



# ILC measurement of SUSY $(g-2)_\mu$

[Sho IWAMOTO](#) (岩本 祥)

[Kavli IPMU](#), the University of Tokyo, JAPAN

22nd Jul. 2014

[ILC Summer Camp 2014](#) @ Sekigane Onsen

Reference)

M. Endo, K. Hamaguchi, S.I. T. Kitahara, T. Moroi [[1310.4496](#)].

昨日

ナイトセッションで  
やった話です



もっかい  
やります

自己

紹介

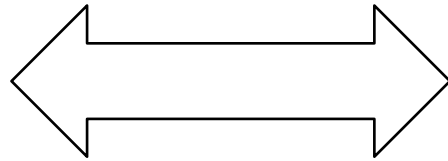
# hep-ph

- 加速器実験

- **LHC**

- ILC

- 宇宙線観測



- 標準模型

- **SUSY**

- 暗黒物質の  
色々なモデル

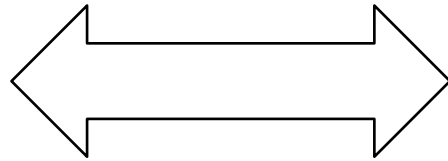
# hep-ph

- 加速器実験

- LHC

- ILC

- 宇宙線観測



- 標準模型

- SUSY

- 暗黒物質の  
色々なモデル



# IPMU

↓ 2014秋

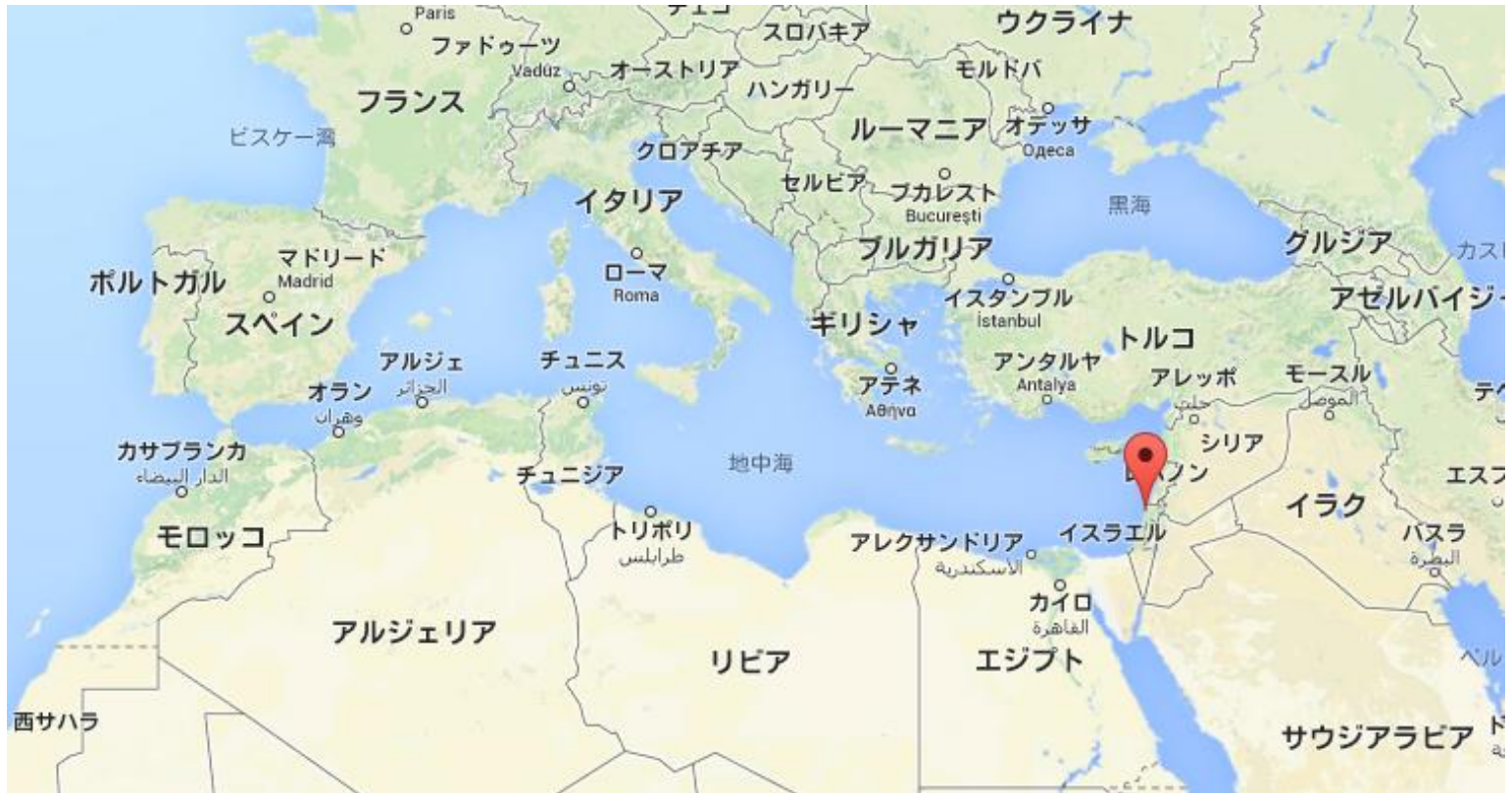




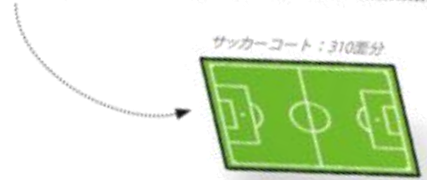
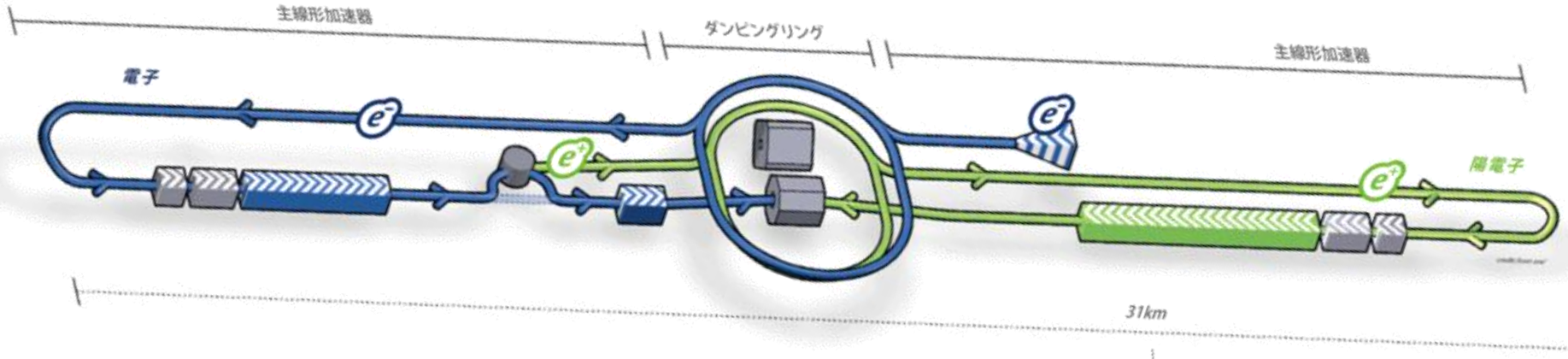
ISRAEL

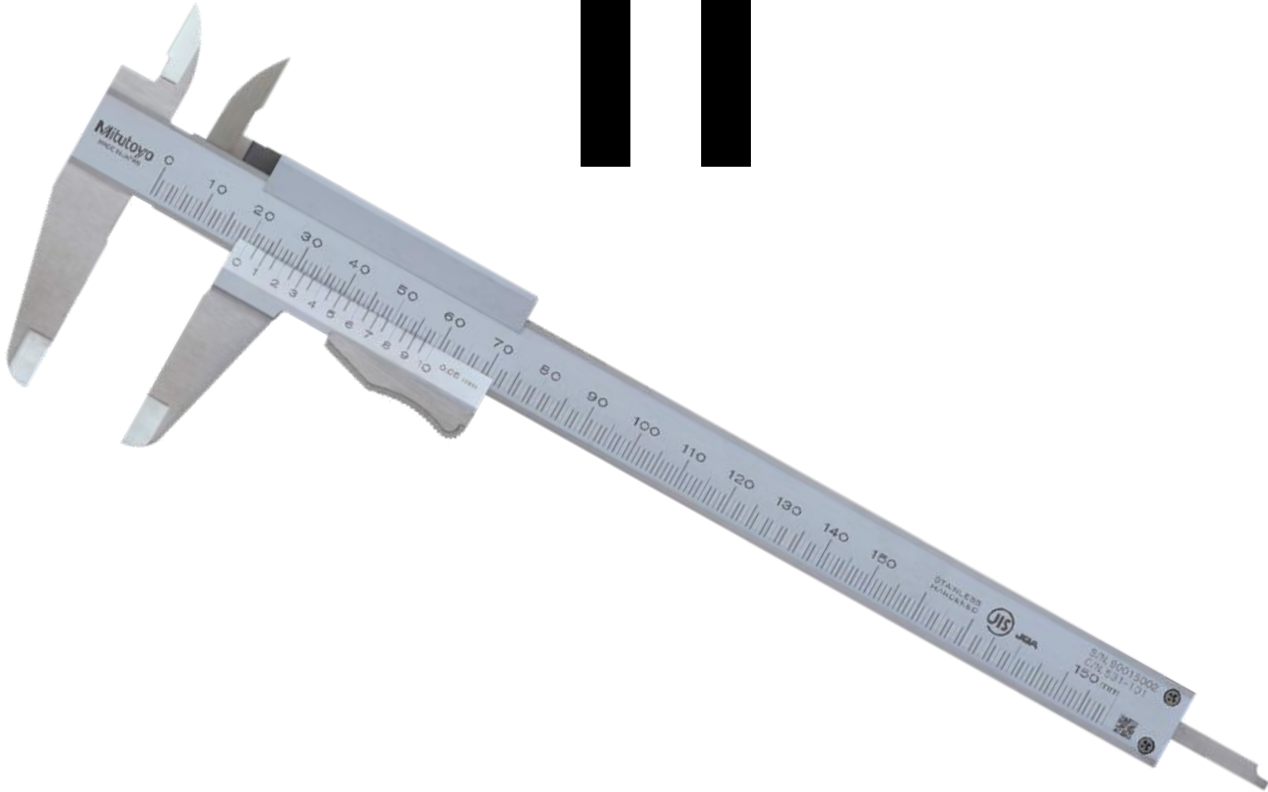
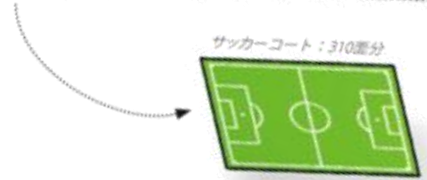
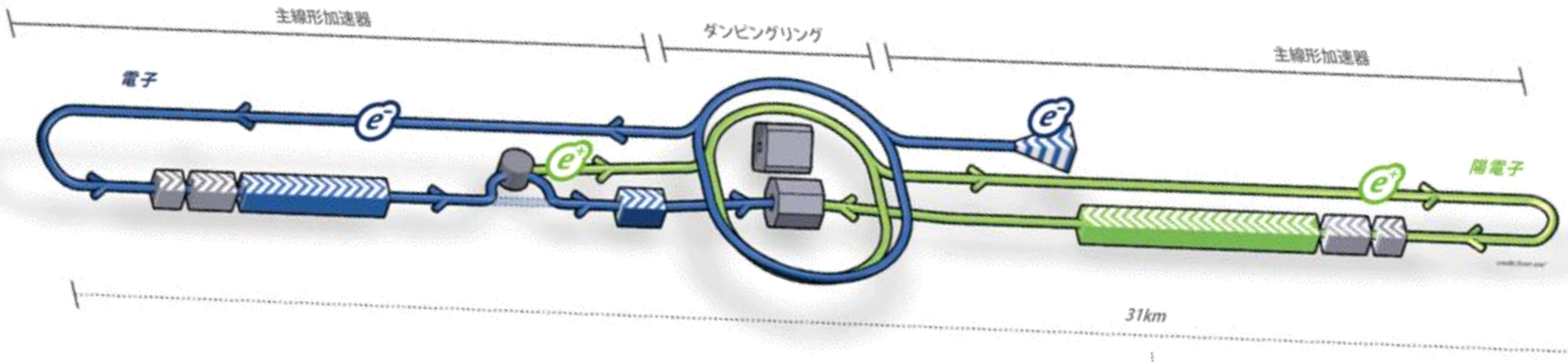
んおにくて

הטכניון  
מכון טכנולוגי  
לישראל

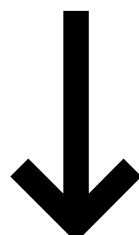


# 本編





ILC = 測定器



何を測る？

◎ Masses

◎ Mixings

◎ Couplings

- ◎ Masses
- ◎ Mixings
- ◎ Couplings
- ◎  $\Delta a_{\mu}^{\text{SUSY}}$



1.  $a_\mu$ ?

2.  $\Delta a_\mu^{\text{SUSY}}$  ?

3. Why measure?

4. How measure?

$a_\mu = \mu$ 粒子の異常磁気MOMENT ( $g - 2$ )

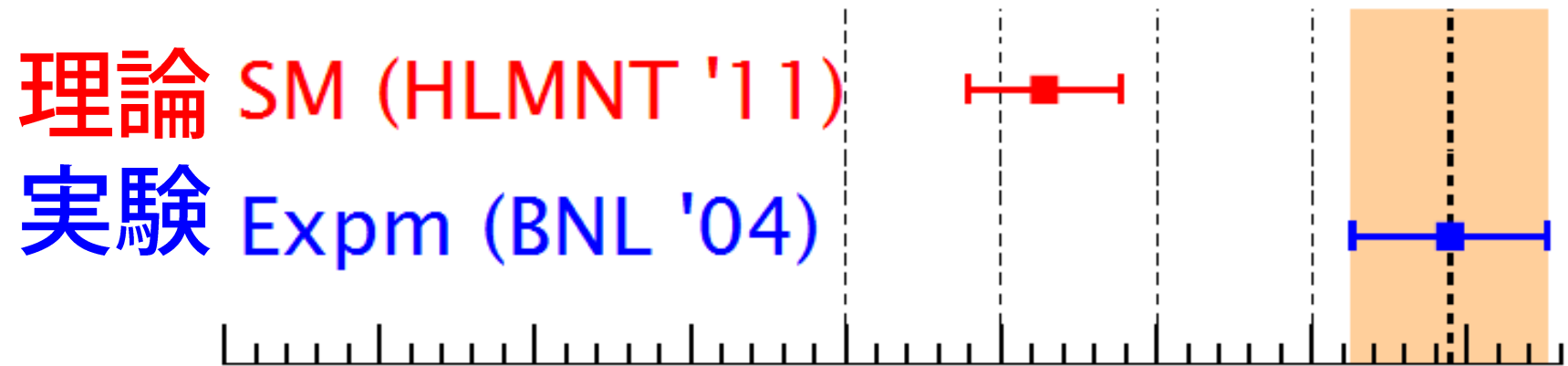
超重要

古典電磁気学 →  $g = 2$

量子電磁気学 →  $g = 2.0023\dots$

$$a_{\mu} = \mu\text{粒子の異常磁気MOMENT } (g - 2)$$

超重要



ズレ

古典電磁気学 →  $g = 2$   
量子電磁気学 →  $g = 2.0023\dots$

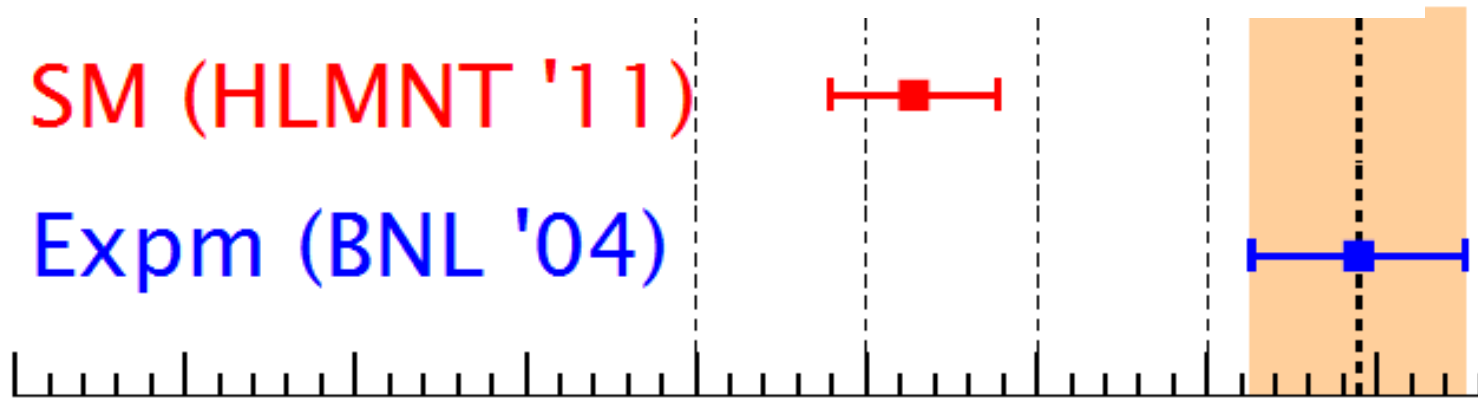
$a_\mu = \mu$ 粒子の異常磁気MOMENT ( $g - 2$ )

# 新物理の証拠

だったらいいなあ

理論 SM (HLMNT '11)

実験 Expm (BNL '04)



ズレ

古典電磁気学 →  $g = 2$

量子電磁気学 →  $g = 2.0023\dots$

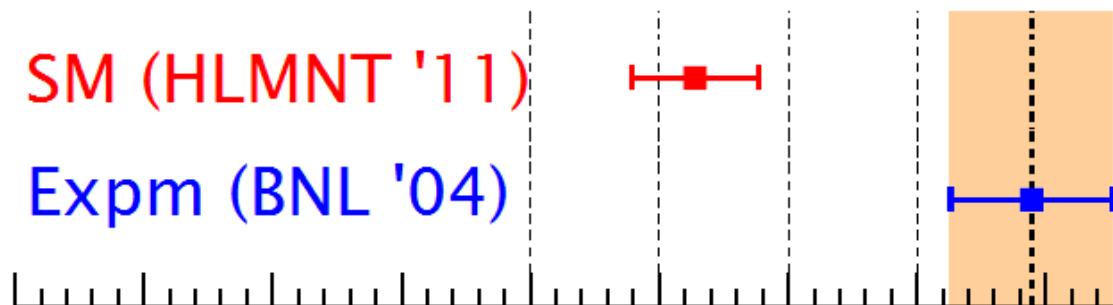
1.  $a_\mu =$  新物理の証拠？

2.  $\Delta a_\mu^{\text{SUSY}}$  ?

3. Why measure?

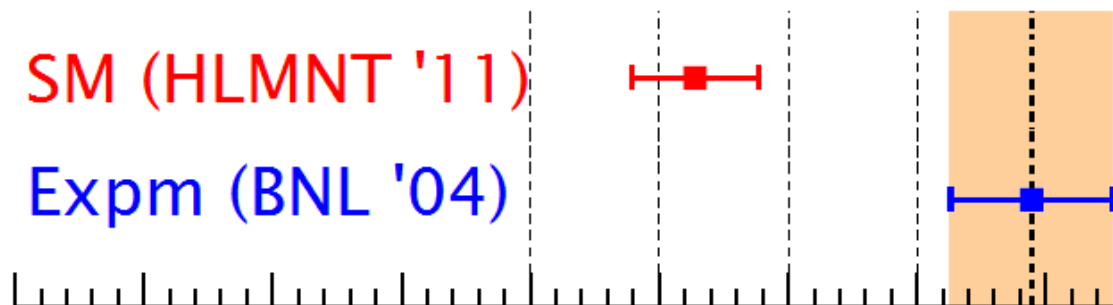
4. How measure?

このズレを説明する理論：



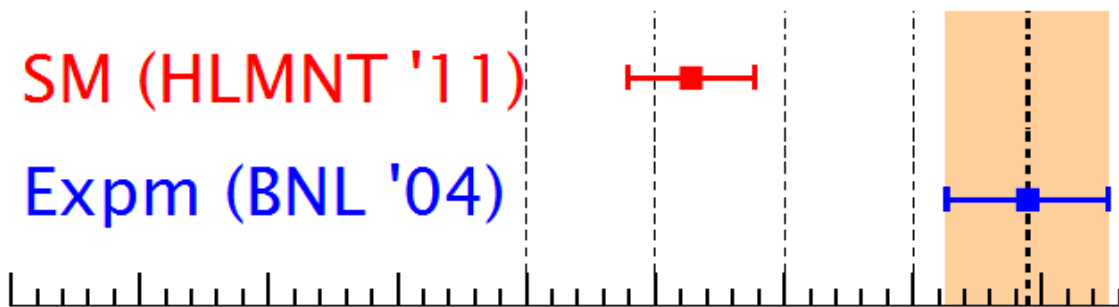
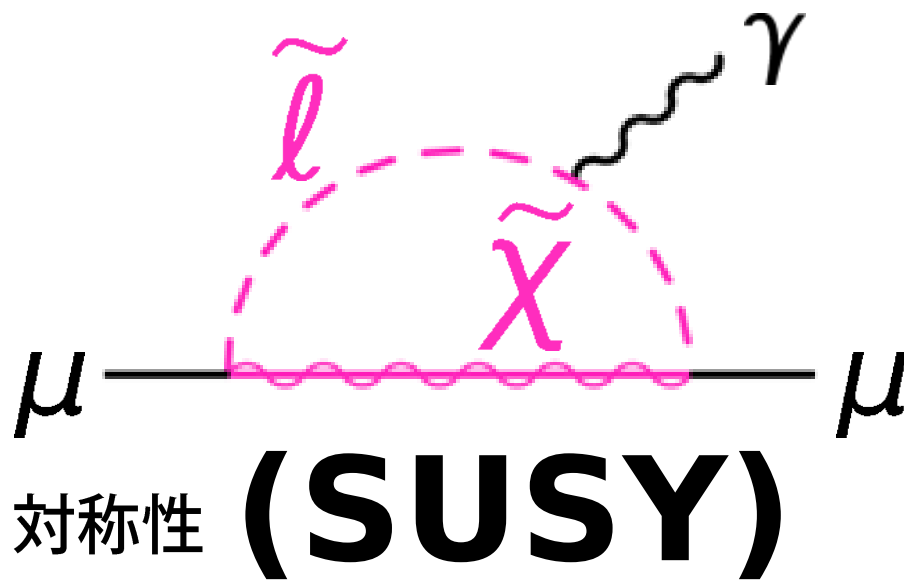
このズレを説明する理論：

# 超対称性 (SUSY)



このズレを説明する理論：

# 超

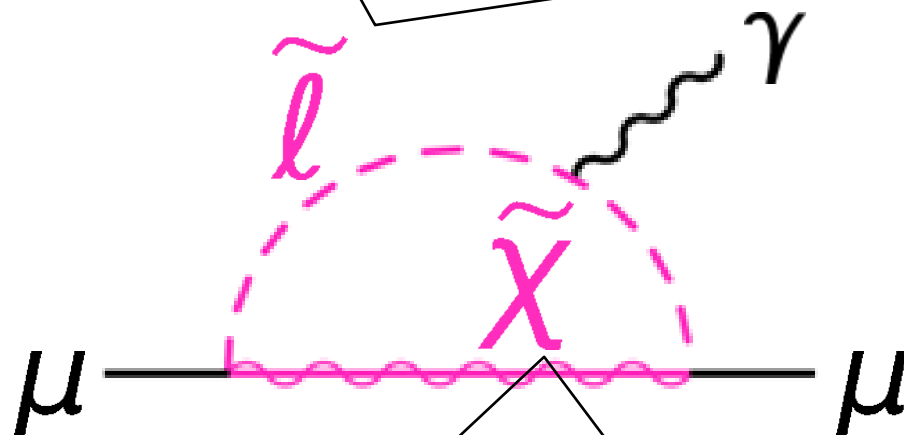




Scalar lepton (slepton)

Lepton ( $e, \mu, \tau$ ) の超対称 partner。

$\tilde{e}, \tilde{\mu}, \tilde{\tau}$  の 3 種類。

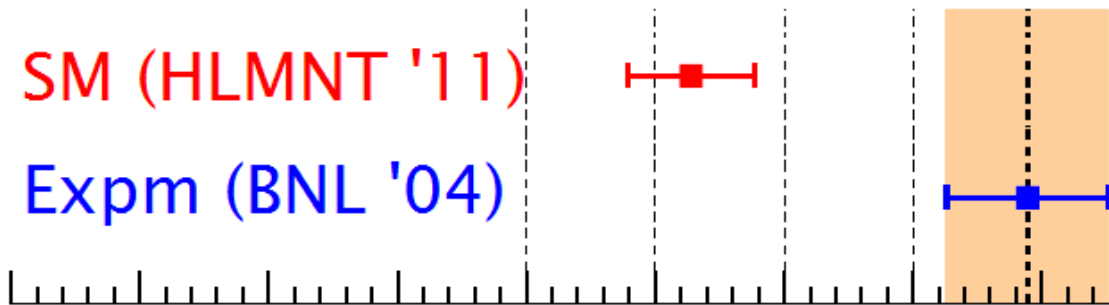
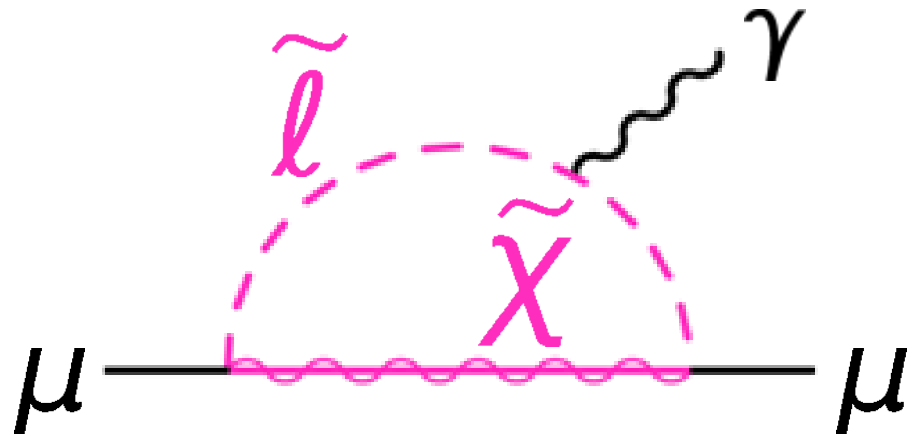


Neutralino。SUSY粒子。

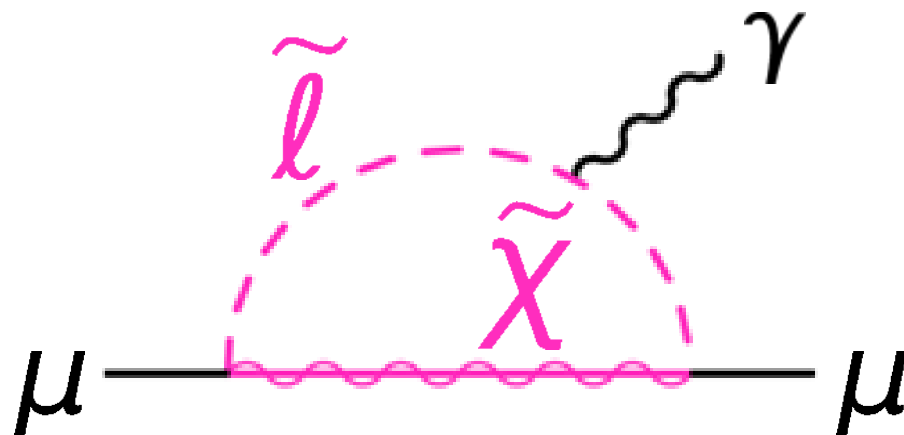
電荷を持たないので、暗黒物質の候補の 1 つ。

なので、すごい。

このズレを説明する理論：



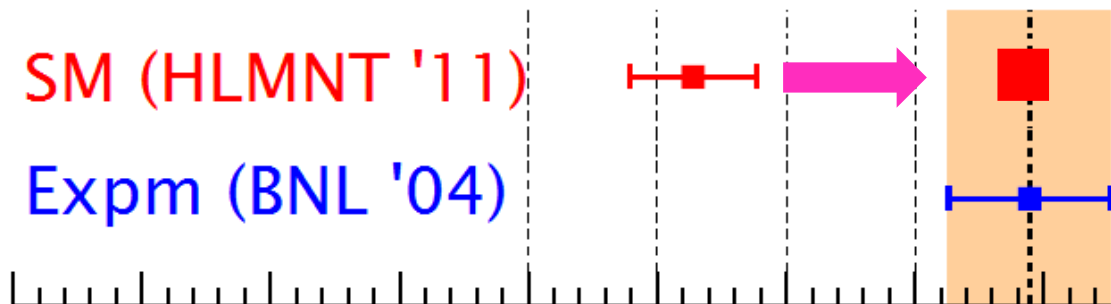
このズレを説明する理論：



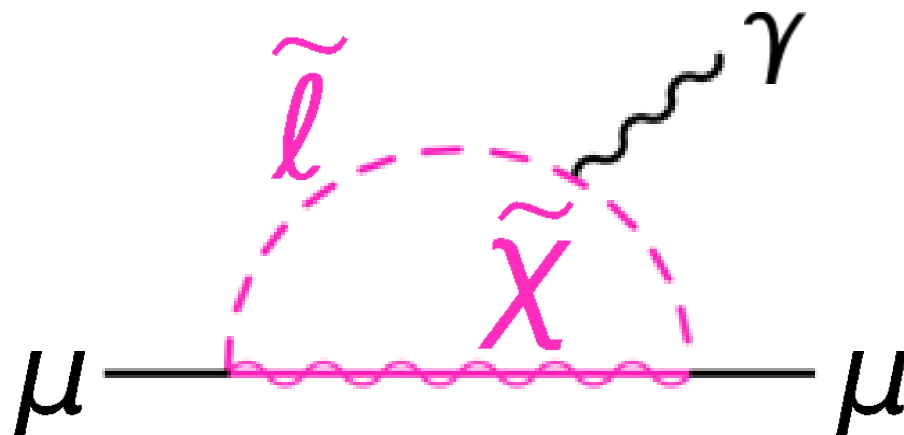
$\tilde{l}, \tilde{\chi} \sim O(100 \text{ GeV})$  なら

SM (HLMNT '11)

Expm (BNL '04)



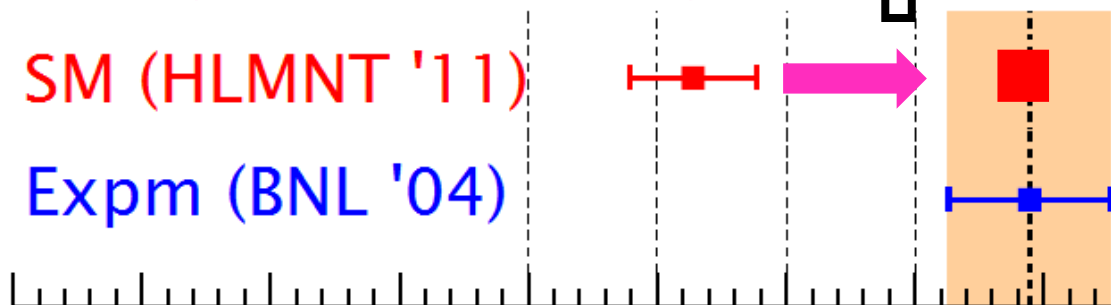
このズレを説明する理論：



$\tilde{l}, \tilde{\chi} \sim O(100 \text{ GeV})$

SM (HLMNT '11)

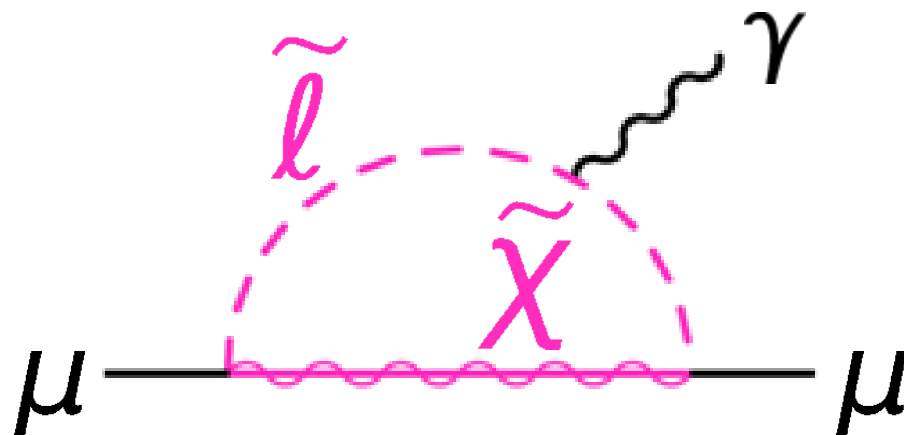
Expm (BNL '04)



このズレを説明する理論：

$\Delta a_{\mu}^{\text{SUSY}}$   
 $\mu$  の見積もりが可能

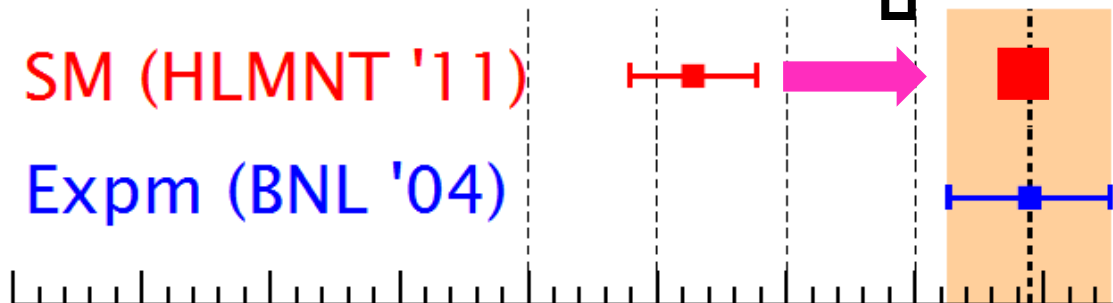
↑  
ILCで見える！  
↑



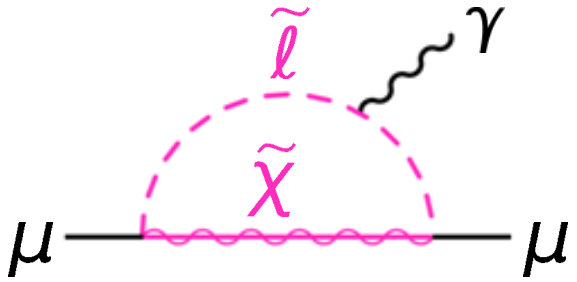
$\tilde{l}, \tilde{\chi} \sim O(100 \text{ GeV})$

SM (HLMNT '11)

Expm (BNL '04)



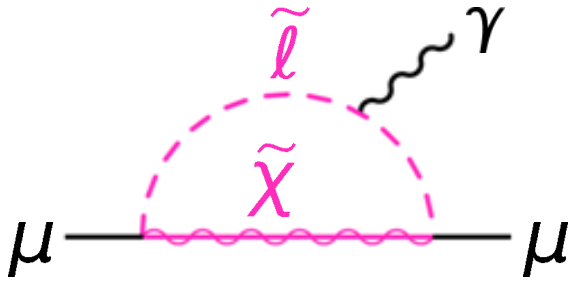
1.  $a_\mu =$  新物理の証拠？

2.  $\Delta a_\mu^{\text{SUSY}} =$  

3. Why measure?

4. How measure?

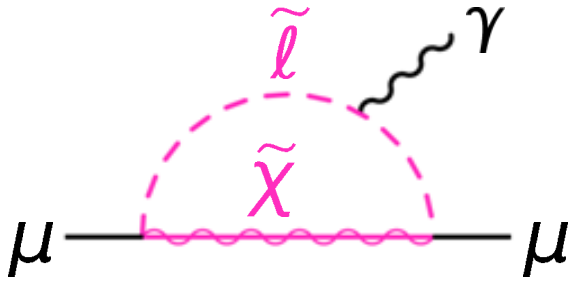
1.  $a_\mu \rightarrow \tilde{l}, \tilde{\chi}$  が見えるかも？

2.  $\Delta a_\mu^{\text{SUSY}} =$  

3. Why measure?

4. How measure?

1.  $a_\mu \rightarrow \tilde{l}, \tilde{\chi}$  が見えるかも？

2.  $\Delta a_\mu^{\text{SUSY}} =$  

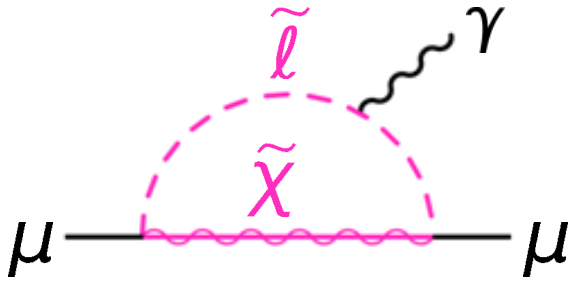
3. Why measure?

そこに物理量があるから。

4. How measure?



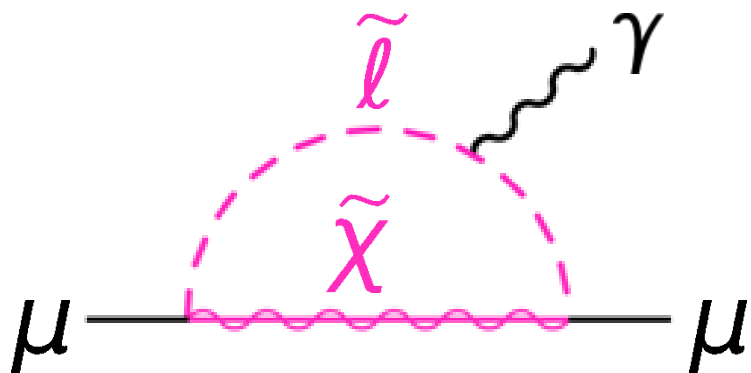
1.  $a_\mu \rightarrow \tilde{l}, \tilde{\chi}$  が見えるかも？

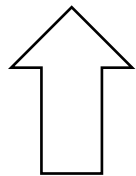
2.  $\Delta a_\mu^{\text{SUSY}} =$  

3. Why measure?

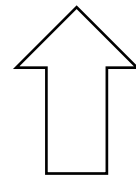
そこに物理量があるから。

4. How measure?

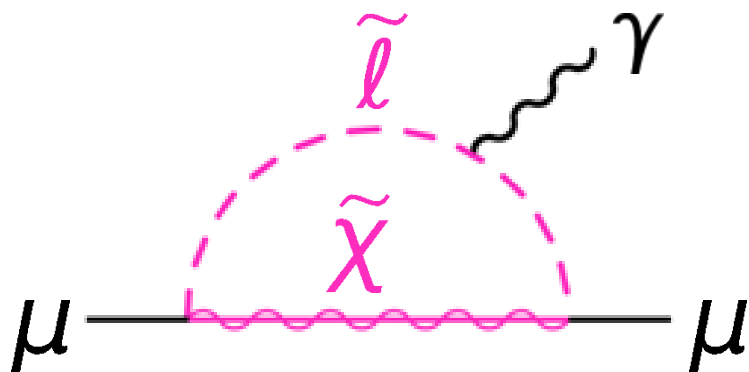
$$\Delta a_{\mu}^{\text{SUSY}} =$$


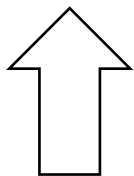


これを“測る”



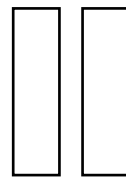
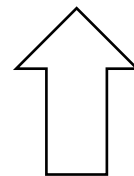
= この diagram にでてくる量を測る

$$\Delta a_{\mu}^{\text{SUSY}} =$$




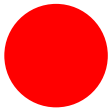
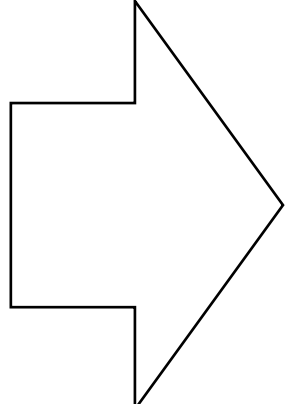
これを“測る”

= この diagram にでてくる量を測る

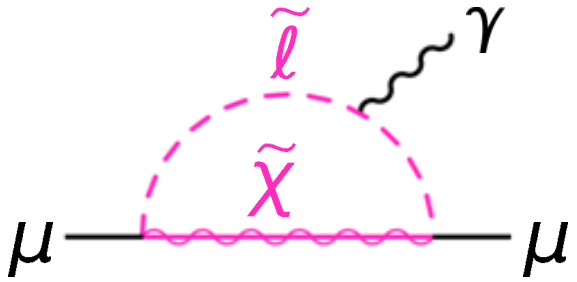


Masses, Mixings, Couplings を測る

$$\Delta a_{\mu}^{\text{SUSY}} =$$

- Mass of  $(\tilde{l}, \tilde{\chi})$
  - Mixing of  $\tilde{l}_L - \tilde{l}_R$
  - Coupling of 
- 
 $\Delta a_{\mu}^{\text{SUSY}}$

1.  $a_\mu \rightarrow \tilde{\ell}, \tilde{\chi}$  が見えるかも？

2.  $\Delta a_\mu^{\text{SUSY}} =$  

3. Why measure?  
そこに物理量があるから。

4. How measure?

→ masses, mixings, couplings.

# 結論

$\Delta a_{\mu}^{\text{SUSY}}$  は、測れます！

→ masses, mixings, couplings.

# ほんとの結論

大きな  $\mu$  parameter によって  $\Delta a_{\mu}^{\text{SUSY}}$  を稼いで  
ずれを説明するシナリオでは

$\tilde{e}_L, \tilde{e}_R, \tilde{\mu}_L, \tilde{\mu}_R, \tilde{\tau}_L, \tilde{\tau}_R, \tilde{\chi}_1^0$   
が全てILCで見つかれば

$\Delta a_{\mu}^{\text{SUSY}}$  の測定精度は約10%

→ masses, mixings, couplings.

# ほんとの結論

大きな  $\mu$  parameter によって  $\Delta a_{\mu}^{\text{SUSY}}$  を稼いで  
ずれを説明するシナリオでは **実は  $\sigma(ee \rightarrow \tilde{\tau}_1 \tilde{\tau}_1)$   
を用いた値**  
 $\tilde{e}_L, \tilde{e}_R, \tilde{\mu}_L, \tilde{\mu}_R, \tilde{\tau}_L, \tilde{\tau}_R, \tilde{\chi}_1^0$   
が全てILCで見つかれば

$\Delta a_{\mu}^{\text{SUSY}}$  の測定精度は約**10%**

- Mass of  $(\tilde{\mu}_L, \tilde{\mu}_R, \tilde{\chi}_1^0)$  → End-point analysis
- Mixing of  $\tilde{\mu}_L - \tilde{\mu}_R$  →  $\sigma(ee \rightarrow \tilde{\tau}_1 \tilde{\tau}_2) \Rightarrow \theta_{\tilde{\tau}}$
- Coupling of ● **これを使えばもっと精度よく測れるはず!**



続きは昨日の  
ナイトセッションで。

**BACKUP**

# ほんとの結論

大きな  $\mu$  parameter によって  $\Delta a_{\mu}^{\text{SUSY}}$  を稼いで  
ずれを説明するシナリオでは

$\tilde{e}_L, \tilde{e}_R, \tilde{\mu}_L, \tilde{\mu}_R, \tilde{\tau}_L, \tilde{\tau}_R, \tilde{\chi}_1^0$   
が全てILCで見つかれば

$\Delta a_{\mu}^{\text{SUSY}}$  の測定精度は約10%

→ masses, mixings, couplings.

$\mu$  parameter: SUSY の parameter の 1 つ。

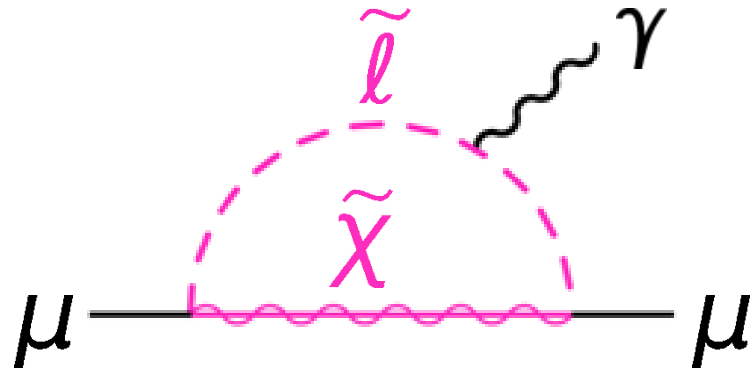
小さい  $\mu$  parameter でかい

かるい  $\tilde{H}^0, \tilde{H}^\pm$  おもい

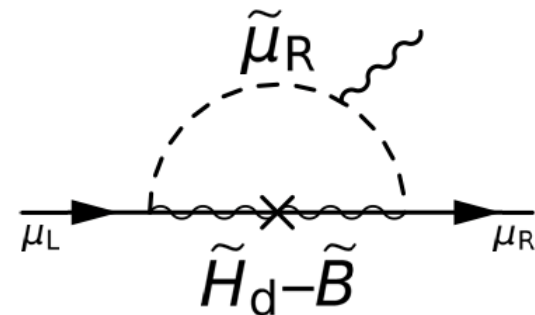
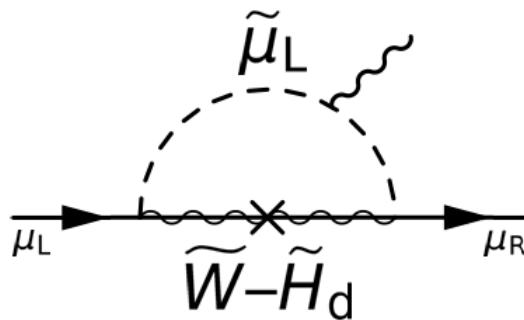
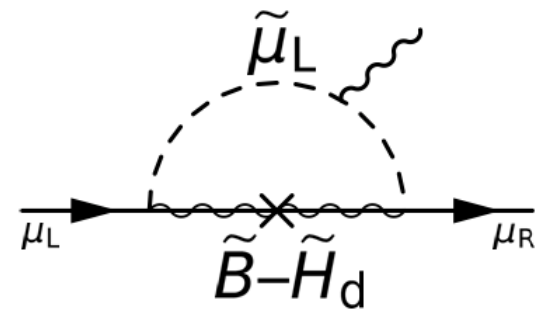
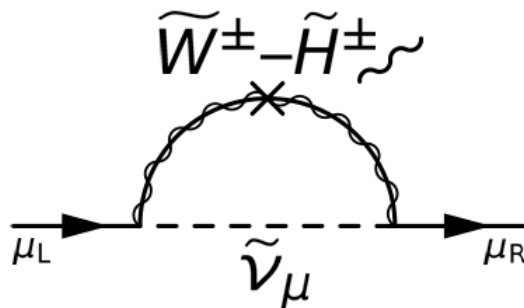
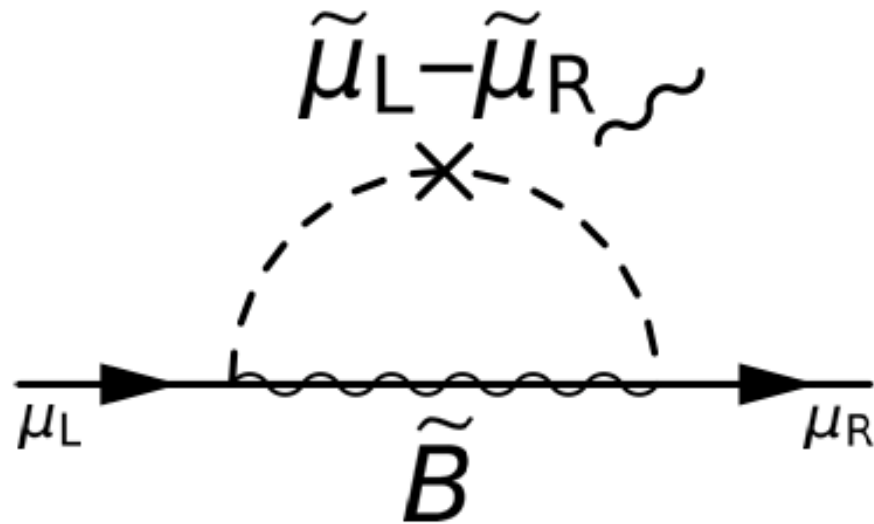
小さい  $\tilde{l}_L - \tilde{l}_R$  mixing でかい

いい naturalness わるい

$$\Delta a_{\mu}^{\text{SUSY}} =$$



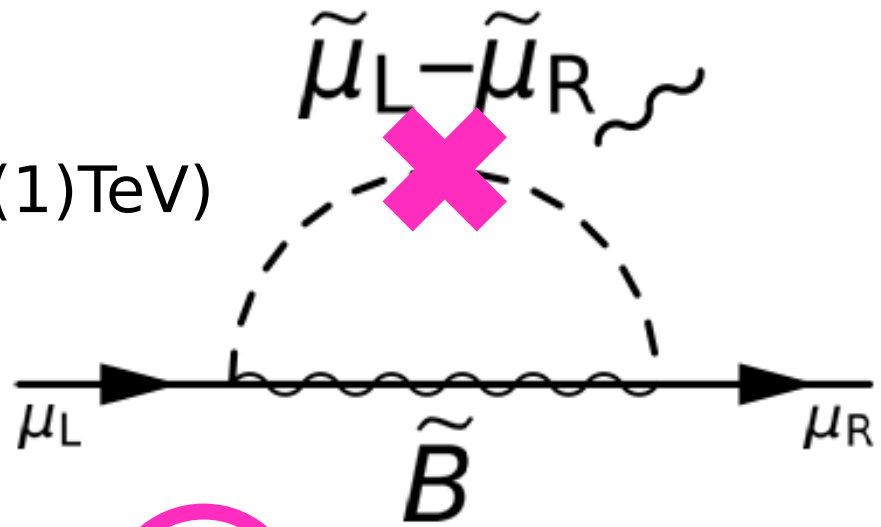
$$\Delta a_{\mu}^{\text{SUSY}} =$$

$$=$$


$\mu$ -param. が**でかい** ( $O(1)\text{TeV}$ )

→ Mixing **でかい**

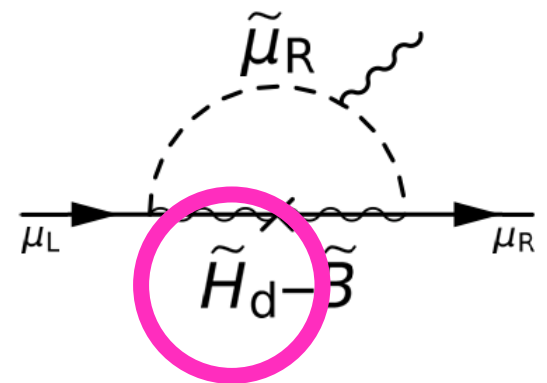
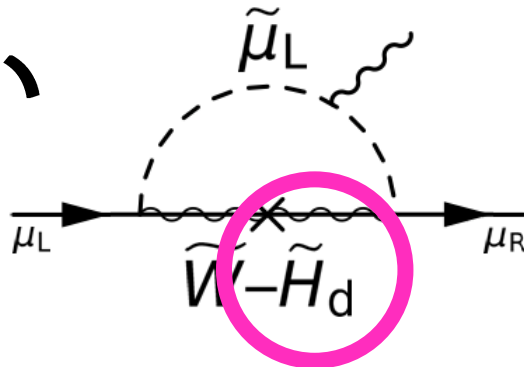
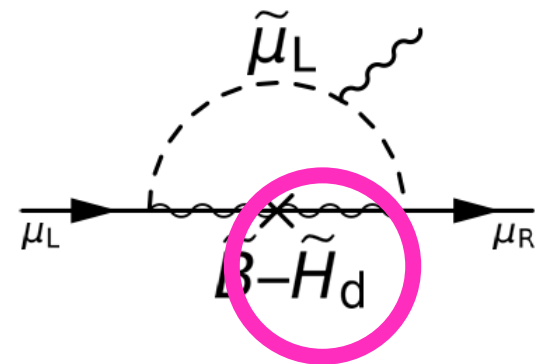
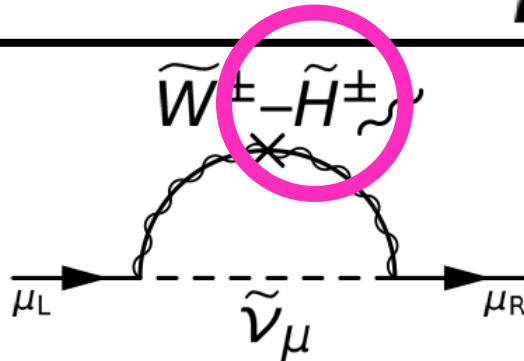
→ この寄与が**でかい**



$\mu$ -param. が**小さい**  
( $O(100)\text{GeV}$ )

→  $\tilde{H}^0, \tilde{H}^\pm$  **軽い**

→ この寄与が**でかい**



# ほんとの結論

大きな  $\mu$  parameter によって  $\Delta a_{\mu}^{\text{SUSY}}$  を稼いで  
ずれを説明するシナリオでは

$$\tilde{e}_L, \tilde{e}_R, \tilde{\mu}_L, \tilde{\mu}_R, \tilde{\tau}_L, \tilde{\tau}_R, \tilde{\chi}_1^0$$

が全てILCで見つかれば

$\Delta a_{\mu}^{\text{SUSY}}$  の測定精度は約10%

→ masses, mixings, couplings.



1.  $a_\mu \rightarrow \tilde{l}, \tilde{\chi}$  が見えるかも？

2.  $\Delta a_\mu^{\text{SUSY}} =$  

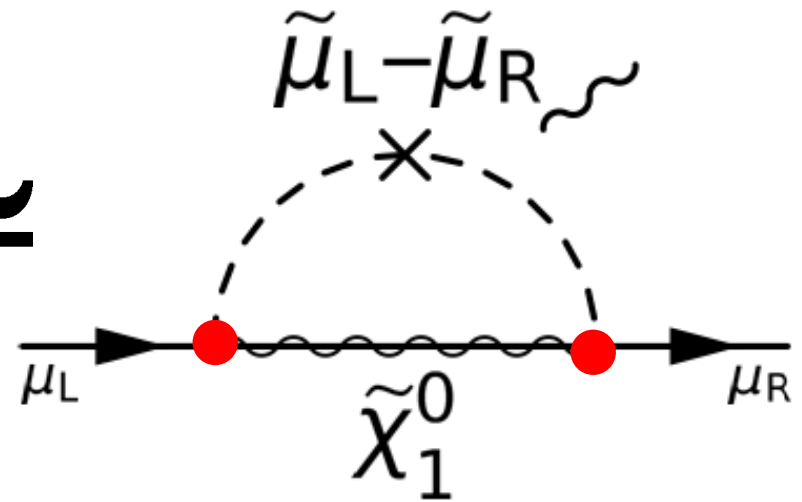
3. Why measure?  
そこに物理量があるから。

4. How measure?

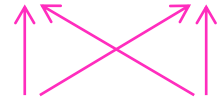
→ masses, mixings, couplings.

$\mu$  param. でかい場合,

$$\Delta a_{\mu}^{\text{SUSY}} \simeq$$



$\tilde{\mu}_1, \tilde{\mu}_2$

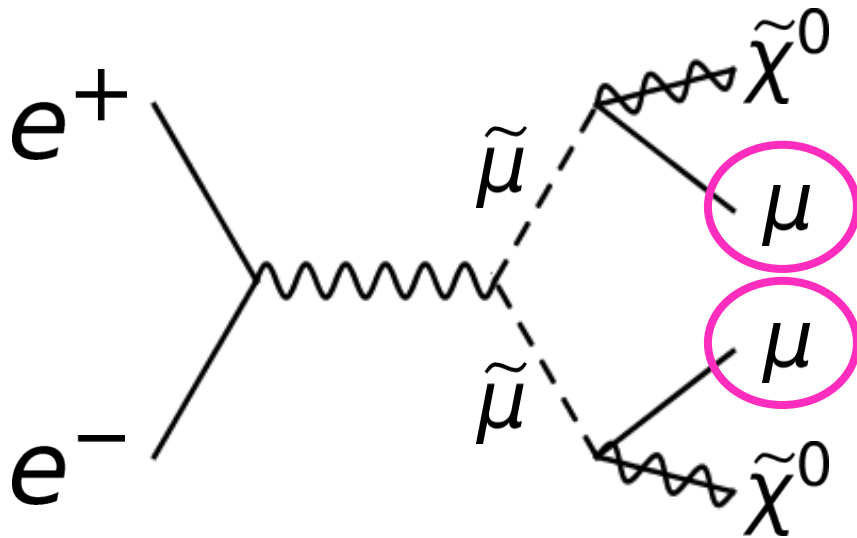
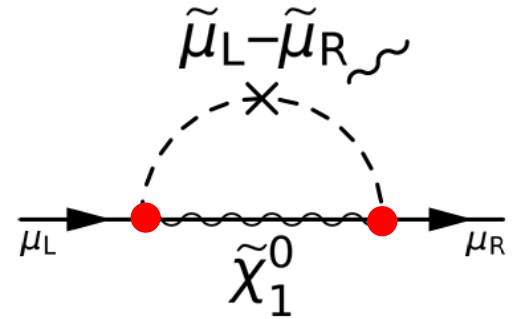


- Mass of  $(\tilde{\mu}_L, \tilde{\mu}_R, \tilde{\chi})$
- Mixing of  $\tilde{\mu}_L - \tilde{\mu}_R$
- Coupling of ●

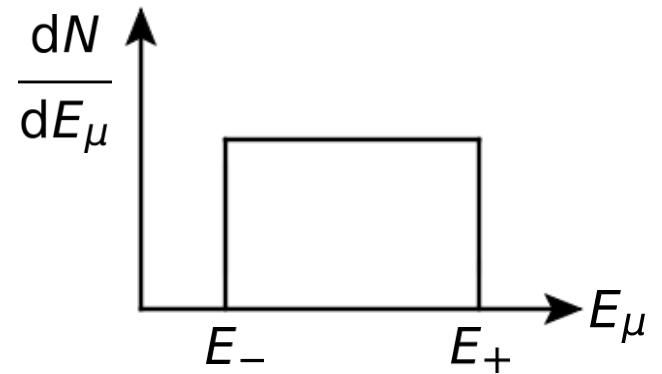
$$\Delta a_{\mu}^{\text{SUSY}}$$

- Mass of  $(\tilde{\mu}_1, \tilde{\mu}_2, \tilde{\chi}_1^0)$

⇒ End-point analysis



⇒



$$E_{\pm} = \frac{\sqrt{S}}{4} (1 \pm \beta) \left( 1 - \frac{m^2}{M^2} \right); \quad \beta = \sqrt{1 - \frac{4M^2}{S}}$$

- Mixing of  $\tilde{\mu}_L - \tilde{\mu}_R$



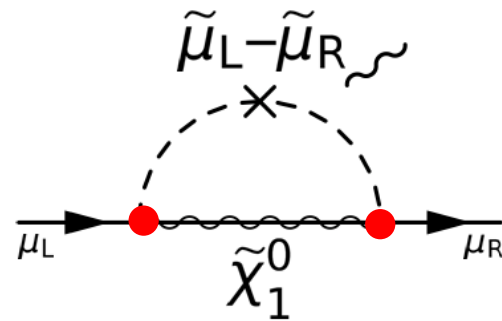
Cross Section

$$\sigma(e^+e^- \rightarrow \tilde{\mu}_1\tilde{\mu}_2)$$

$$\propto \sin 2\theta_{\tilde{\mu}}$$

...小さすぎて見えない

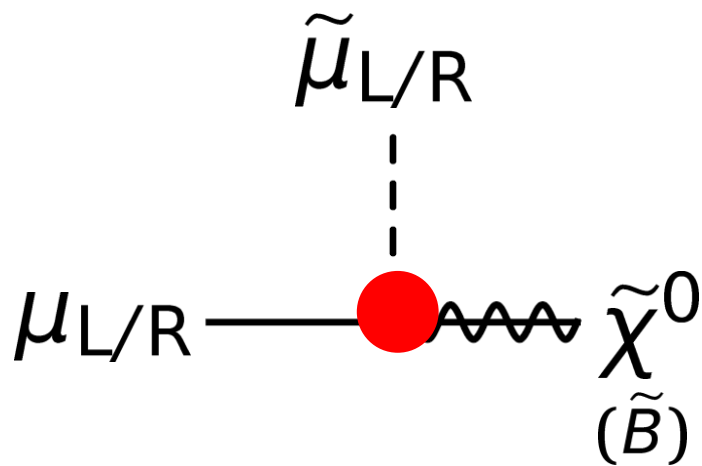
$$(\theta_{\mu} = O(10^{-2}) \Rightarrow \sigma \sim O(0.1) \text{ fb})$$



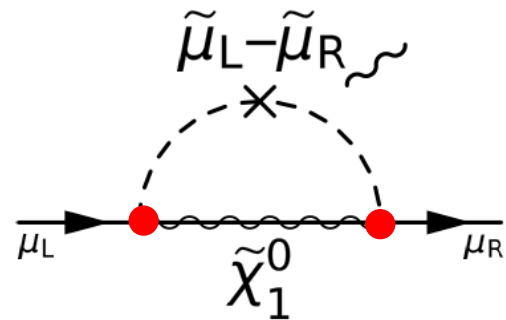
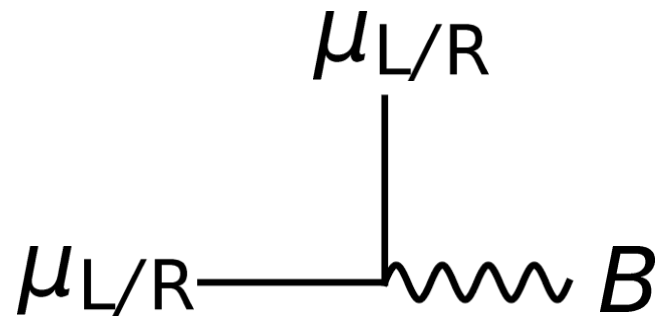
実は, *だいたい*  $\theta_{\tilde{\ell}} \propto m_{\ell}$

$$\Rightarrow \sigma(e^+e^- \rightarrow \tilde{\tau}_1\tilde{\tau}_2) \Rightarrow \theta_{\tilde{\tau}} \Rightarrow \theta_{\tilde{\mu}}$$

- Coupling of ●



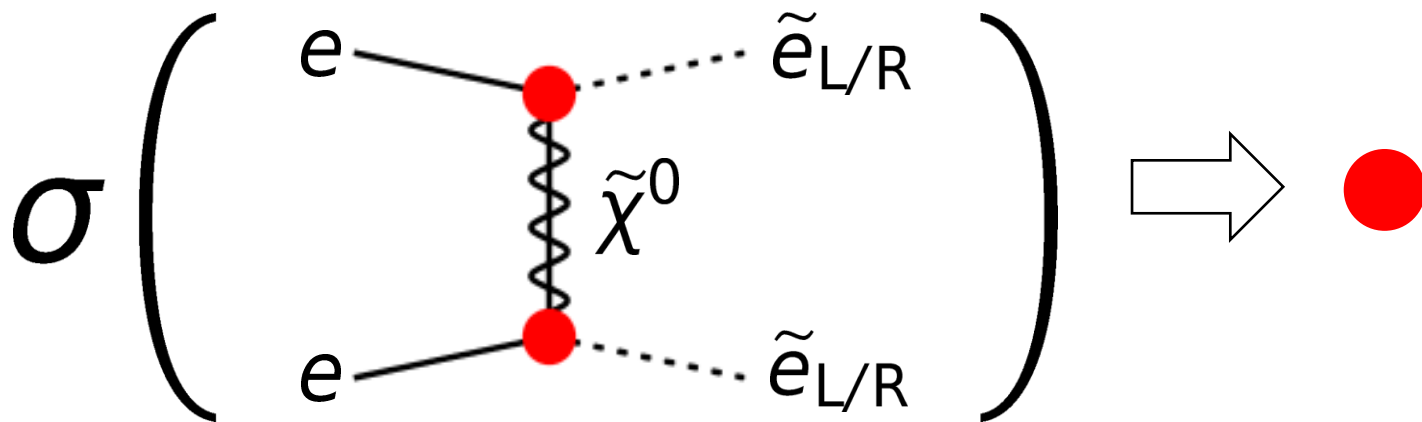
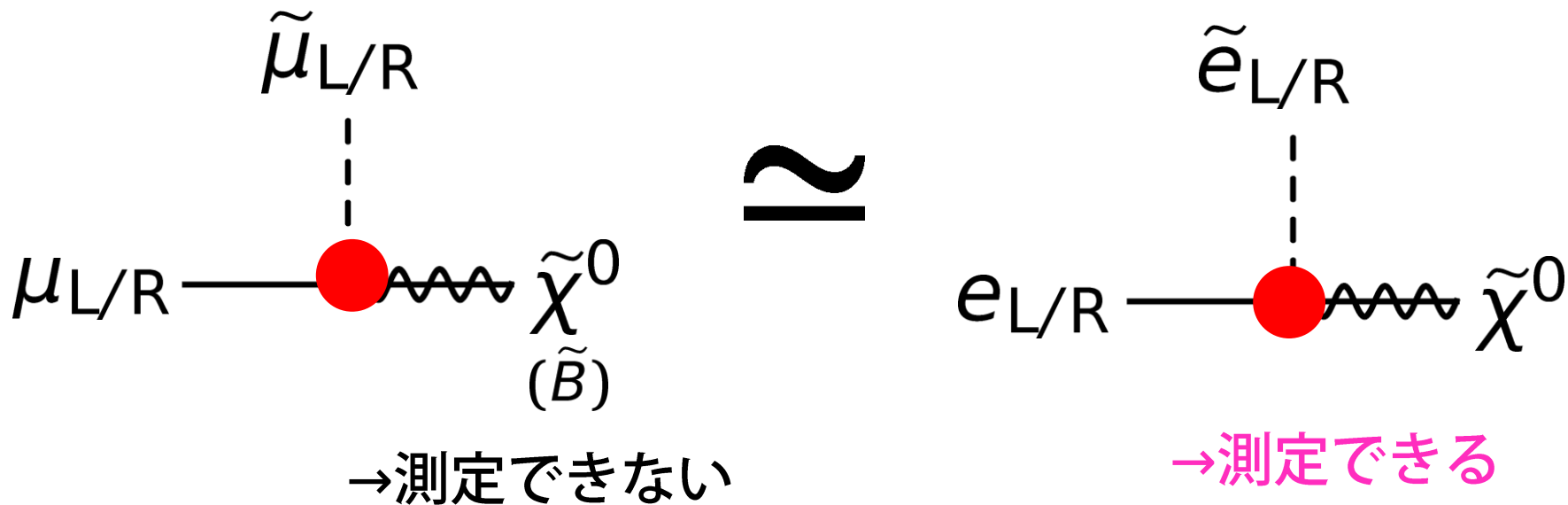
$\neq$



(∵他のSUSY粒子の量子効果)

→実験で測定すべき

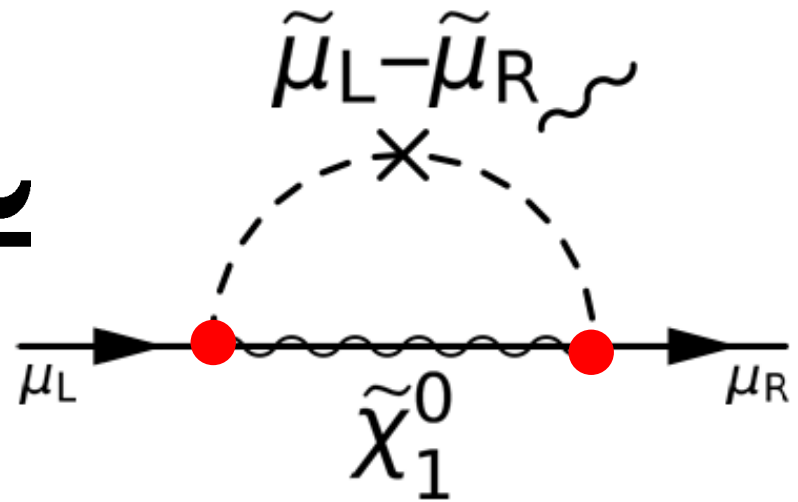
- Coupling of 



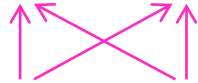
$\mu$  param. でかい場合,

$$\Delta a_{\mu}^{\text{SUSY}} \approx$$

$\approx$



$\tilde{\mu}_1, \tilde{\mu}_2$



- Mass of  $(\tilde{\mu}_L, \tilde{\mu}_R, \tilde{\chi}_1^0)$  → End-point analysis
- Mixing of  $\tilde{\mu}_L - \tilde{\mu}_R$  →  $\sigma(ee \rightarrow \tilde{\tau}_1 \tilde{\tau}_2) \Rightarrow \theta_{\tilde{\tau}}$
- Coupling of ● →  $ee \rightarrow \tilde{e}\tilde{e}$  ( $t$ -channel)

# ほんとの結論

大きな  $\mu$  parameter によって  $\Delta a_{\mu}^{\text{SUSY}}$  を稼いで  
ずれを説明するシナリオでは

$$\tilde{e}_L, \tilde{e}_R, \tilde{\mu}_L, \tilde{\mu}_R, \tilde{\tau}_L, \tilde{\tau}_R, \tilde{\chi}_1^0$$

が全てILCで見つかれば

$\Delta a_{\mu}^{\text{SUSY}}$  の測定精度は約10%

- Mass of  $(\tilde{\mu}_L, \tilde{\mu}_R, \tilde{\chi}_1^0)$  → End-point analysis
- Mixing of  $\tilde{\mu}_L - \tilde{\mu}_R$  →  $\sigma(ee \rightarrow \tilde{\tau}_1 \tilde{\tau}_2) \Rightarrow \theta_{\tilde{\tau}}$
- Coupling of ● →  $ee \rightarrow \tilde{e}\tilde{e}$  ( $t$ -channel)



# ほんとの結論

大きな  $\mu$  parameter によって  $\Delta a_{\mu}^{\text{SUSY}}$  を稼いで  
ずれを説明するシナリオでは **実は  $\sigma(ee \rightarrow \tilde{\tau}_1 \tilde{\tau}_1)$   
を用いた値**  
 $\tilde{e}_L, \tilde{e}_R, \tilde{\mu}_L, \tilde{\mu}_R, \tilde{\tau}_L, \tilde{\tau}_R, \tilde{\chi}_1^0$   
が全てILCで見つかれば

$\Delta a_{\mu}^{\text{SUSY}}$  の測定精度は約**10%**

- Mass of  $(\tilde{\mu}_L, \tilde{\mu}_R, \tilde{\chi}_1^0)$  → End-point analysis
- Mixing of  $\tilde{\mu}_L - \tilde{\mu}_R$  →  $\sigma(ee \rightarrow \tilde{\tau}_1 \tilde{\tau}_2) \Rightarrow \theta_{\tilde{\tau}}$
- Coupling of ● **これを使えばもっと精度よく測れるはず!**

# Message

# $a_\mu$ のずれ $\rightarrow$ 新物理 ( $\Delta a_\mu^{\text{SUSY}}$ ) の示唆

- $\mu = \mathcal{O}(100) \text{ GeV}$   
 $\rightarrow \mathcal{O}(100) \text{ GeV } \tilde{\chi}^+$

- $\mu = \mathcal{O}(1) \text{ TeV}$   
 $\rightarrow \text{large } \tilde{\ell}\text{-mixing}$

$\Delta a_\mu^{\text{SUSY}}$  は測れるかもしれません

# $a_\mu$ のずれ $\rightarrow$ 新物理 ( $\Delta a_\mu^{\text{SUSY}}$ ) の示唆

- $\mu = \mathcal{O}(100) \text{ GeV}$   
 $\rightarrow \mathcal{O}(100) \text{ GeV } \tilde{\chi}^+$

- $\mu = \mathcal{O}(1) \text{ TeV}$   
 $\rightarrow$  large  $\tilde{\ell}$ -mixing

これを測るのおもしろそう！

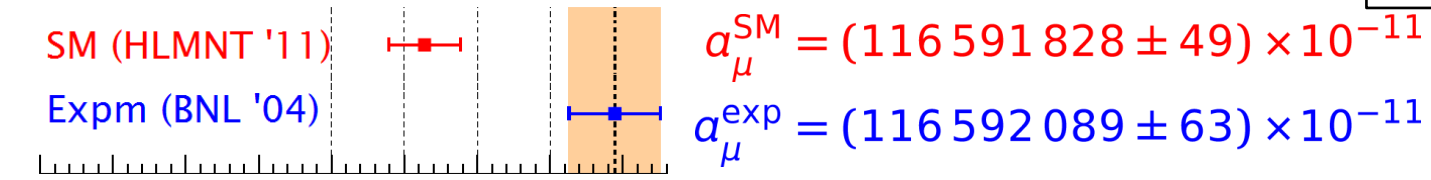
$\Delta a_\mu^{\text{SUSY}}$  は測れるかもしれません

# MORE BACKUP

# Muon $g-2$ Problem

$$\left( a_\mu := \frac{g_\mu - 2}{2} \right)$$

Muon  $g-2$  (anomalous magnetic moment)



Hagiwara, Liao, Martin, Nomura, Teubner [[1105.3149](#)]

**3.3 $\sigma$  discrepancy**

**New Physics?**

can be explained with **SUSY**.

Lopez, Nanopoulos, Wang [[ph/9308336](#)]  
Chattopadhyay, Nath [[ph/9507386](#)]  
Moroi [[ph/9512396](#)]

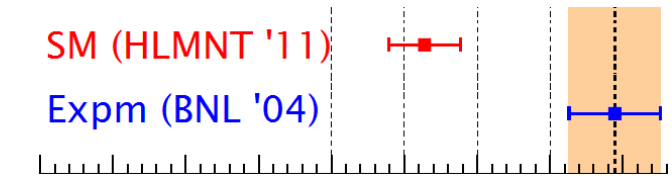
## SUSY!

- ✓ Dark matter problem,
- ✓ Hierarchy problem,
- ✓ Muon  $g-2$  problem,
- ✓ Grand unification,
- ✓ **will be discovered at LHC.**

# Muon $g-2$ Problem

$$\left( a_\mu := \frac{g_\mu - 2}{2} \right)$$

Muon  $g-2$  (anomalous magnetic moment)



$$a_\mu^{\text{SM}} = (116\,591\,828 \pm 49) \times 10^{-11}$$

$$a_\mu^{\text{exp}} = (116\,592\,089 \pm 63) \times 10^{-11}$$

Hagiwara, Liao, Martin, Nomura, Teubner [[1105.3149](#)]

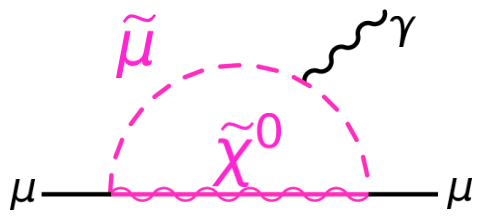
**3.3 $\sigma$  discrepancy**

**New Physics?**

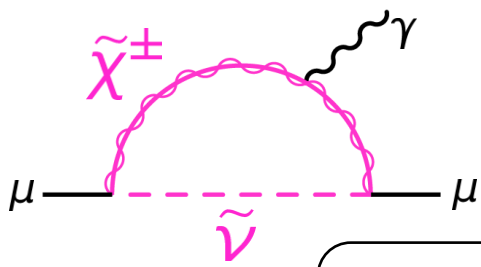
can be explained with **SUSY**.

Lopez, Nanopoulos, Wang [[ph/9308336](#)]  
 Chattopadhyay, Nath [[ph/9507386](#)]  
 Moroi [[ph/9512396](#)]

$$\left[ \rightsquigarrow (\tilde{\chi}^0, \tilde{\mu}) \text{ or } (\tilde{\chi}^\pm, \tilde{\nu}) = \mathcal{O}(100)\text{GeV} \right]$$



$$a_\mu^{\text{SUSY}}(\tilde{\chi}^0, \tilde{\mu}) \approx \frac{g_Y^2}{(4\pi)^2} \frac{m_\mu^2}{m_{\text{soft}}^2} \text{sgn}(\mu) \tan \beta + \dots,$$



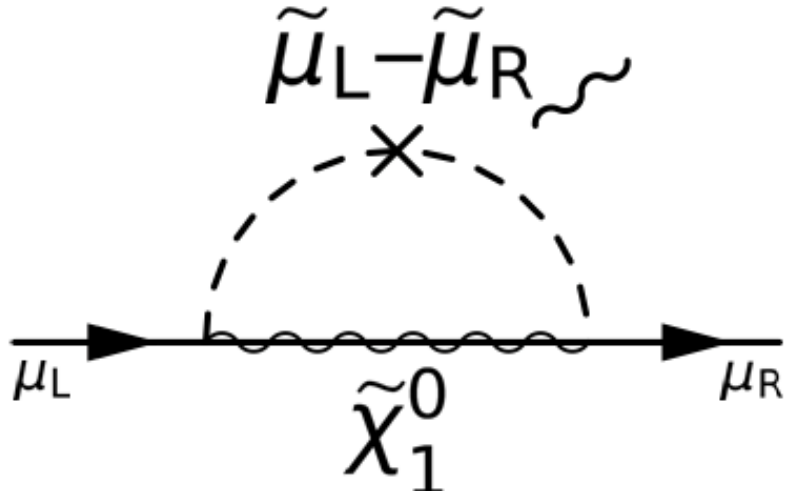
$$a_\mu^{\text{SUSY}}(\tilde{\chi}^\pm, \tilde{\nu}) \approx \frac{g_2^2}{(4\pi)^2} \frac{m_\mu^2}{m_{\text{soft}}^2} \text{sgn}(\mu) \tan \beta.$$

$W \ni \mu H_u H_d$  (Higgsino mass term),  $\tan \beta = \frac{\langle H_u \rangle}{\langle H_d \rangle}$ ,  
 $m_{\text{soft}}$  : SUSY-particle mass-scale,  $g_i$  : Gauge couplings.

## What keys should we collect?

- Let's measure  $a_\mu^{\text{SUSY}}$ !

## What should be measured?

$$a_\mu^{\text{SUSY}} \approx$$


The diagram shows a muon line (solid line with arrows) going from left to right, labeled  $\mu_L$  and  $\mu_R$ . A dashed line loop is attached to the muon line, with a cross on it. The loop is labeled  $\tilde{\mu}_L - \tilde{\mu}_R$  and  $\tilde{\chi}_1^0$ .

$$\left( \propto \frac{m_\mu \cdot M_{LR}^2 \cdot \tilde{g}_L \tilde{g}_R}{(\text{mass})^3} \right)$$



## What keys should we collect?

- Let's measure  $\alpha_{\mu}^{\text{SUSY}}$ !

## What should be measured?

➤ Mass of  $\tilde{\mu}_1, \tilde{\mu}_2, \tilde{\chi}_0^1$

➤ Mixing  $M_{LR}^2$

➤ Coupling  $\tilde{g}_L, \tilde{g}_R$

$\neq g_Y$  because

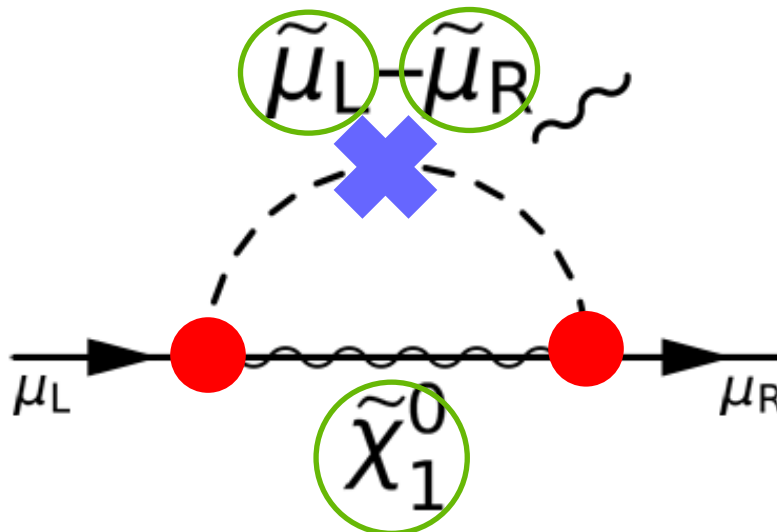
- SUSY effect.
- $\tilde{\chi}_1^0 \neq$  "pure"  $\tilde{B}$ .

$$\mathcal{M}_{\tilde{\mu}}^2 = \begin{pmatrix} m_L^2 & M_{LR}^2 \\ M_{LR}^2 & m_R^2 \end{pmatrix}$$

$$(M_{LR}^2 \simeq m_{\mu} \mu \tan \beta)$$

$$M_{LR}^2 = -\frac{1}{2} (m_2^2 - m_1^2) \sin 2\theta$$

$\alpha_{\mu}^{\text{SUSY}} \simeq$

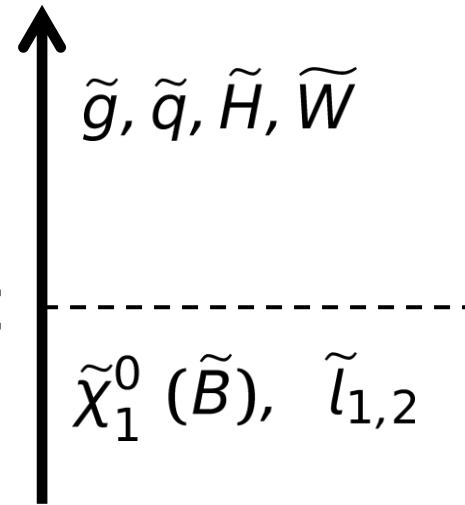


$$\left( \propto \frac{m_{\mu} \cdot M_{LR}^2 \cdot \tilde{g}_L \tilde{g}_R}{(\text{mass})^3} \right)$$

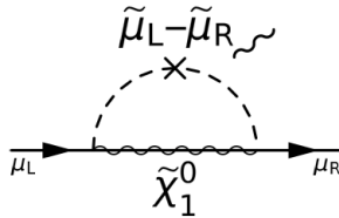
$\tilde{e}_1, \tilde{\mu}_1$	$\tilde{e}_2, \tilde{\mu}_2$	$\tilde{\tau}_1$	$\tilde{\tau}_2$	$\tilde{\chi}_1^0$	$\mu \tan \beta$
126	200	108	210	90	$6.1 \times 10^3$

[in GeV]

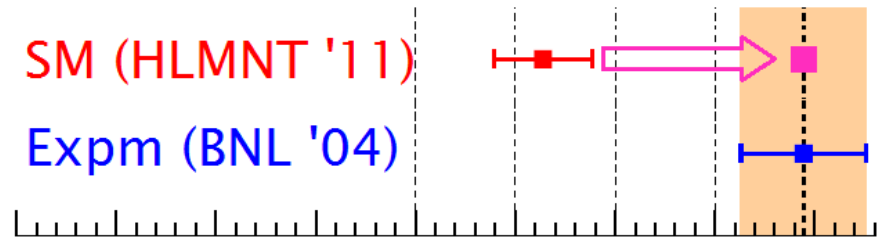
ILC



$\rightsquigarrow \sin \theta_{\tilde{\mu}} = 0.027, \sin \theta_{\tilde{\tau}} = 0.36,$



$\approx 2.6 \times 10^{-9}$



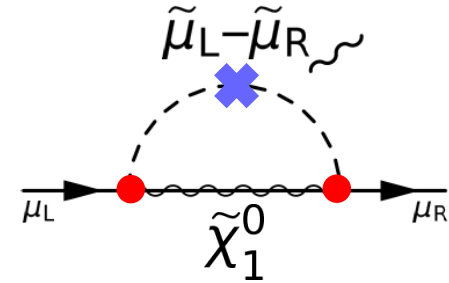
- Satisfies LEP/LHC constraints.
- Close to SPS1a(')

→ We can consult Previous works! Don't call us lazy :)

## How can we measure

- Mass of  $\tilde{\mu}_1, \tilde{\mu}_2, \tilde{\chi}_0^1$
- Mixing  $M_{LR}^2$
- Coupling  $\tilde{g}_L, \tilde{g}_R$  ?

and How accurately?



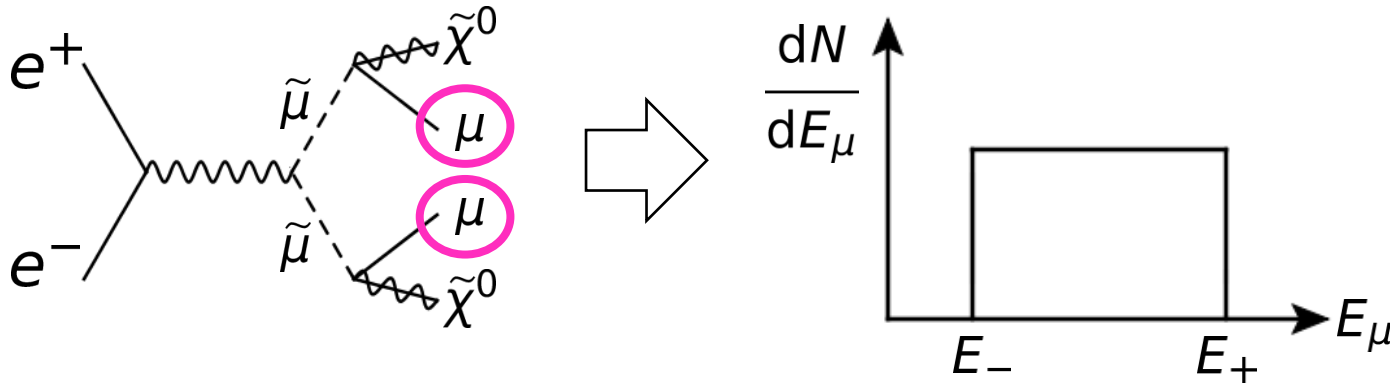
# How can we measure

- Mass of  $\tilde{\mu}_1, \tilde{\mu}_2, \tilde{\chi}_0^1$

End-point analysis  $\rightarrow \Delta m_{\tilde{\mu}}, \Delta m_{\text{LSP}} \sim 100\text{--}200 \text{ MeV}$   
 (dominated by **stat.** unc.) ( $\sim 0.1\%$ )

@  $\sqrt{s} = 500 \text{ GeV}$ ,  $\int \mathcal{L} = 500 \text{ fb}^{-1}$

[ILC-TDR Vol.2 Sec.7.5.4]



$$E_{\pm} = \frac{\sqrt{S}}{4} (1 \pm \beta) \left( 1 - \frac{m^2}{M^2} \right); \quad \beta = \sqrt{1 - \frac{4M^2}{S}}$$

$$\begin{aligned} \sin \theta_{\tilde{\mu}} &= 0.027, & M_{LR}^2 &= -\frac{1}{2} (m_2^2 - m_1^2) \sin 2\theta \\ \sin \theta_{\tilde{\tau}} &= 0.36, \end{aligned}$$

# How can we measure

➤ Mixing  $M_{LR}^2$

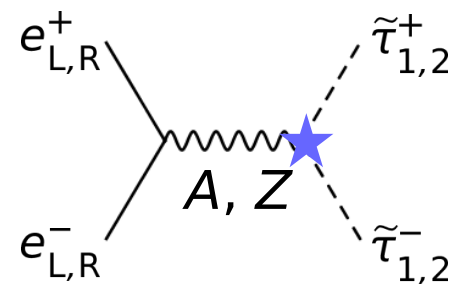
$$\mathcal{M}_{\tilde{\mu}}^2 = \begin{pmatrix} m_L^2 & M_{LR}^2 \\ M_{LR}^2 & m_R^2 \end{pmatrix}$$

$(M_{LR}^2 \simeq m_\mu \mu \tan \beta)$

$$\sigma(e^+ e^- \rightarrow \tilde{\tau}_A^+ \tilde{\tau}_B^-)$$

⇒  $\tilde{\tau}$  mixing  $M_{LR}^2(\tilde{\tau})$  measured.

⇒  $M_{LR}^2 = \frac{m_\mu}{m_\tau} M_{LR}^2(\tilde{\tau})$  (as long as  $A_{\tilde{\tau}} \simeq 0$ .)



$$\begin{aligned} \sigma(e^+ e^- \rightarrow \tilde{\tau}_i \tilde{\tau}_j) &= \frac{8\pi\alpha^2}{3s} v^3 \left[ c_{ij}^2 \frac{\Delta_Z^2}{\sin^4 2\theta_W} (\mathcal{P}_{-+} L^2 + \mathcal{P}_{+-} R^2) \right. \\ &\quad \left. + \delta_{ij} \frac{1}{16} (\mathcal{P}_{-+} + \mathcal{P}_{+-}) + \delta_{ij} c_{ij} \frac{\Delta_Z}{2 \sin^2 2\theta_W} (\mathcal{P}_{-+} L + \mathcal{P}_{+-} R) \right]; \end{aligned}$$

$$v^2 = [1 - (m_{\tilde{\tau}_i} + m_{\tilde{\tau}_j})^2/s][1 - (m_{\tilde{\tau}_i} - m_{\tilde{\tau}_j})^2/s], \quad \Delta_Z = s/(s - m_Z^2),$$

$$c_{11/22} = \frac{1}{2} [L + R \pm (L - R) \cos 2\theta_{\tilde{\tau}}],$$

$$c_{12} = c_{21} = \frac{1}{2} (L - R) \sin 2\theta_{\tilde{\tau}},$$

$$L = -\frac{1}{2} + \sin^2 \theta_W,$$

$$R = \sin^2 \theta_W.$$

$$\sin \theta_{\tilde{\mu}} = 0.027, \quad M_{LR}^2 = -\frac{1}{2} (m_2^2 - m_1^2) \sin 2\theta$$

$$\sin \theta_{\tilde{\tau}} = 0.36,$$

## How can we measure

➤ Mixing  $M_{LR}^2$

$$\sigma(e^+ e^- \rightarrow \tilde{\tau}_A^+ \tilde{\tau}_B^-)$$

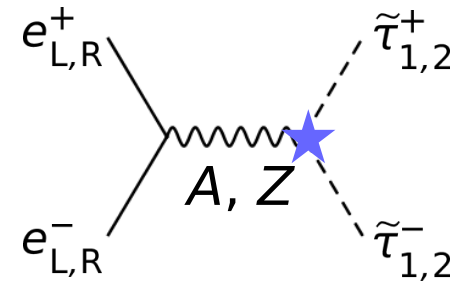
⇒  $\tilde{\tau}$  mixing  $M_{LR}^2(\tilde{\tau})$  measured.

$$\Rightarrow M_{LR}^2 = \frac{m_\mu}{m_\tau} M_{LR}^2(\tilde{\tau}) \quad (\text{as long as } A_{\tilde{\tau}} \simeq 0.)$$

$$\sigma(\tilde{\tau}_A \tilde{\tau}_B) = \begin{cases} (\dots) + (\dots) \cos 2\theta + (\dots) \cos^2 2\theta & (A = B) \\ (\dots) \sin^2 2\theta & (A \neq B) \end{cases}$$

$$\mathcal{M}_{\tilde{\mu}}^2 = \begin{pmatrix} m_L^2 & M_{LR}^2 \\ M_{LR}^2 & m_R^2 \end{pmatrix}$$

$$(M_{LR}^2 \simeq m_\mu \mu \tan \beta)$$



# How can we measure

➤ Mixing  $M_{LR}^2$

$$\begin{aligned} \sin \theta_{\tilde{\mu}} &= 0.027, & M_{LR}^2 &= -\frac{1}{2} (m_2^2 - m_1^2) \sin 2\theta \\ \sin \theta_{\tilde{\tau}} &= 0.36, & & \end{aligned}$$

$$\sigma(\tilde{\tau}_A \tilde{\tau}_B) = \begin{cases} (\dots) + (\dots) \cos 2\theta + (\dots) \cos^2 2\theta & (A = B) \\ (\dots) \sin^2 2\theta & (A \neq B) \end{cases}$$

$$\sigma(e^+ e^- \rightarrow \tilde{\tau}_A^+ \tilde{\tau}_B^-)$$

⇒  $\tilde{\tau}$  mixing  $M_{LR}^2(\tilde{\tau})$  measured.

$$\Rightarrow M_{LR}^2 = \frac{m_\mu}{m_\tau} M_{LR}^2(\tilde{\tau}) \quad (\text{as long as } A_{\tilde{\tau}} \simeq 0.)$$

$$\Delta M_{LR}^2 = \Delta m_{\tilde{\tau}1} \oplus \Delta m_{\tilde{\tau}2} \oplus \Delta \sigma \left( \tilde{\tau}_A^+ \tilde{\tau}_B^- \right)$$

↓

~ 0.1%

↓

~ 3%

⌈ @500 GeV, 500 fb<sup>-1</sup>;  
(P<sub>+</sub>, P<sub>-</sub>) = (-0.3, +0.8) ⌋

ILC-TDR Vol.2 Sec.7.5.4 (and Bechtle, Berggren, et al. [0908.0876])

[All values are for the sample mass spectrum.] **71** / 26

# How can we measure

➤ Mixing  $M_{LR}^2$

$$\begin{aligned} \sin \theta_{\tilde{\mu}} &= 0.027, & M_{LR}^2 &= -\frac{1}{2} (m_2^2 - m_1^2) \sin 2\theta \\ \sin \theta_{\tilde{\tau}} &= 0.36, & & \end{aligned}$$

$$\sigma(\tilde{\tau}_A \tilde{\tau}_B) = \begin{cases} (\dots) + (\dots) \cos 2\theta + (\dots) \cos^2 2\theta & (A = B) \\ (\dots) \sin^2 2\theta & (A \neq B) \end{cases}$$

$$\Delta\sigma(\tilde{\tau}_1 \tilde{\tau}_1) = \frac{1.8 \text{ fb}}{54 \text{ fb}} = 3.4\% \quad \Rightarrow \quad \Delta M_{LR}^2 = 12\%$$

(stat. dominated)

Not precise...

$$\Delta M_{LR}^2 = \Delta m_{\tilde{\tau}_1} \oplus \Delta m_{\tilde{\tau}_2} \oplus \Delta\sigma(\tilde{\tau}_A^+ \tilde{\tau}_B^-)$$

$\downarrow$   
 $\sim 0.1\%$

$\downarrow$   
 $\sim 3\%$

$\left[ \begin{array}{l} @500 \text{ GeV}, 500 \text{ fb}^{-1}; \\ (P_+, P_-) = (-0.3, +0.8) \end{array} \right]$

ILC-TDR Vol.2 Sec.7.5.4 (and Bechtle, Berggren, et al. [0908.0876])

[All values are for the sample mass spectrum.] **72**/<sub>26</sub>



$$\begin{aligned} \sin \theta_{\tilde{\mu}} &= 0.027, & M_{LR}^2 &= -\frac{1}{2} (m_2^2 - m_1^2) \sin 2\theta \\ \sin \theta_{\tilde{\tau}} &= 0.36, & & \end{aligned}$$

## How can we measure

➤ Mixing  $M_{LR}^2$

$$\sigma(\tilde{\tau}_A \tilde{\tau}_B) = \begin{cases} (\dots) + (\dots) \cos 2\theta + (\dots) \cos^2 2\theta & (A = B) \\ (\dots) \sin^2 2\theta & (A \neq B) \end{cases}$$

$$\Delta\sigma(\tilde{\tau}_1 \tilde{\tau}_1) = \frac{1.8 \text{ fb}}{54 \text{ fb}} = 3.4\% \quad \Rightarrow \quad \Delta M_{LR}^2 = 12\%$$

(stat. dominated)

Not precise...

$$\Delta\sigma(\tilde{\tau}_1 \tilde{\tau}_2) = \frac{??? \text{ fb}}{2.7 \text{ fb}} = \dots \quad \rightarrow \text{ should be studied!}$$

$$\Delta M_{LR}^2 = \Delta m_{\tilde{\tau}_1} \oplus \Delta m_{\tilde{\tau}_2} \oplus \Delta\sigma(\tilde{\tau}_A^+ \tilde{\tau}_B^-)$$

$\downarrow$   
 $\sim 0.1\%$

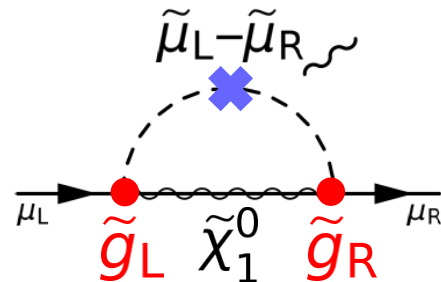
$\downarrow$   
 $\sim 3\%$

$\left[ \begin{array}{l} @500 \text{ GeV}, 500 \text{ fb}^{-1}; \\ (P_+, P_-) = (-0.3, +0.8) \end{array} \right]$

ILC-TDR Vol.2 Sec.7.5.4 (and Bechtle, Berggren, et al. [0908.0876])

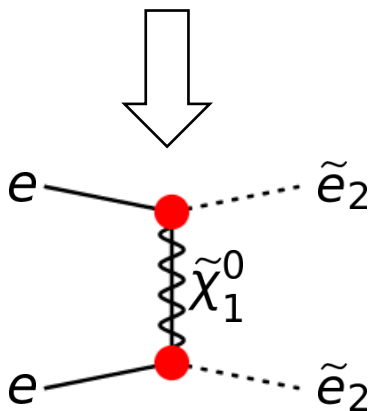
# How can we measure

➤ Coupling  $\tilde{g}_L, \tilde{g}_R$



$$\mu_R \text{---} \tilde{g}_R \text{---} \tilde{\chi}_1^0 \text{---} \tilde{\mu}_R = e_R \text{---} \tilde{g}_R^{(e)} \text{---} \tilde{\chi}_1^0 \text{---} \tilde{e}_R (\approx \tilde{e}_2) + \tilde{H}^0\text{-contribution} (\propto Y_\mu)$$

measured via



< 0.4% contrib.  
for  $\tilde{H} > 500 \text{ GeV}$

$$\Delta\sigma \sim \frac{4.7 \text{ fb}}{316 \text{ fb}} = 1.5\% \rightsquigarrow \Delta\tilde{g}_R^{(e)} \sim \underline{0.4\%}$$

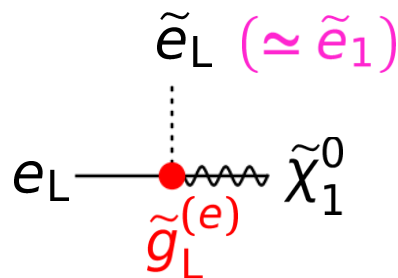
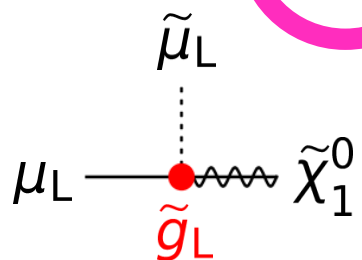
$$\therefore \Delta\tilde{g}_R \lesssim 1\%$$

Freitas, Kalinowski, et al. [[ph/0211108](#)]  
 Freitas, Manteuffel, Zerwas [[ph/0310382](#)]  
 Kilian, Zerwas [[ph/0601217](#)]

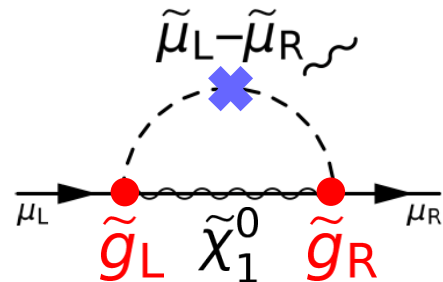
(@500 GeV,  $500 \text{ fb}^{-1}$ ;  
 $(P_+, P_-) = (-0.3, +0.8)$ )

# How can we measure

➤ Coupling  $\tilde{g}_L, \tilde{g}_R$

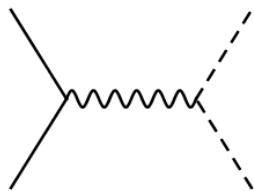


+  $\tilde{H}^0$ -contribution  
( $\propto Y_\mu$ )

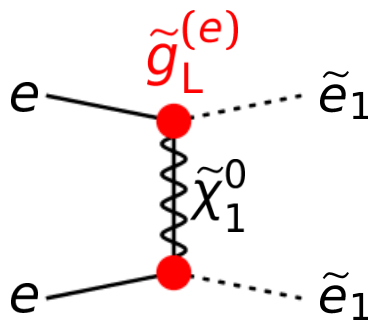


However

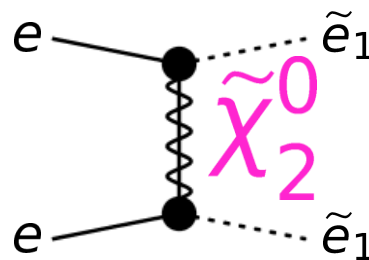
$$\sigma(\tilde{e}_L \tilde{e}_L) \approx$$



+

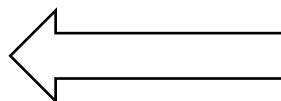


+



↓ ( $\simeq \tilde{W}$ )

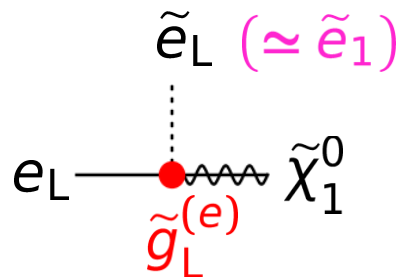
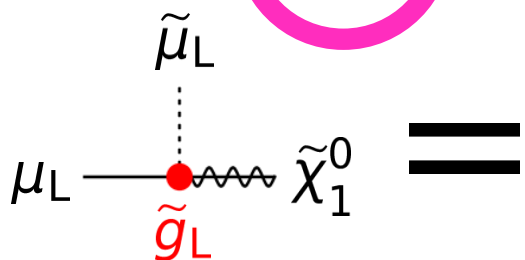
We use  $\sigma(\tilde{e}_L \tilde{e}_R)$ .



cannot be neglected.

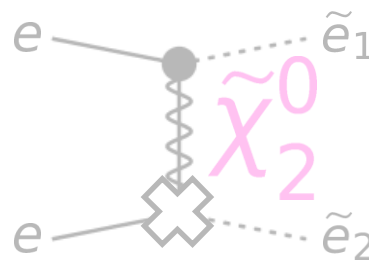
# How can we measure

➤ Coupling  $\tilde{g}_L, \tilde{g}_R$

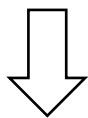


$\tilde{H}^0$ -contribution  
( $\propto Y_\mu$ )

< 0.9% contrib.  
for  $\tilde{M}, \tilde{W} > 500$  GeV



$$\sigma(\tilde{e}_L \tilde{e}_R) \approx$$

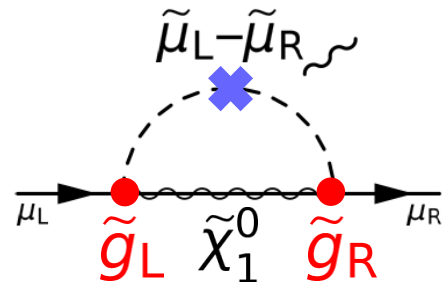


$$\Delta\sigma \sim \text{???}\% \quad (\sigma = 5.5 \text{ fb})$$

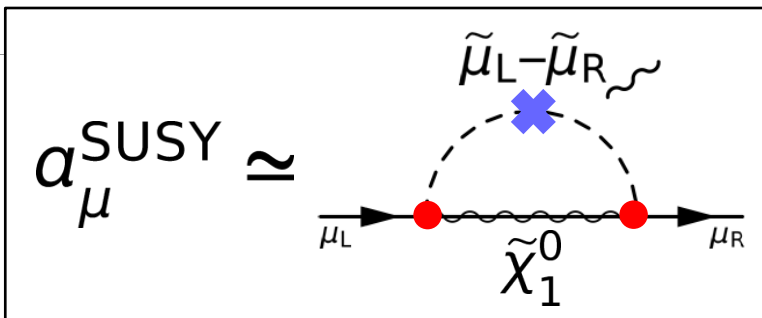
should be studied...

Here we use  $\Delta\tilde{g}_L^{(e)} \sim \underline{\text{a few}\%}$

$\therefore \Delta\tilde{g}_L \sim 1 + \text{a few}\%$



# Summary



$\therefore \Delta a_{\mu}^{\text{SUSY}} = 13\%$

Mass of  $\tilde{\mu}_1, \tilde{\mu}_2, \tilde{\chi}_1^0$

Mixing  $M_{LR}^2$

coupling  $\tilde{g}_L, \tilde{g}_R$

$e^+e^- \rightarrow \tilde{\mu}^+\tilde{\mu}^-$   
end-point

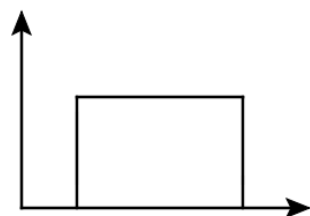
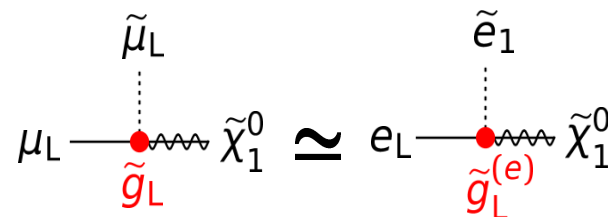
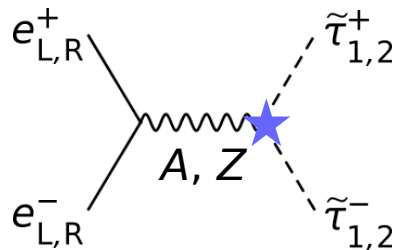
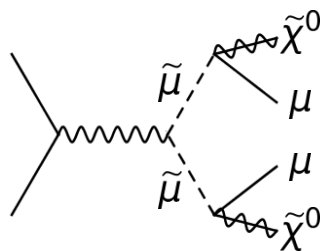
$\sigma(ee \rightarrow \tilde{\tau}_1\tilde{\tau}_1)$

$\sigma(ee \rightarrow \tilde{e}_R\tilde{e}_R),$   
 $\sigma(ee \rightarrow \tilde{e}_L\tilde{e}_R)$

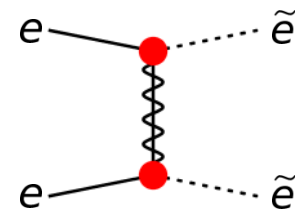
$\rightarrow \sim 0.1\%$

$\rightarrow \sim 12\%$

$\rightarrow$  R:  $\sim 1\%$   
L: (a few + 1)%

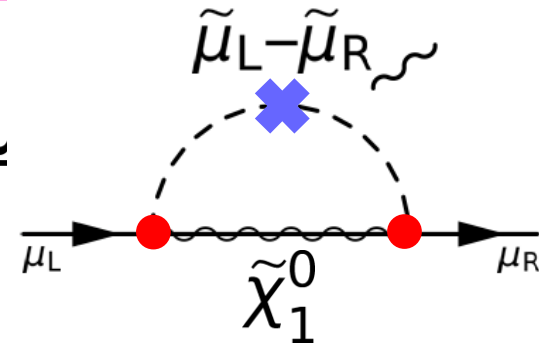


$M_{LR}^2 \simeq m_{\mu} \mu \tan \beta$   
 $\simeq \frac{m_{\mu}}{m_{\tau}} M_{LR}^2(\tilde{\tau})$



## For the scenario

- $\tilde{g}, \tilde{q}, \tilde{H}, \tilde{W} \gg 100 \text{ GeV}$ ,
- $\tilde{e}, \tilde{\mu}, \tilde{\tau} < \text{ILC reach}$ ,



$a_{\mu}^{\text{SUSY}}$  can reconstructed via

Mass of  $\tilde{\mu}_1, \tilde{\mu}_2, \tilde{\chi}_0^1$

$e^+e^- \rightarrow \tilde{\mu}^+\tilde{\mu}^-$   
end-point

Mixing  $M_{LR}^2$

$\sigma(ee \rightarrow \tilde{\tau}_1\tilde{\tau}_1)$

coupling  $\tilde{g}_L, \tilde{g}_R$

$\sigma(ee \rightarrow \tilde{e}_R\tilde{e}_R)$ ,  
 $\sigma(ee \rightarrow \tilde{e}_L\tilde{e}_R)$

with the precision **13%** (at our sample point).

can be improved if we use  
 $\sigma(ee \rightarrow \tilde{\tau}_1\tilde{\tau}_2)$ .

Largely depends  
on **mixing**.