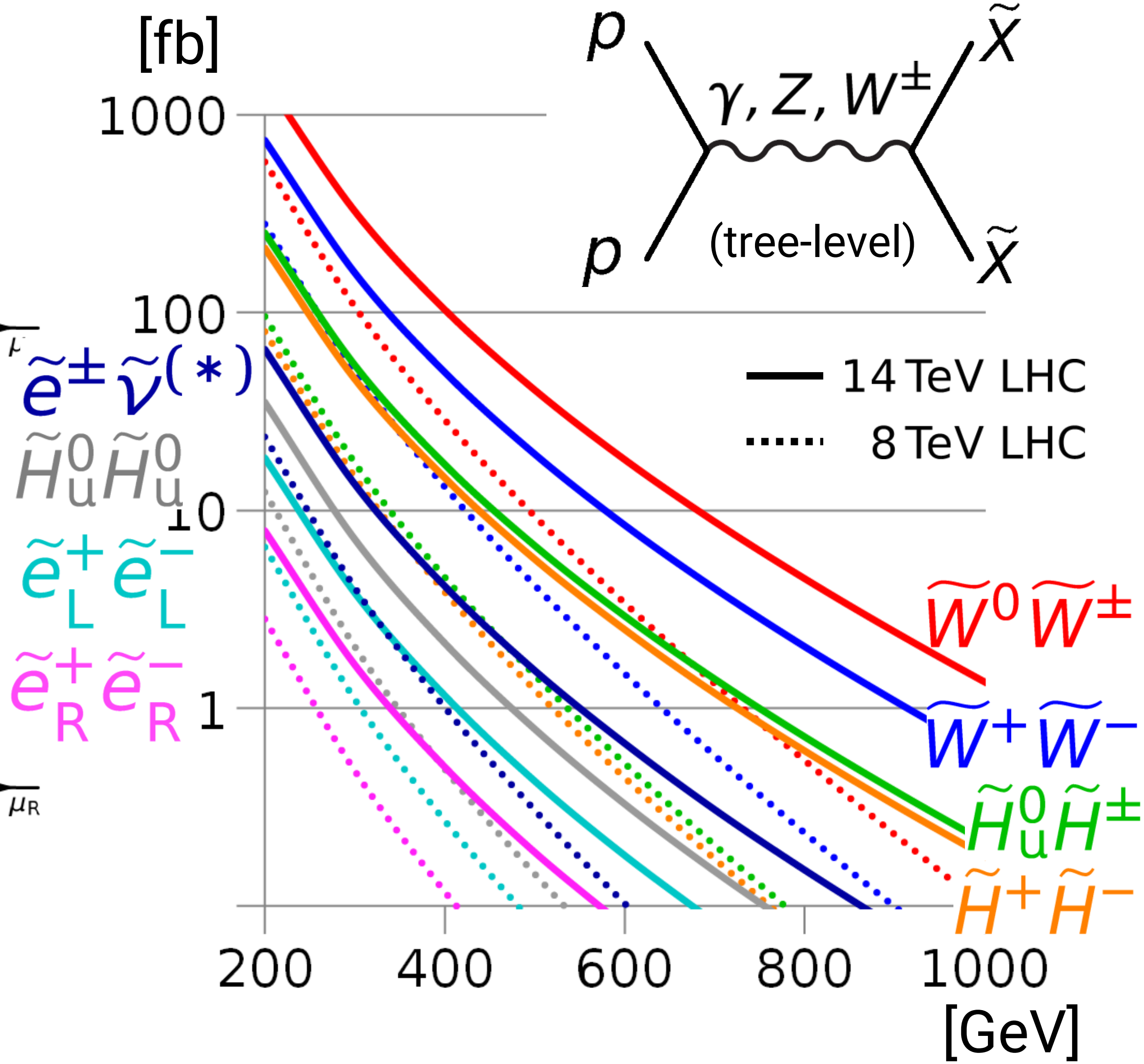
- Overview
- Collider physics
- Dark Matter

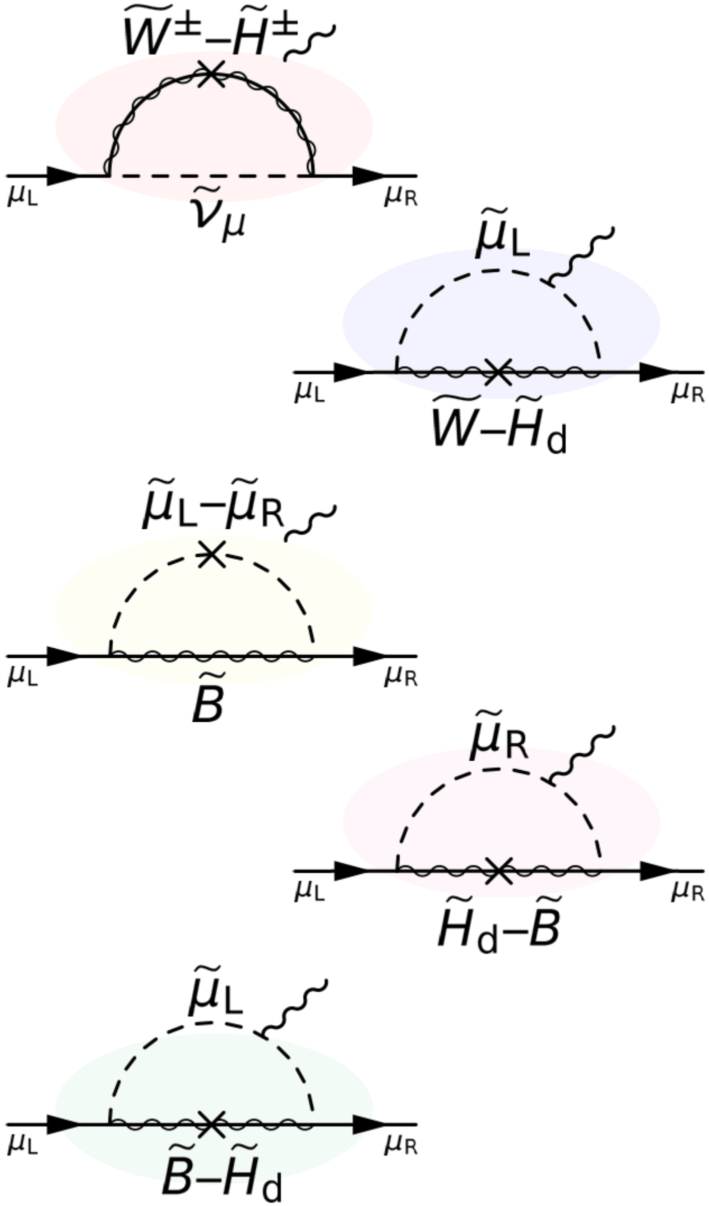
# 3. Discussion for each scenario

- "Chargino" scenario: multi-lepton signature is promising.
- "Pure-bino" scenario: di-lepton, but production not sufficient.
- "BHR" or "BHL": multi-tau, combined with direct detections.

# What can we expect at the LHC?



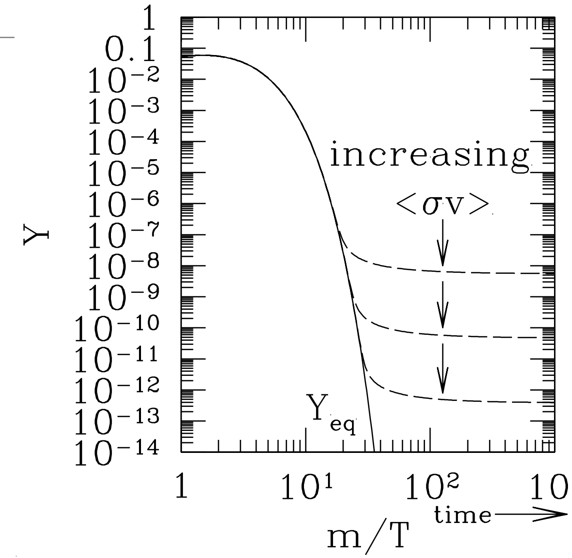






- $(g-2)_\mu$ always requires $\widetilde{\chi}^0$
  → good DM candidate!

- Relic Density?
  → depends on thermal history of Univ.
  - too much → some mechanism to reduce.
  - too little → late production or other DM.

→ Let's discuss simplest case!




- Simplest $\tilde{\chi}^0$-DM scenario
  - DM was in thermal equilibrium → freeze-out.

    $\cdots\ \langle\sigma v\rangle_{\mathrm{DM\,DM\to any}} \sim 3\times10^{-26}\,\mathrm{cm^3/s}$
  - No other component of DM.
- If $\tilde{\chi}^0$ is almost...
  - pure-Bino → almost no interaction → over-abundant.
  - pure-Higgsino → $m_{\mathrm{LSP}}$ ~ 1TeV for correct abundance.
  - pure-Wino → $m_{\mathrm{LSP}}$ ~ 2.5TeV for correct abundance.
- Possibilities:
  - Bino-like + some mechanism to reduce the relic density
    (≲ 500GeV)
  - Higgsino DM, or Bino−Higgsino mixed DM ("well-tempered scenario")
    (~1TeV) (≲ 1TeV)
  - Bino−Wino mixed DM.
    (≲ 2.5TeV)

Figure from Gelmini and Gondolo, 1009.3690

- Simplest $\tilde{\chi}^0$-DM scenario

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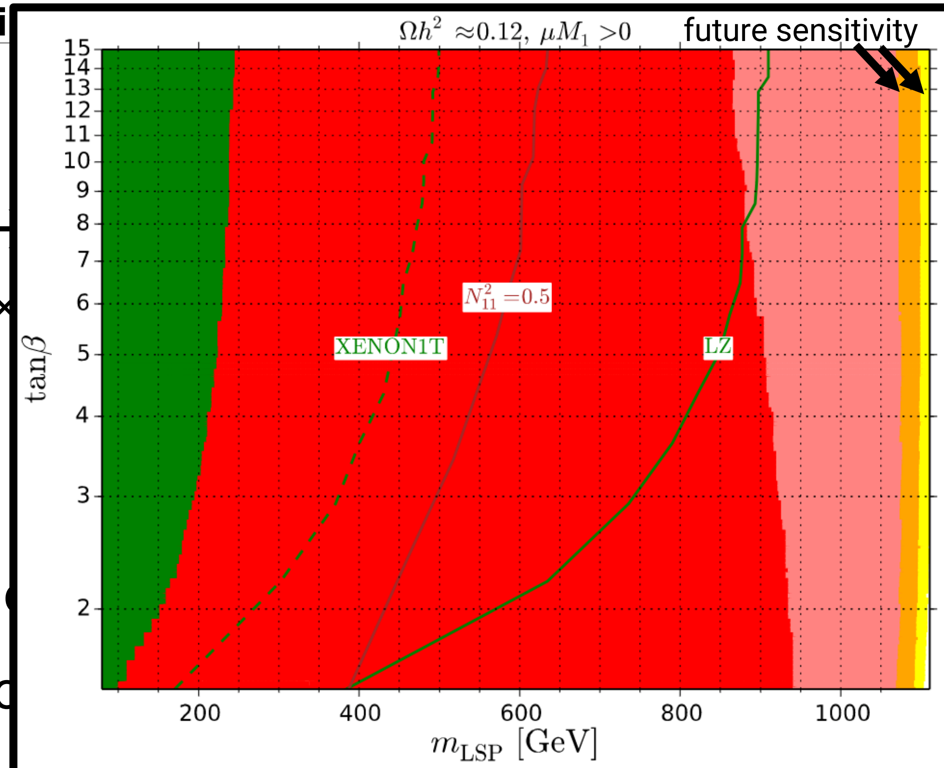
> ➢ **Bino−slepton co-annihilation**
> ($m_{\tilde{B}} \simeq m_{\tilde{l}}$)
>
> ➢ *H*- or *Z*-resonance ("funnel")
> ($m_{\tilde{B}} = m_H/2$ or $m_Z/2$)
>
> ➢ MSSM4G
> Abdullah, Feng [1510.06089],
> Abdullah, Feng, SI, Lillard [1608.00283]




➢ pure-Wino → $m_{\rm LS}$ 5TeV for correct abundance.

- Possibilities:

➢ Bino-like + some mechanism to reduce the relic density
(≲ 500GeV)

➢ Higgsino DM, or Bino−Higgsino mixed DM ("well-tempered scenario")
(~1TeV)

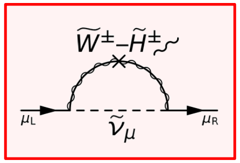
**almost excluded by XENON1T**

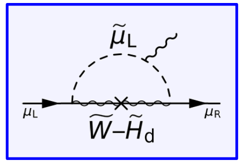
Badziak, Olechowski, Szczerbiak [1701.05869]

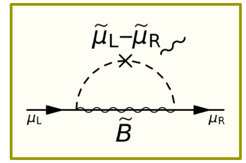
➢ Bino−Wino mixed DM.

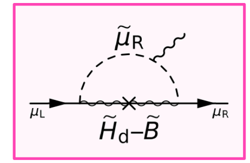
**theoretically not nice**

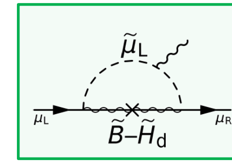
Figure from Ge... and Gondolo, 1009.3690

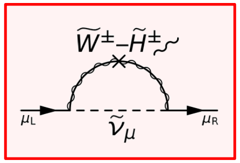
$a_\mu^{\mathrm{SUSY}} \simeq$ 

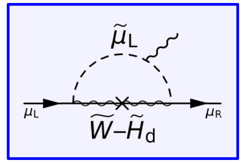
$+$ 

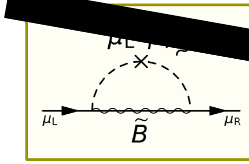
$+$ 

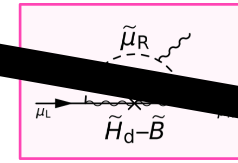
$+$ 

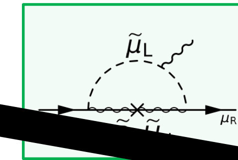
$+$ 


$a_\mu^{\rm SUSY} \simeq$ 

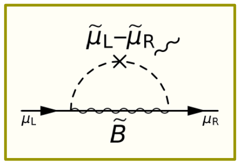
\+ 

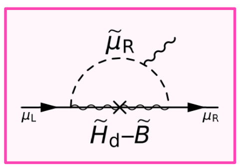
\+ 

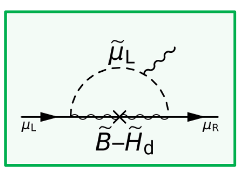
\+ 

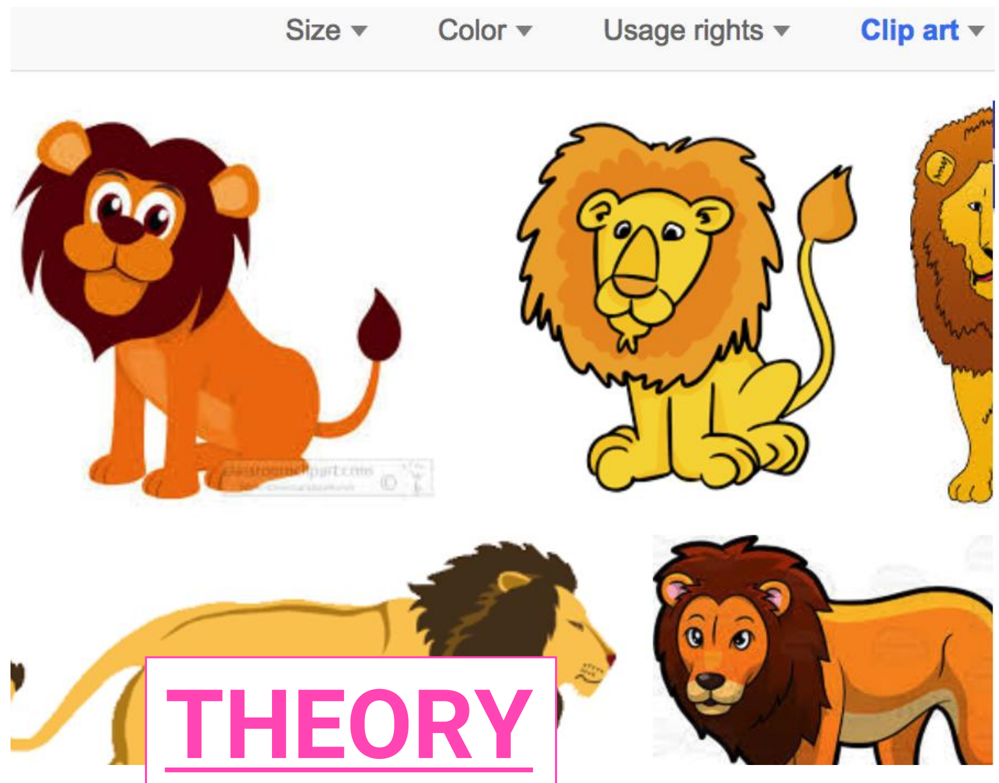
\+ 

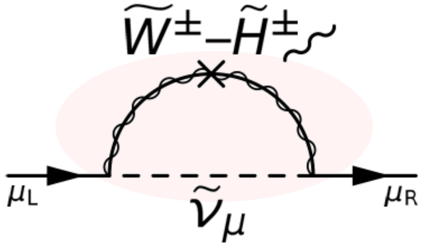

$\sim 0$

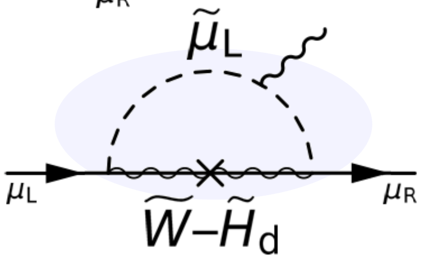
$a_\mu^{\rm SUSY} \simeq$ 

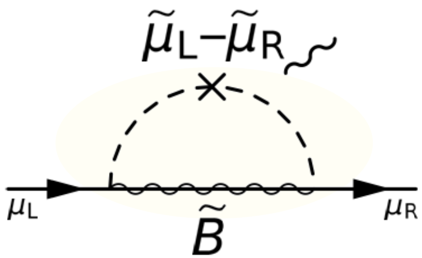

$a_\mu^{\rm SUSY} \simeq$ 

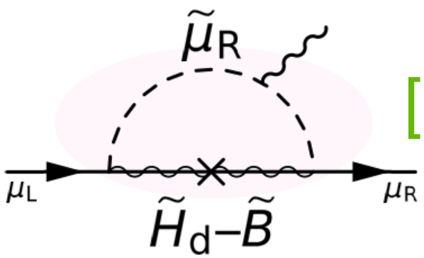

$a_\mu^{\rm SUSY} \simeq$ 

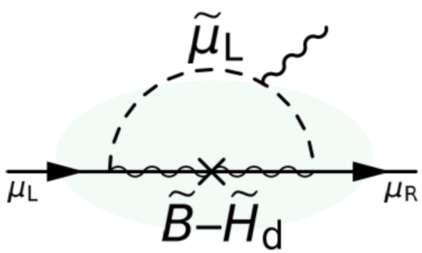






[C] $\frac{g_2^2 m_\mu^2}{8\pi^2}\,\frac{M_2\mu\tan\beta}{m^4_{[illegible]}}\cdot F_a\left(\frac{M_2}{m_{[illegible]}},\frac{\mu}{m_{[illegible]}}\right)$

**tend to be large/dominant**

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[B] $\frac{g_Y^2 m_\mu^2}{8\pi^2}$ … $\frac{m_{\widetilde{\mu}_R}}{M_1}$

**μ-enhancement**

[BHR] $-\frac{g_Y^2 m_\mu^2}{8\pi^2}$ … $\left(\frac{M_1}{m_{\widetilde{\mu}_R}},\frac{\mu}{m_{\widetilde{\mu}_R}}\right)$

**negative**

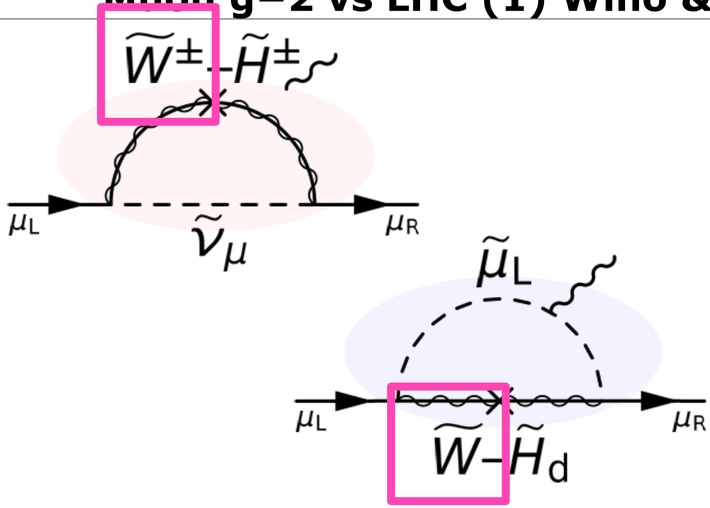
[BHL] $\frac{g_Y^2 m_\mu^2}{16\pi^2}$ … $\frac{\mu}{m_{\widetilde{\mu}_L}}$

**nothing special**

$F_a, F_b$ are loop functions and positive.

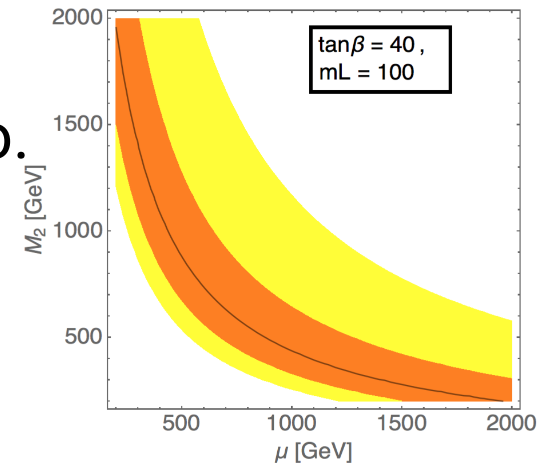
$$F_a(x,y)=\frac{1}{2}\frac{C_1(x^2)-C_1(y^2)}{x^2-y^2},\qquad F_b(x,y)=-\frac{1}{2}\frac{N_2(x^2)-N_2(y^2)}{x^2-y^2};$$

$$C_1(x)=\frac{3-4x+x^2+2\log x}{(1-x)^3},\qquad N_2(x)=\frac{1-x^2+2x\log x}{(1-x)^3}.$$



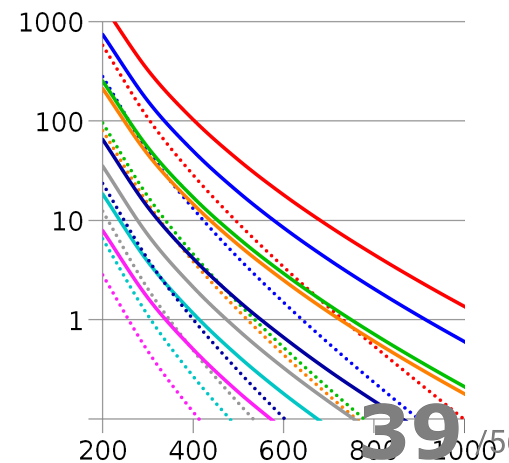

$$\frac{g_2^2 m_\mu^2}{8\pi^2}\frac{M_2\mu\tan\beta}{m_{\widetilde{\nu}_\mu}^4}\cdot F_a\left(\frac{M_2}{m_{\widetilde{\nu}_\mu}},\frac{\mu}{m_{\widetilde{\nu}_\mu}}\right)$$

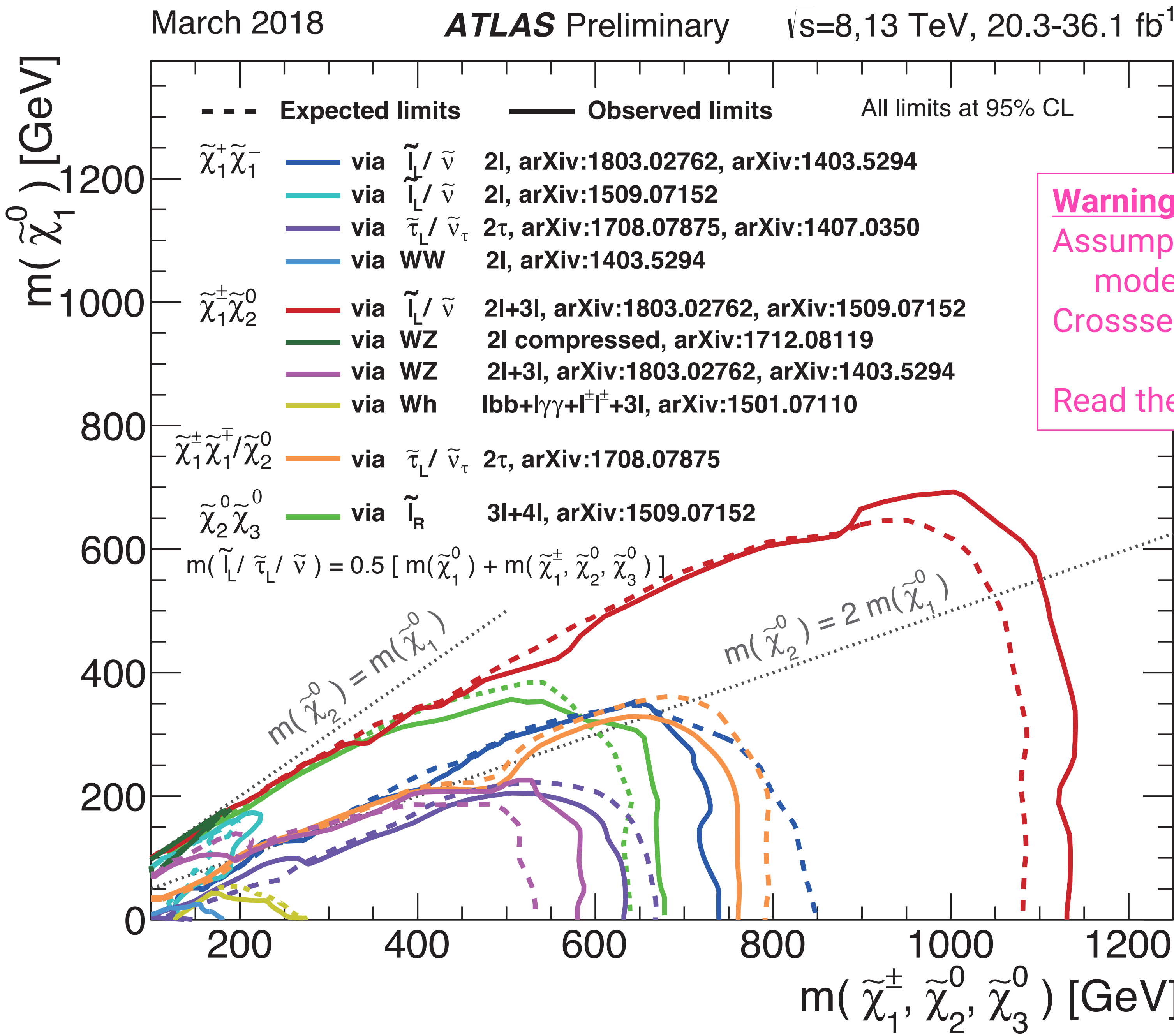
$$-\frac{g_2^2 m_\mu^2}{16\pi^2}\frac{M_2\mu\tan\beta}{m_{\widetilde{\mu}_L}^4}\cdot F_b\left(\frac{M_2}{m_{\widetilde{\mu}_L}},\frac{\mu}{m_{\widetilde{\mu}_L}}\right)$$




- Wino&Higgsino < TeV → chargino scenario.
  - $\propto g_2^2$ → relevant particles $\lesssim 1$ TeV
  - DM: not considered here
    - $(g-2)_\mu \Leftarrow (\widetilde{W},\widetilde{H},\widetilde{\mu}_L)$; DM $\Leftarrow (\widetilde{l}_L,\widetilde{B})$ ... "orthogonal"
    - co-annihilation or resonance may work.
      ($m_{\widetilde{B}} \simeq m_{\widetilde{l}}$) ($m_{\widetilde{B}} \simeq m_Z/2$ or $m_h/2$)
  - LHC: Wino pair-production

$$\sigma(pp\to\widetilde{W}\widetilde{W})_{14\,\mathrm{TeV}} \sim 50\,\mathrm{fb}\ \ @\ m_{\widetilde{W}} = 500\,\mathrm{GeV}$$
$$1.5\,\mathrm{fb}\qquad\qquad 1\,\mathrm{TeV}$$





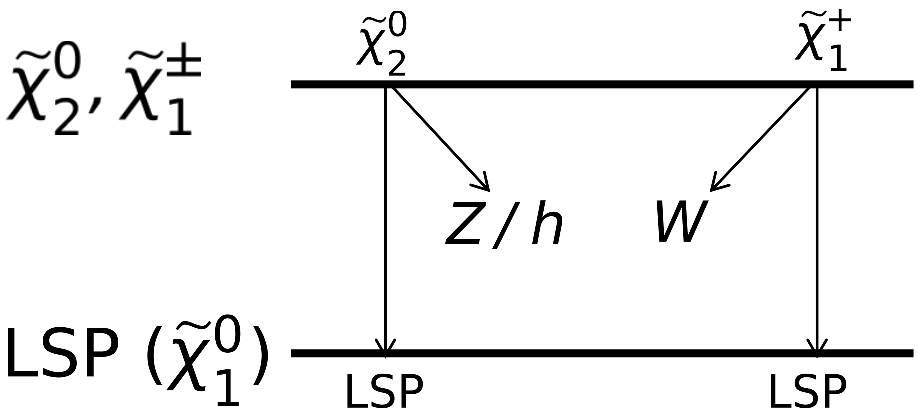

**Warning**

Assumptions for model simplification!

Crosssection is for $pp \to \widetilde{W}^0\widetilde{W}^+$!!

Read the papers!

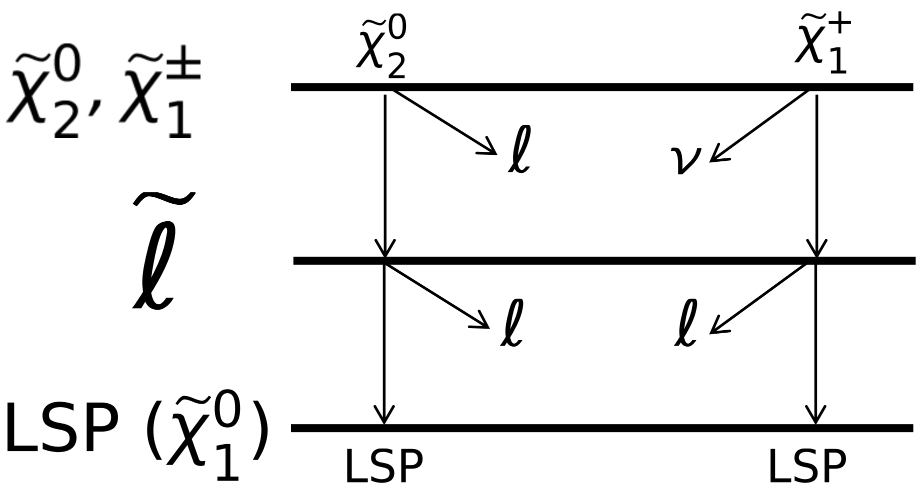
$pp \to \tilde{\chi}^0 \tilde{\chi}^+$ $(\widetilde{W}^0 \widetilde{W}^+$ or $\widetilde{H}^0 \widetilde{H}^+)$; then?




$\tilde{\chi}^0_2 \tilde{\chi}^+_1 \to ZW / hW + \text{mET}$

$(\to 3\ell + \text{mET})$

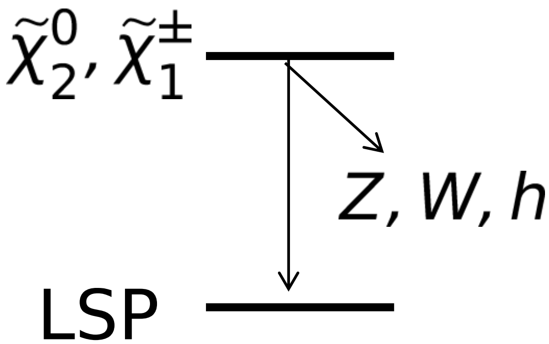
but $Z$-like leptons

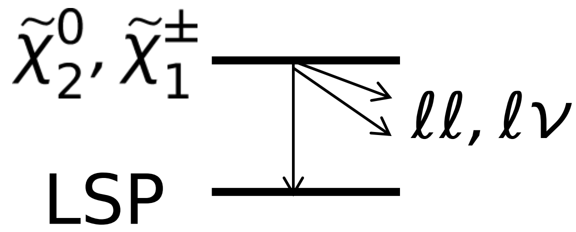


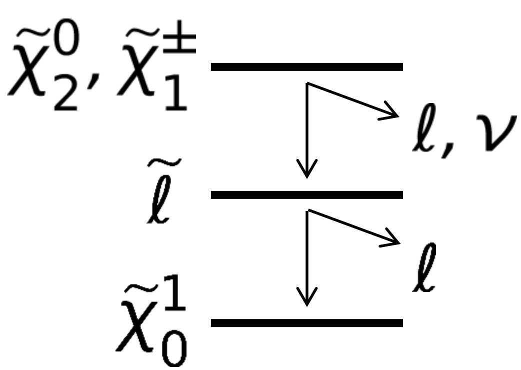

$\tilde{\chi}^0_2 \tilde{\chi}^+_1 \to 3\ell + \text{mET}$

**$Z$-unlike**

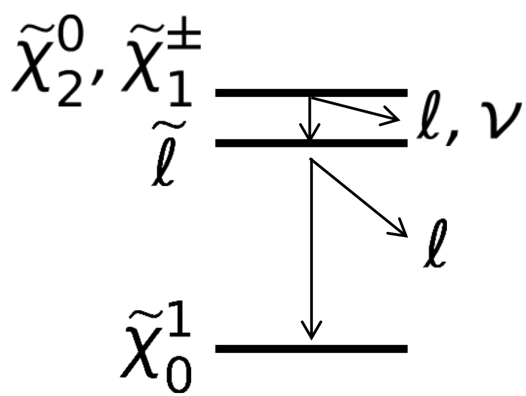
$pp \to \tilde{\chi}^0 \tilde{\chi}^+$ ($\widetilde{W}^0 \widetilde{W}^+$ or $\widetilde{H}^0 \widetilde{H}^+$) ; then?



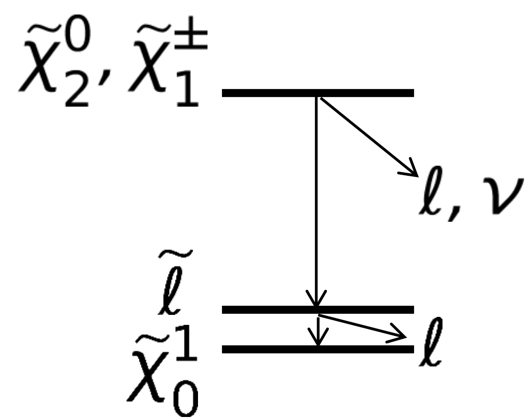




$x_\ell \sim 0.5$

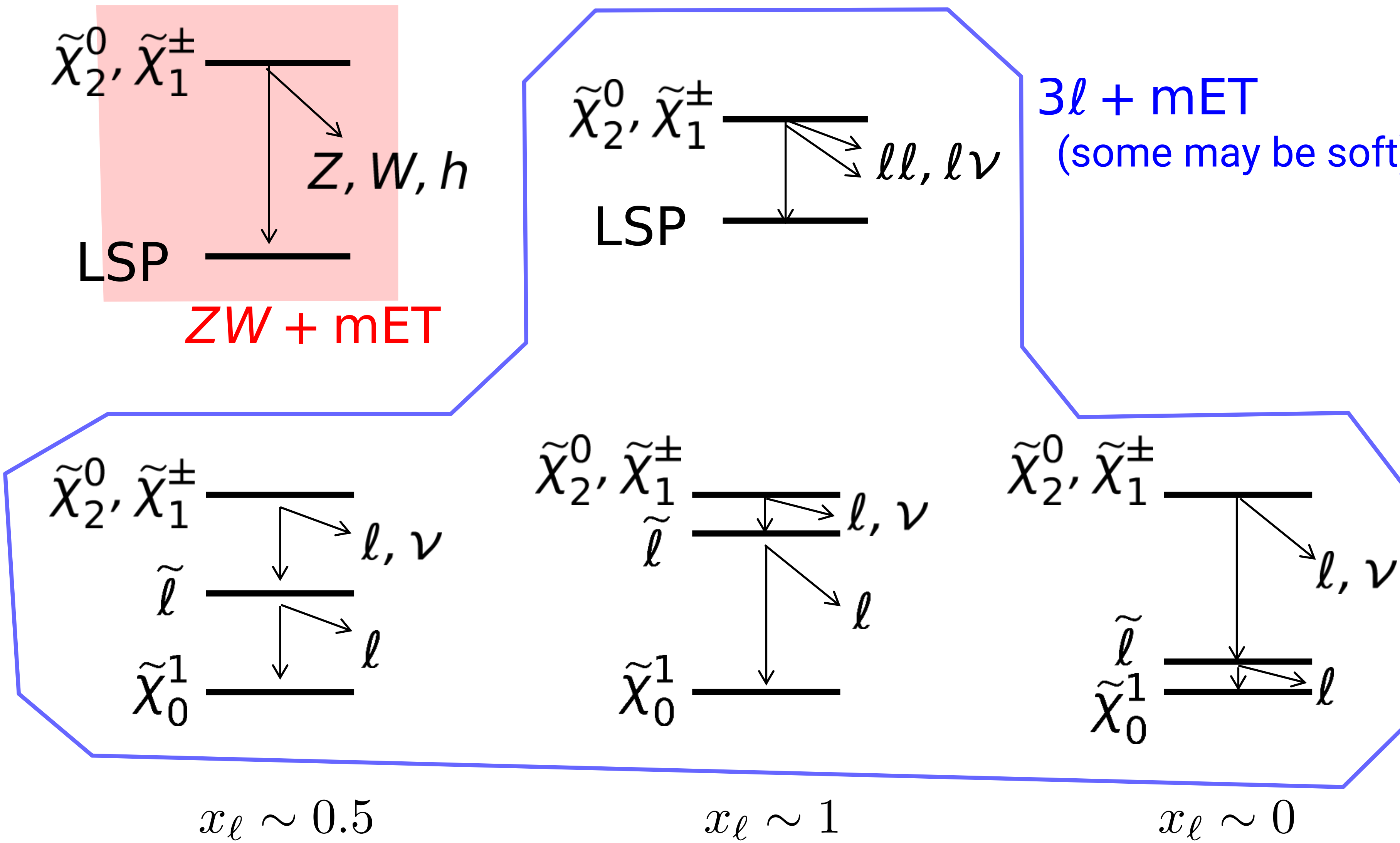


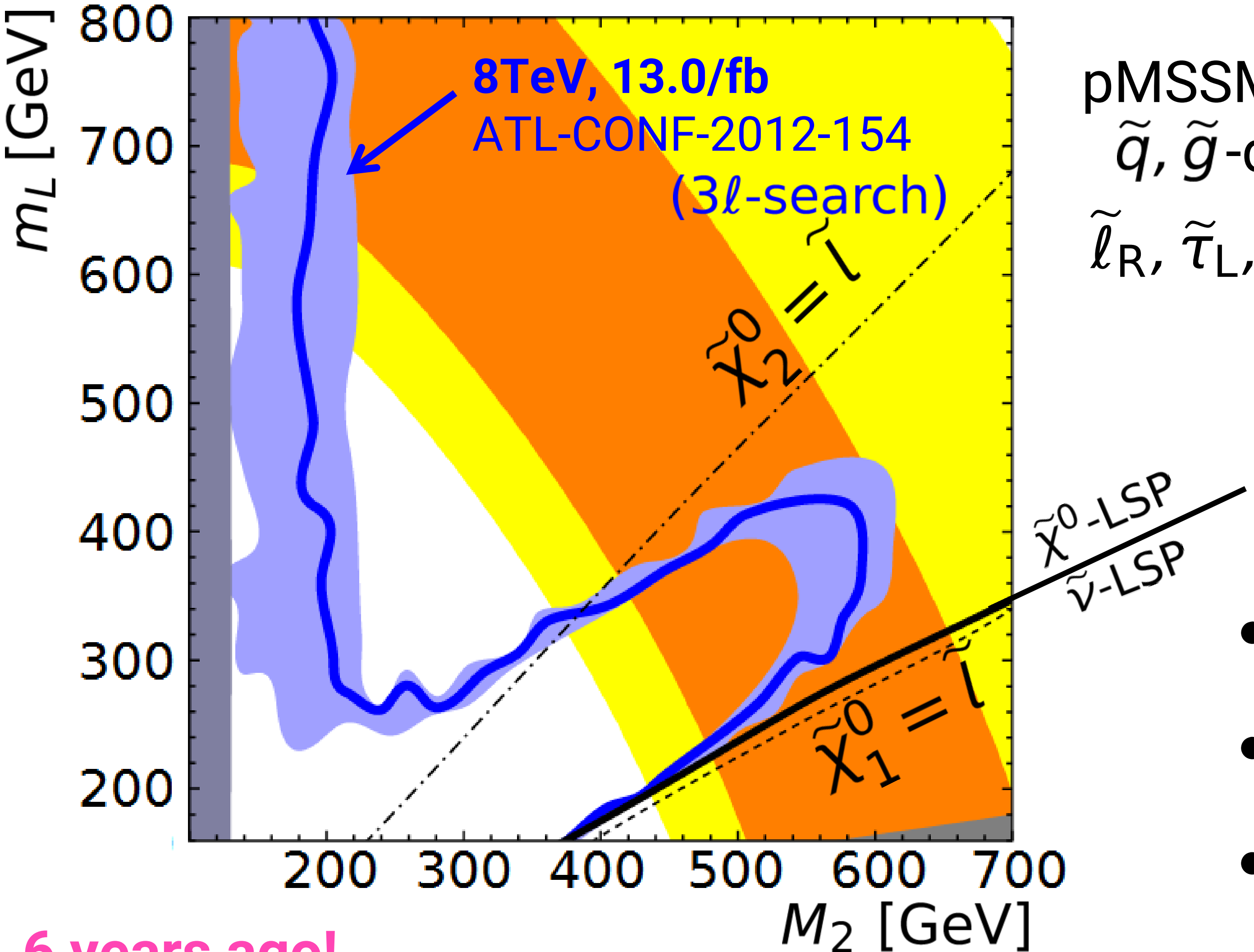

$x_\ell \sim 1$




$x_\ell \sim 0$

$pp \to \tilde{\chi}^0 \tilde{\chi}^+$ ($\widetilde{W}^0 \widetilde{W}^+$ or $\widetilde{H}^0 \widetilde{H}^+$); then?

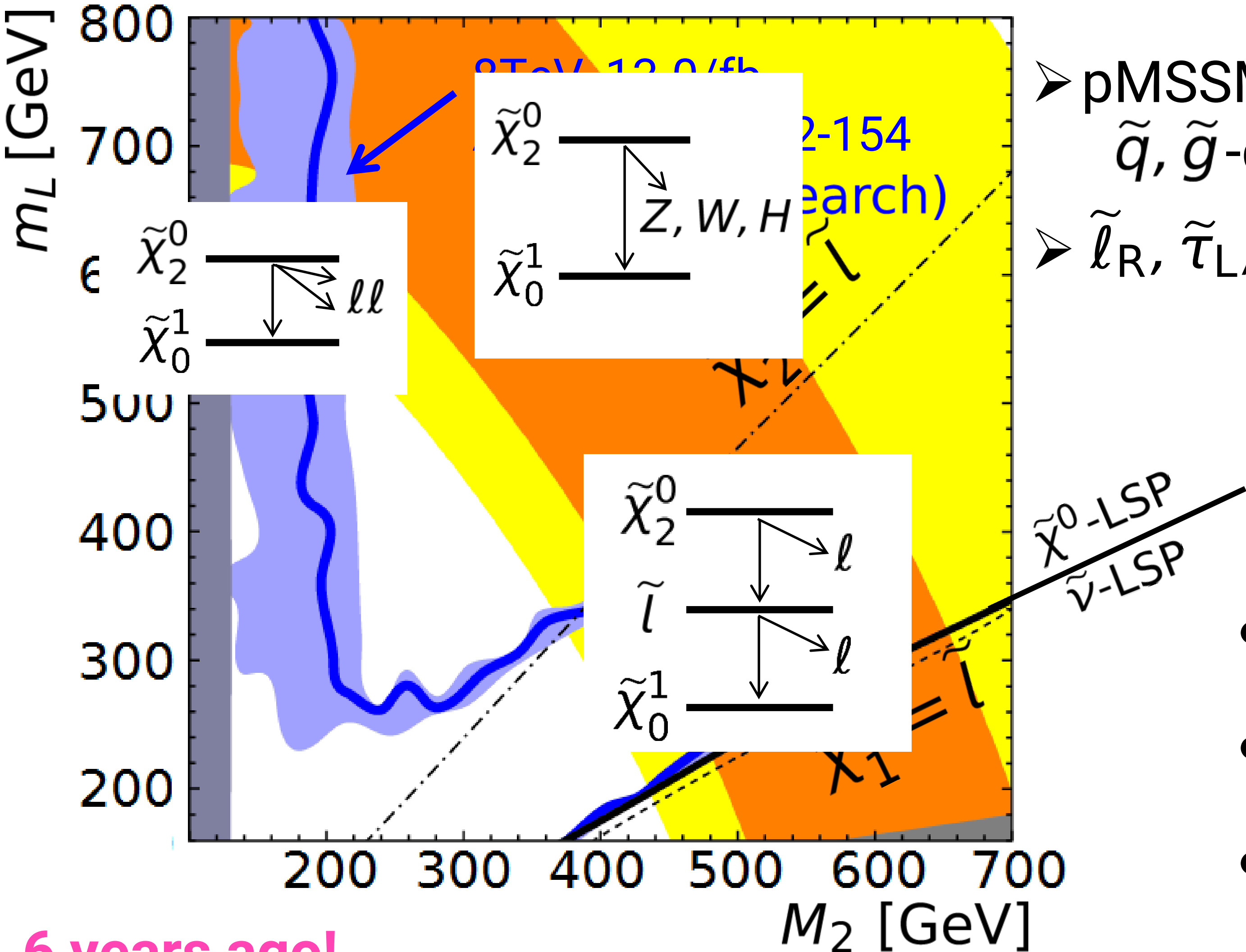





6 years ago!

pMSSM w.
$\tilde{q}, \tilde{g}$-decoupled.
$\tilde{\ell}_R, \tilde{\tau}_L, \tilde{\tau}_R$ also decoupled.

- $\tan\beta = 40$
- $M_1 = M_2/2$
- $\mu = M_2$

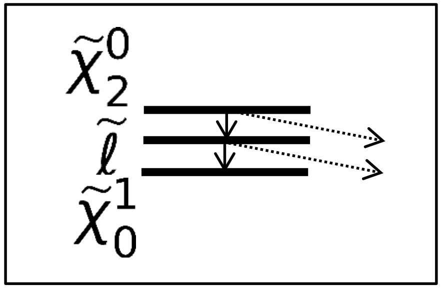


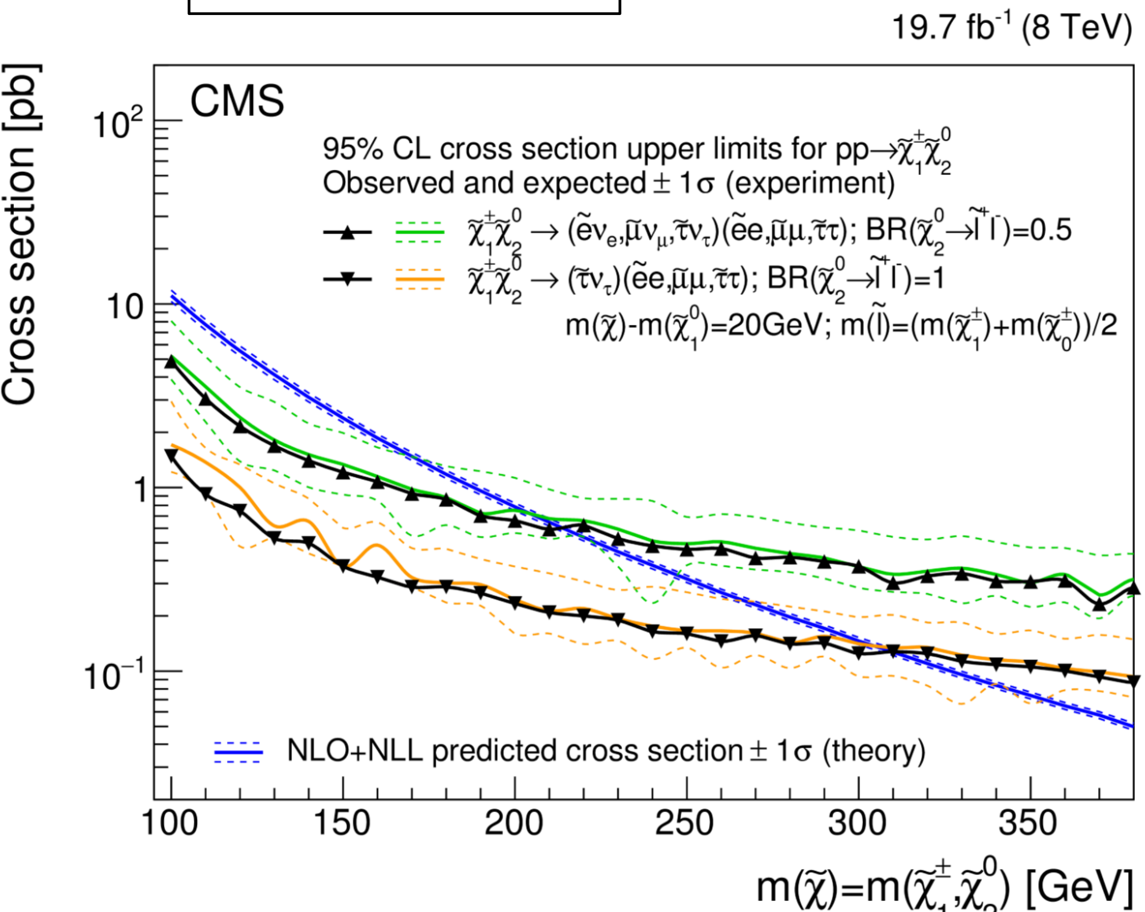

6 years ago!

- pMSSM w. $\tilde{q}, \tilde{g}$-decoupled.
- $\tilde{\ell}_R, \tilde{\tau}_L, \tilde{\tau}_R$ also decoupled.

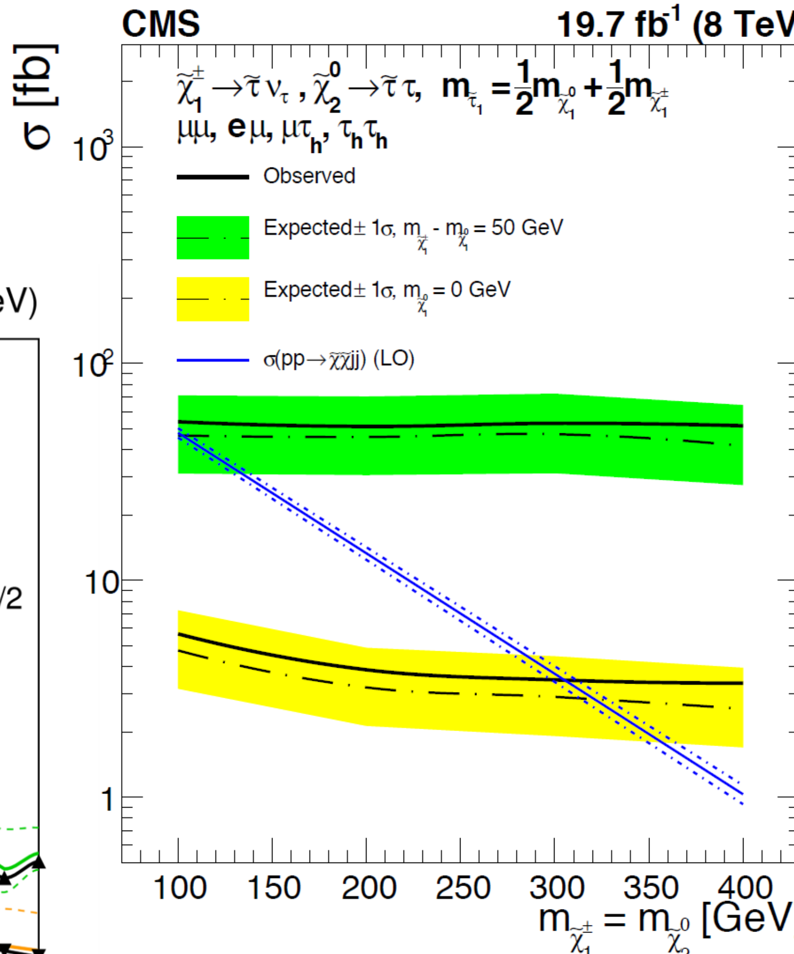
- $\tan\beta = 40$
- $M_1 = M_2/2$
- $\mu = M_2$

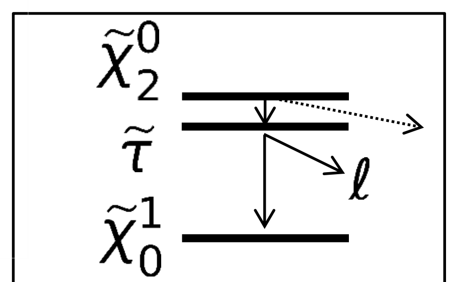
- ISR + 2-lepton [1512.08002]



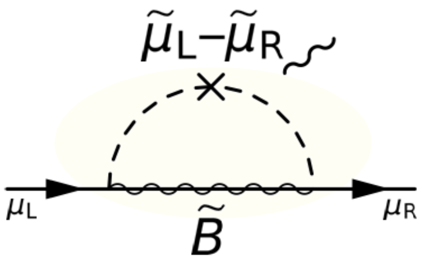



- VBF + 2-lepton [1508.07628]



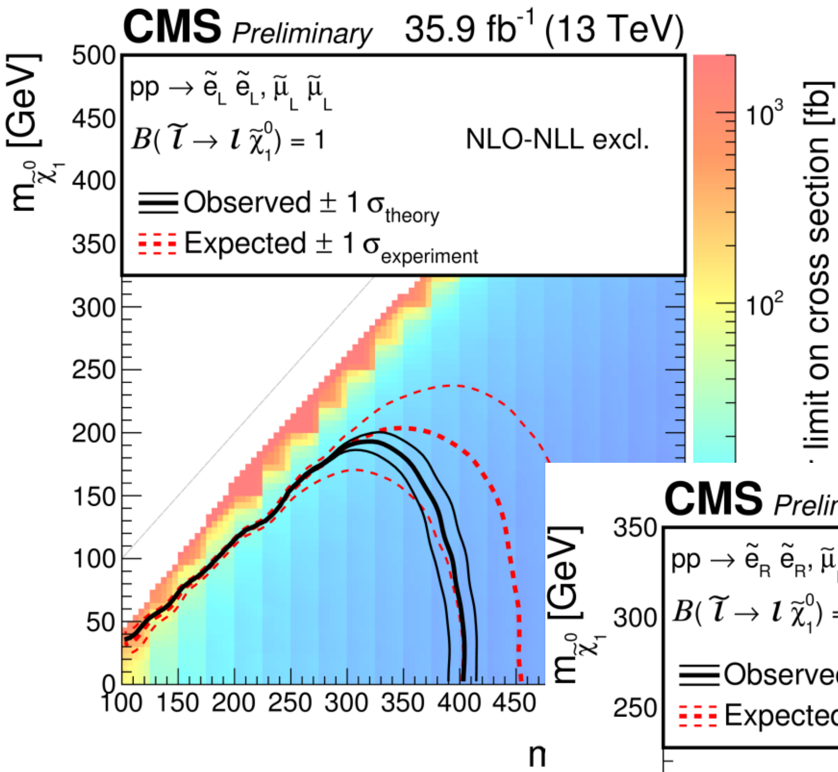
- Higgsino > TeV → pure-Bino scenario.
  - $\mu$-enhancement v.s. vacuum stability
  - DM: not considered here ("orthogonal")
    - co-annihilation or resonance may work.

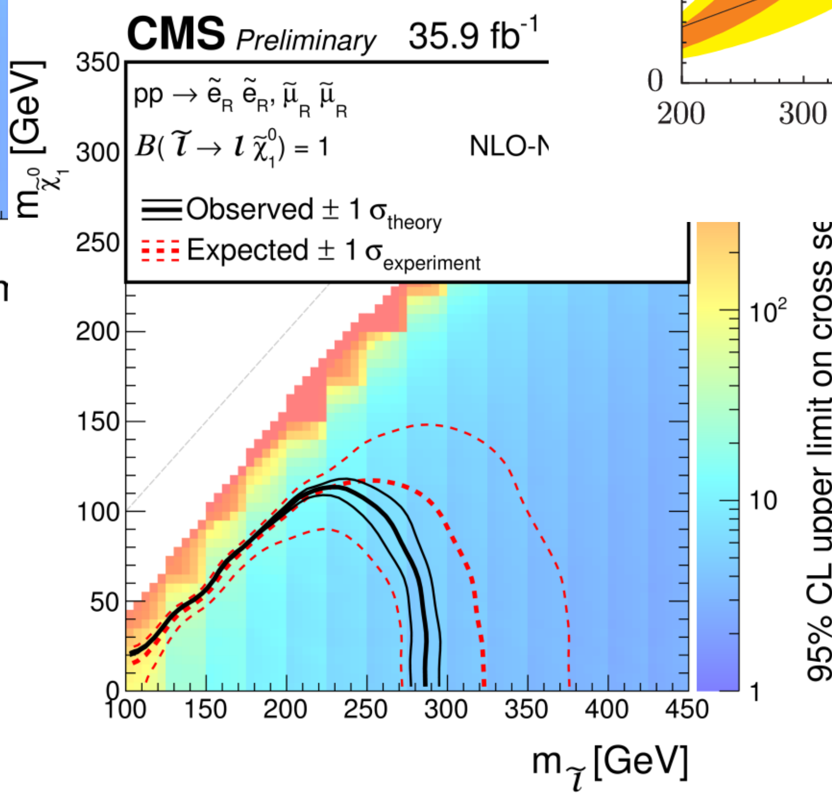


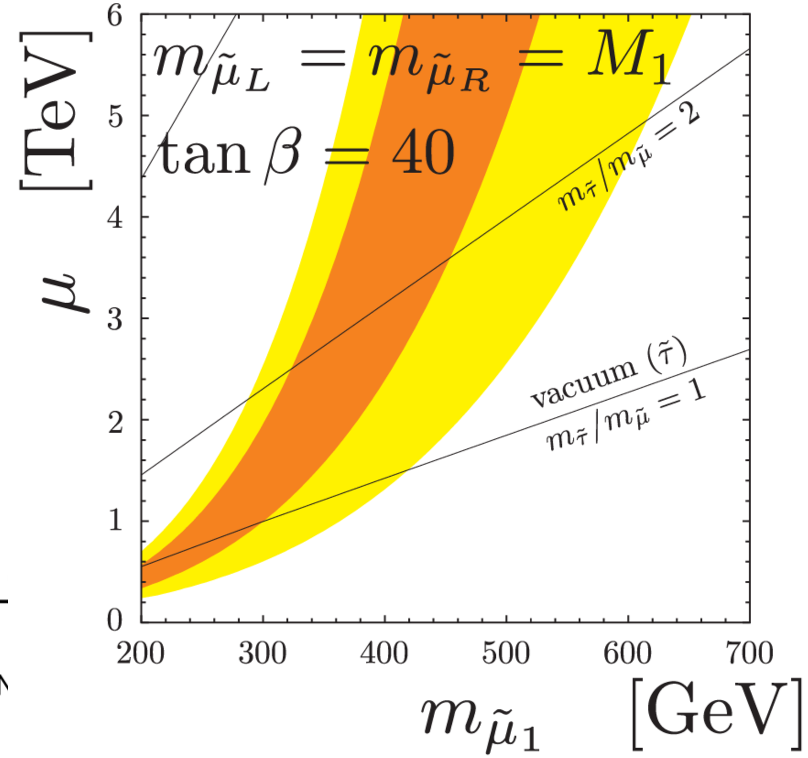

[B] $\frac{g_Y^2 m_\mu^2}{8\pi^2}$ **$\mu$-enhancement** $, \frac{m_{\tilde\mu_R}}{M_1}\Big)$

  - LHC: only slepton pair-production
    - small cross section: 0.47 (0.18) fb for 500 GeV $\tilde\ell_L$ ($\tilde\ell_R$)
    - "di-lepton + missing" signature ... not easy.

# Muon $g-2$ vs LHC (2) Pure-bino contribution results in slepton pair-production

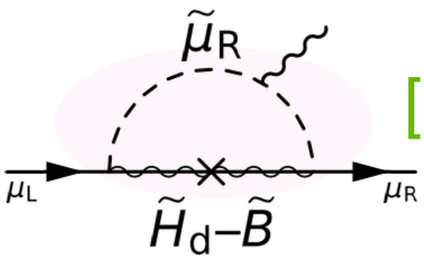


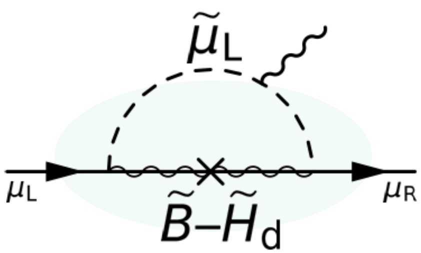

top-right figure from Endo, Hamaguchi, Kitahara, Yoshinaga [1309.3065]

- Wino >> TeV & Higgsino < TeV → BHL or BHR scenario.
  ($\mu > 0$) ($\mu < 0$)
  - $\propto g_Y^2$ → relevant particles $\lesssim 500\,\mathrm{GeV}$
  - LHC: $pp \to \tilde{H}^+\tilde{H}^0, \tilde{H}^+\tilde{H}^-$ "not much, but enough"
  - DM: ~~Bino−Higgsino mixing~~ (excl. by XENON1T), bino−slepton co-annihilation, resonance.
    → future works




[BHR]
$$-\frac{g_Y^2 m_\mu^2}{8\pi^2}\frac{M_1\mu\tan\beta}{m_{\tilde{\mu}_R}^4}\cdot F_b\left(\frac{M_1}{m_{\tilde{\mu}_R}},\frac{\mu}{m_{\tilde{\mu}_R}}\right)$$

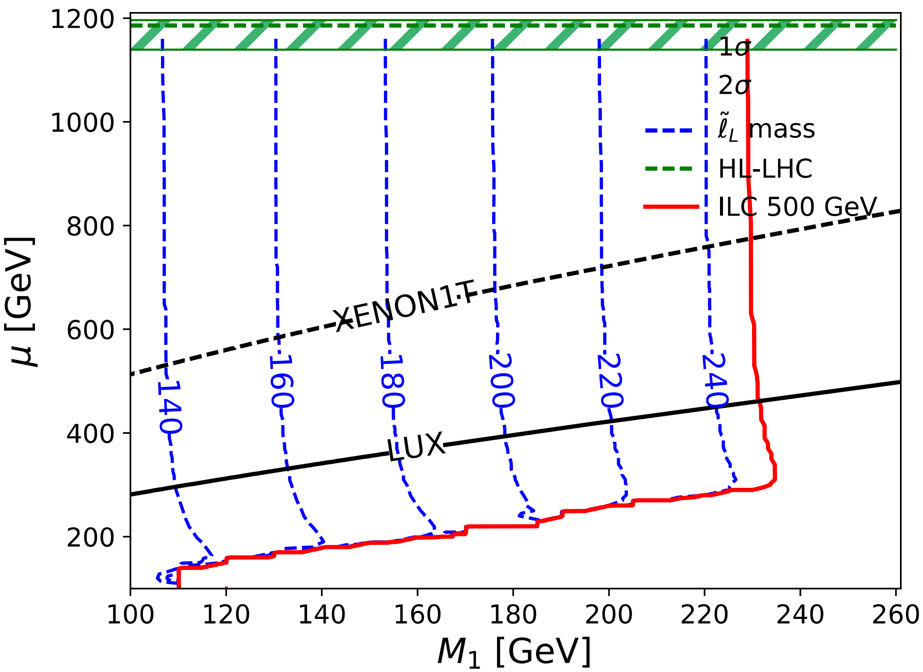
[BHL]
$$\frac{g_Y^2 m_\mu^2}{16\pi^2}\frac{M_1\mu\tan\beta}{m_{\tilde{\mu}_L}^4}\cdot F_b\left(\frac{M_1}{m_{\tilde{\mu}_L}},\frac{\mu}{m_{\tilde{\mu}_L}}\right)$$

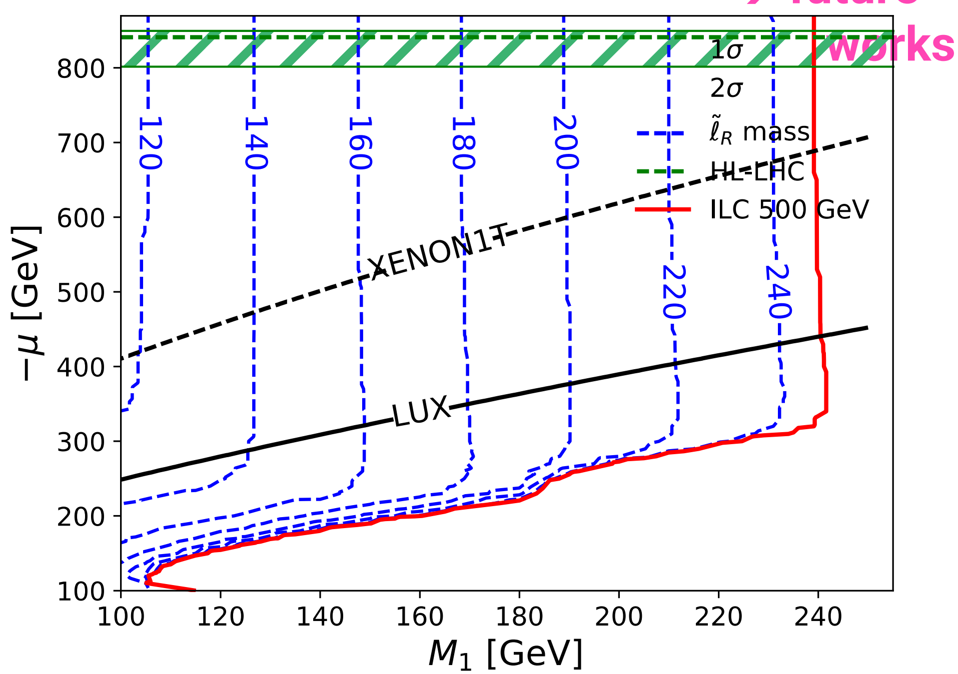


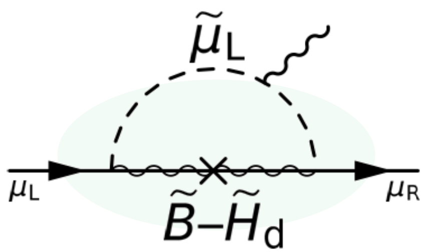

$F_a, F_b$ are loop functions and positive.

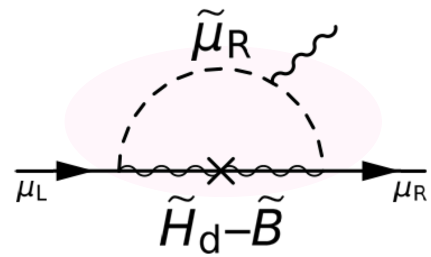
$$F_a(x,y) = \frac{1}{2}\frac{C_1(x^2)-C_1(y^2)}{x^2-y^2}, \qquad F_b(x,y) = -\frac{1}{2}\frac{N_2(x^2)-N_2(y^2)}{x^2-y^2};$$
$$C_1(x) = \frac{3-4x+x^2+2\log x}{(1-x)^3}, \qquad N_2(x) = \frac{1-x^2+2x\log x}{(1-x)^3}.$$

- Wino >> TeV & Higgsino < TeV → BHL or BHR scenario.
  ($\mu > 0$) ($\mu < 0$)
  - $\propto g_Y^2$ → relevant particles $\lesssim 500\,\mathrm{GeV}$
  - LHC: $pp \to \widetilde{H}^+\widetilde{H}^0, \widetilde{H}^+\widetilde{H}^-$ "not much, but enough"
  - DM: ~~Bino–Higgsino mixing~~ (excl. by XENON1T), **bino−slepton co-annihilation**, resonance. → future works



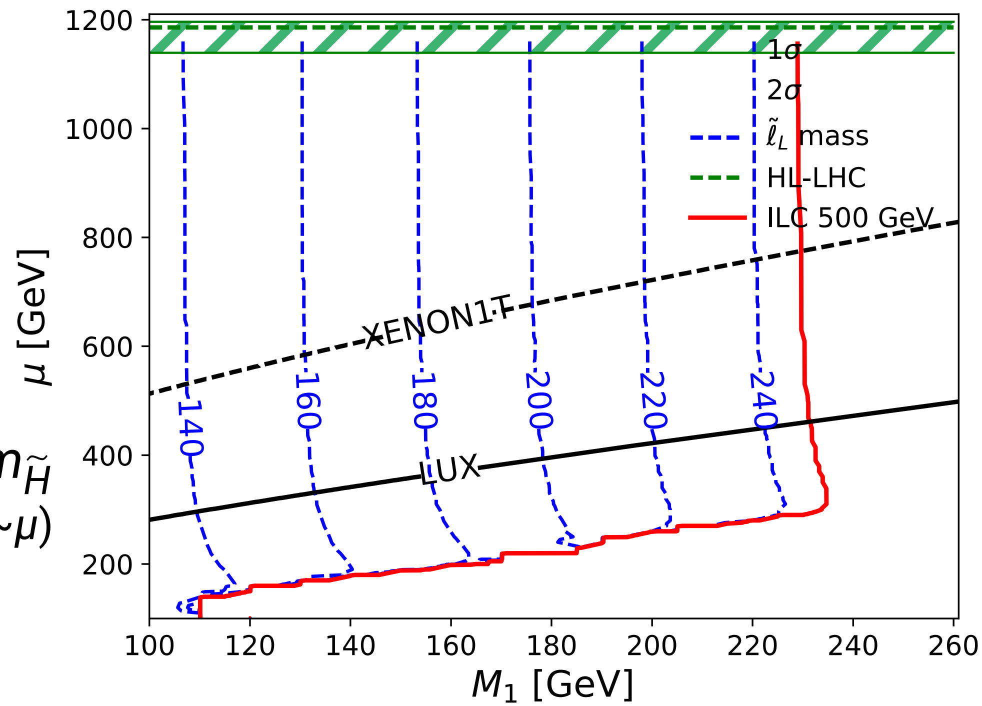



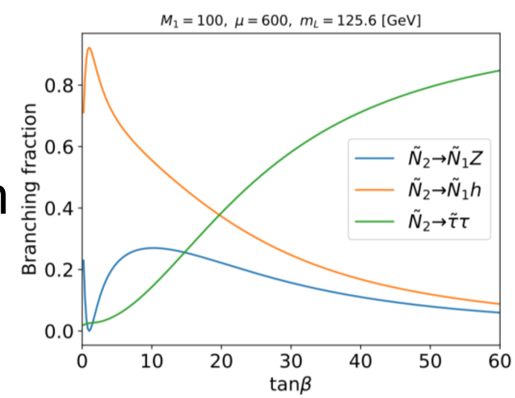
- Bino−slepton (stau) co-annihilation → $m_{\tilde{\nu}_\tau}$ (or $m_{\tilde{\tau}_R}$) $\simeq m_{\tilde{B}}$.
- We assumed:
  - slepton universality,
  - DM density is realized at each point in the plots.

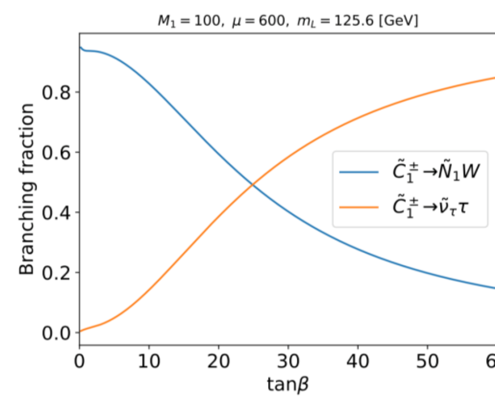
  → $m_{\tilde{B}}$ $(\sim M_1)$ $\simeq$ ($m_{\tilde{\tau}_R}$ or $m_{\tilde{\nu}_\tau}$) $\lesssim m_{\tilde{\mu}} < m_{\tilde{H}}$ $(\sim \mu)$

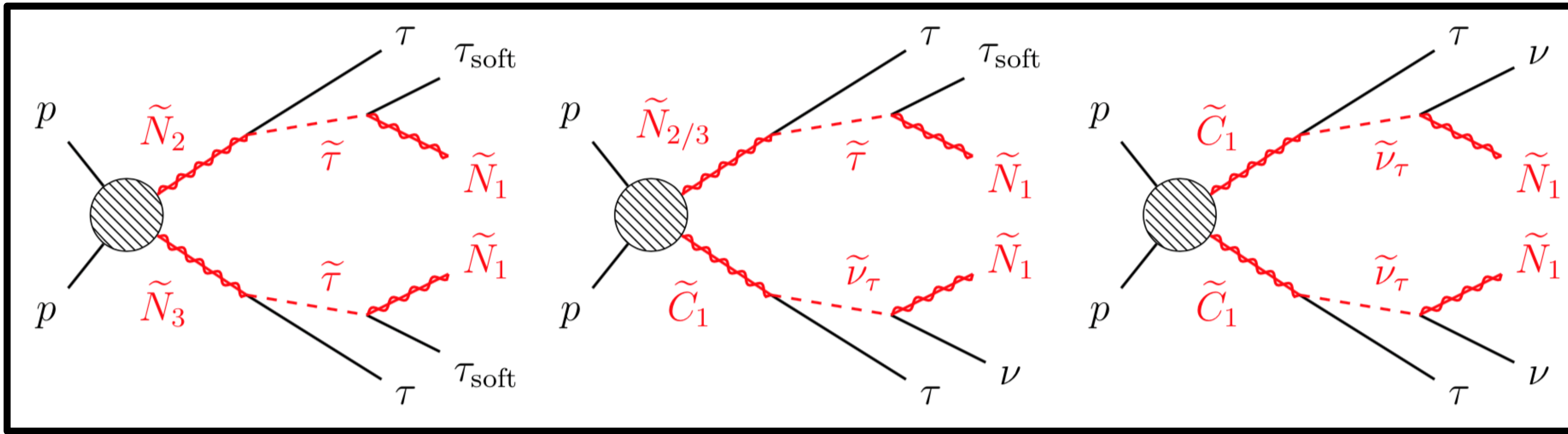


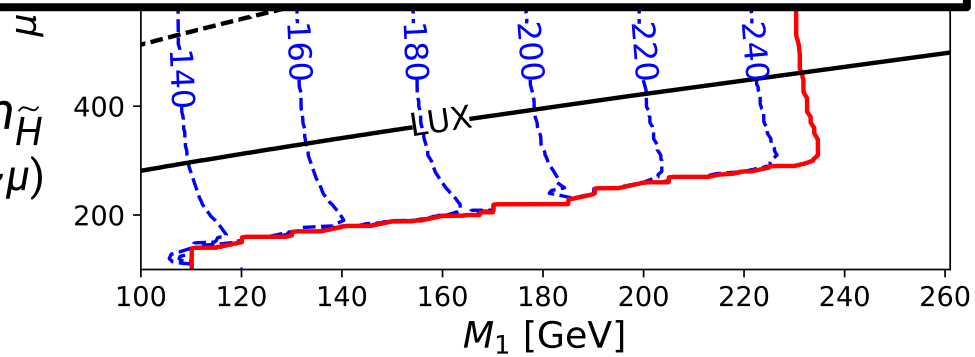

- HL-LHC?
  - $pp \to \tilde{H}^+\tilde{H}^0,\ \tilde{H}^+\tilde{H}^-$
  - $\tilde{H}^0 \to \tau\tilde{\tau},\ \ \tilde{H}^+ \to \tau\tilde{\nu}_\tau$ due to tan$\beta$ + mass spectrum

  → multi-tau signature





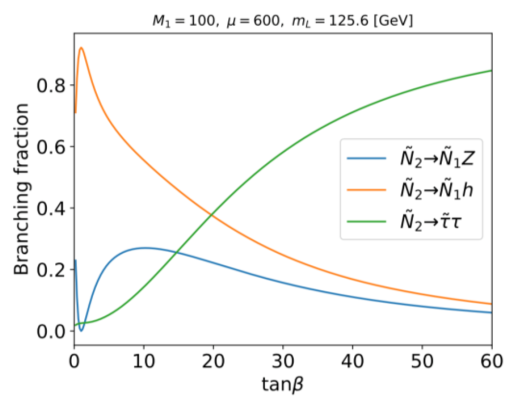



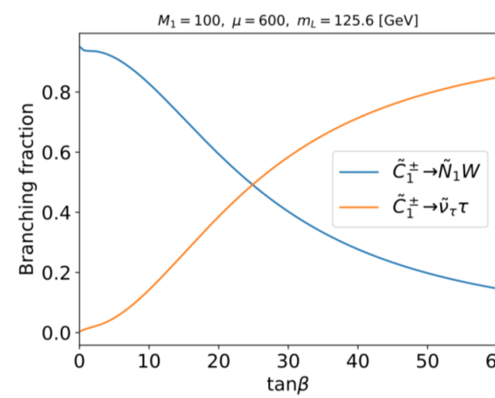

→ $m_{\tilde{B}} \simeq (m_{\tilde{\tau}_R}$ or $m_{\tilde{\nu}_\tau}) \lesssim m_{\tilde{\mu}} < m_{\tilde{H}}$
$(\sim M_1)$ $(\sim \mu)$

■ HL-LHC?

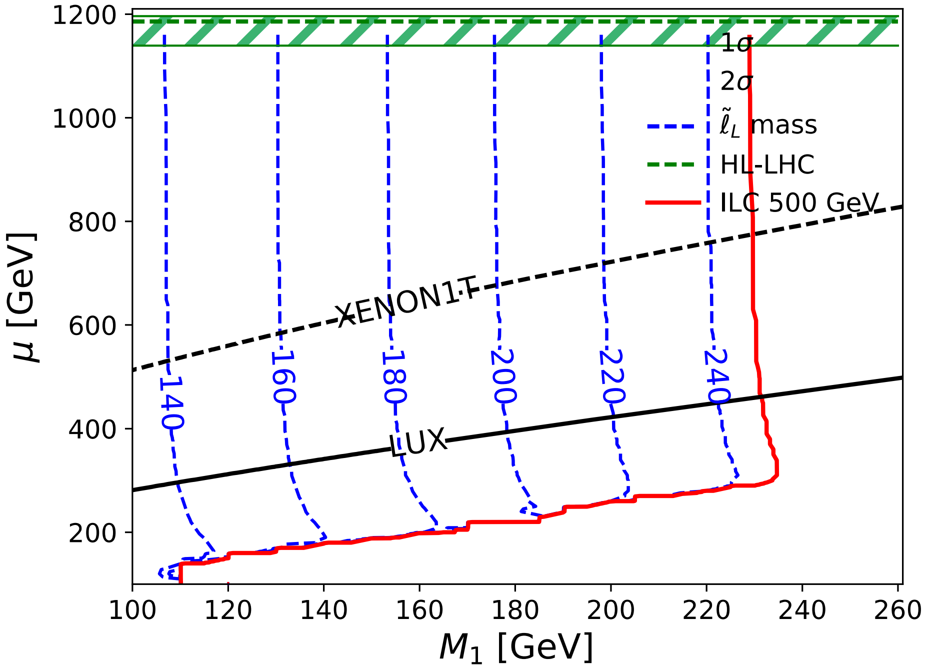
- $pp \to \tilde{H}^+\tilde{H}^0,\ \tilde{H}^+\tilde{H}^-$
- $\tilde{H}^0 \to \tau\tilde{\tau},\ \ \tilde{H}^+ \to \tau\tilde{\nu}_\tau$
  due to tan$\beta$ + mass spectrum

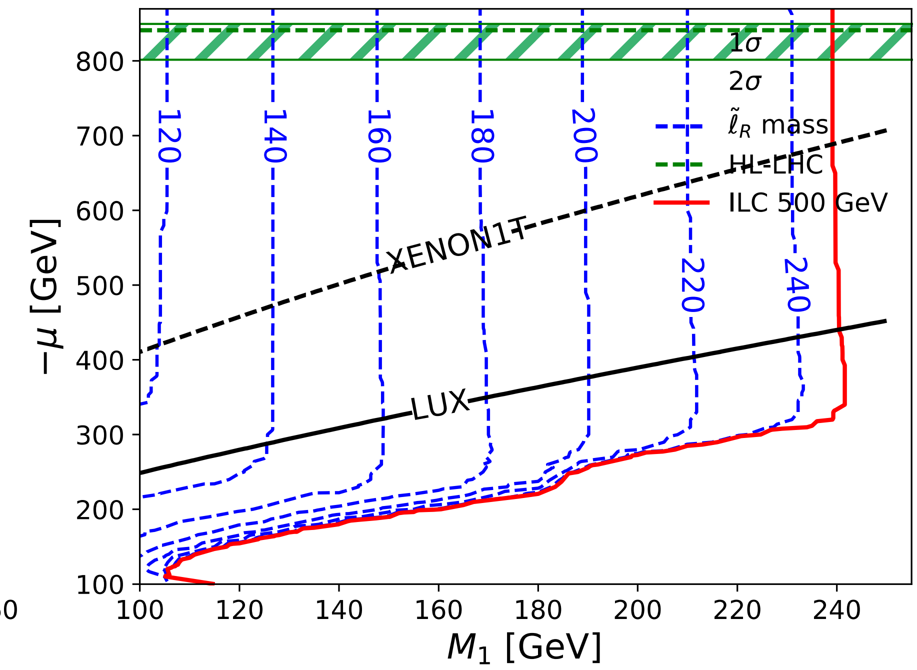
→ multi-tau signature
"2τ (+ soft) + missing"

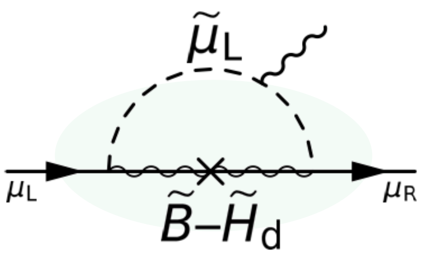


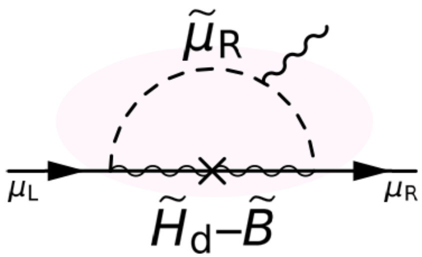



- Wino >> TeV & Higgsino < TeV → BHL ($\mu > 0$) or BHR ($\mu < 0$) scenario.
  - DM: Bino−stau co-annihilation → $m_{\tilde{B}} \simeq (m_{\tilde{\tau}_R}$ or $m_{\tilde{\nu}_\tau}) \lesssim m_{\tilde{\mu}} < m_{\tilde{H}}$
  - DM has small Higgsino component → **LUX/XENON1T** constraint.
  - LHC: $pp \to \tilde{H}^+\tilde{H}^0, \tilde{H}^+\tilde{H}^-;\ \tilde{H} \to \tau + \cdots$ **"2τ+missing"** signature

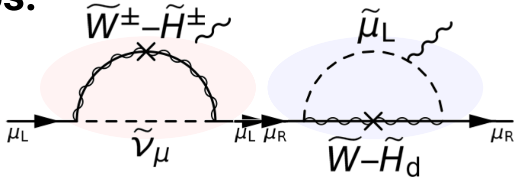
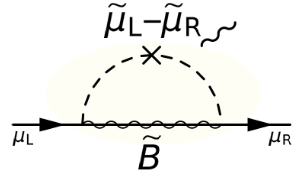
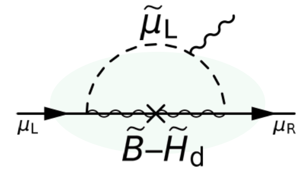
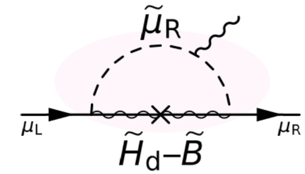


**Simplest scenarios:**

| "chargino" | "pure-Bino" | BHL / BHR |
|---|---|---|
|  |  |   |
| $\widetilde{W}, \widetilde{H} \lesssim$ TeV | $\widetilde{H} >$ TeV | $\widetilde{H} <$ TeV |



**DM:**

"orthogonal" (determined by $m_{\widetilde{B}}$)

... coannihilation / resonance

($m_{\widetilde{B}} \simeq m_{\tilde{l}}$) ($m_{\widetilde{B}} \simeq m_Z/2$ or $m_h/2$)

coannihilation / resonance

we discussed

future work

**Collider:**

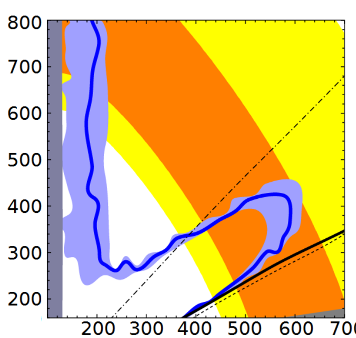
multi-lepton
→ promising

**("stay tuned!")**

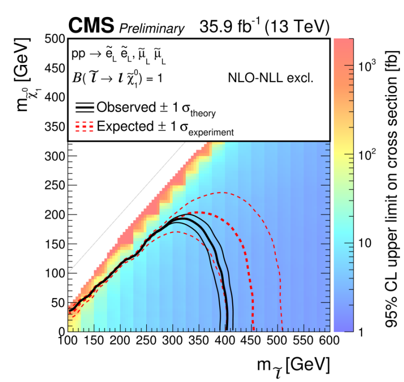
di-lepton
→ **difficult @LHC**

Higgsino → multi-tau

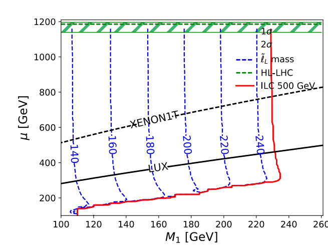
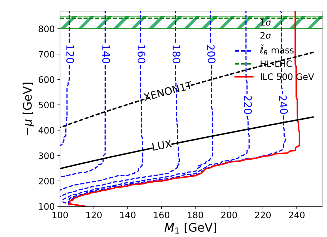
"**covered@HL-LHC if we seriously consider the relic density**"



Endo, Hamaguchi, ***SI***, Yoshinaga [1303.4256]




(cf. Endo, Hamaguchi, Kitahara, Yoshinaga [1309.3065])


Endo, Hamaguchi, ***SI***, Yanagi [1704.05287]