MAGISTERIAL THESIS

Supersymmetry without R-Parity: Its Phenomenology

（R-Parity の保存を課さない超対称理論の現象論的側面）

岩本　祥

[Sho IWAMOTO]

Submitted in February 2010
Modified in March 2010

Department of Physics, Graduate School of Science,

The University of Tokyo

This thesis is a brief summary of what I learned, and a verbose explanation of what I studied, in the master's course.

PREFACE

I majored the particle physics, especially its phenomenological aspect. In my two years, I have learned the Standard Model, the supersymmetry and the minimal supersymmetric standard model, and the foundation of cosmology, collider physics, and grand unified theories, and finally studied cosmological constraints on R-parity violating parameters to write and submit a paper [1] with 遠藤 基 (Motoi ENDO) and 濱口 幸一 (Koichi HAMAGUCHI).

This thesis centers what I did (with the collaborators) in the paper, with some reviews of the R-parity. Also I present a brief summary of what I learned in the Appendices.

ACKNOWLEDGMENT

— Ego Omnia Debeo Factorem Terrae et Vniversitatis
Visibilivm Omnivm et Invisibilivm —

I am obliged to my adviser 濱口 幸一 for various suggestions, discussions, a collaboration, and encouragement with warmheartedness.

Also I am very grateful to 柳田 勉 for his pointing suggestions, advice, and support, and 遠藤 基 for a collaboration and a great deal of advice. I thank all the members of particle physics theory group at the University of Tokyo, and of 駒場 particle theory group, especially my eight colleagues.

Finally I express my sincere gratitude to my family, for all their support in spiritual and material terms.

ABSTRACT

We investigate in detail the R-parity violating SUSY, especially the constraints on its parameters. The constraints are mainly obtained from collider experiments, and they are of order 10^{-3}–10^{-4}. However, we found that, if lepton flavor violating processes are strong enough to equilibrate the lepton flavor asymmetry in the early universe, which is naturally expected in various models, the present baryon–antibaryon asymmetry brings us much more stringent constraints of order 10^{-6}–10^{-7}.

* * *

「超対称理論」とは、標準模型の内包する不自然さ「階層性問題」を解決するための有力な理論である。しかし標準模型をそのまま拡張すると、陽子の寿命が極めて短くなるという破滅が訪れる。ゆえに通常は R-parity という対称性（制約）を導入して陽子崩壊を回避し、幸福を実現する。

しかし、実は R-parity ではない、より弱い制約であっても、陽子崩壊の阻止が可能である。そのとき、制約を緩めた結果として新しい結合（相互作用）が導入されるが、それら新しい結合の大きさは加速器実験などの結果から量的に制限がかかっており、結合の大きさは比較的小さくなければならない。

この修士論文では、超対称標準模型に対して R-parity より弱い制約を課した模型について、その現象論的側面、とりわけ、上述の「実験的制限」を議論している。

また、もしも初期宇宙で lepton の flavor が十分大きく破れている場合には、更に厳しい制限が得られ、その制限は昨年再誕した LHC 実験において超対称理論を検証するという試みにも大きな役割を果たすと期待される。これらの点についても論じている。

REVISION LOG

2010.02.08 : Official Revision
2010.03.06 : Modification in App. C.1.3
2010.03.08 : Modification in Sec. 4.2 (but not in Sec. 4.2.1) and Figs. 4.1– 4.3
Adding some information to the Bibliography

Contents

Chapter 1

Prelude

◆SUSY and R-parity

We have the Standard Model, which describes almost all physics below the energy scale 100GeV. Although it is still under verification, especially the existence of the Higgs boson, the experiments held in the Large Hadron Collider (LHC) will work out the answer soon, which will be a declaration of the triumph of our philosophy.

However, the Standard Model contains one "unnaturalness," the hierarchy problem. The Higgs boson, a sole weird particle in the Standard Model, receives a large mass correction $\Delta m^2 \sim \left(10^{19}\text{GeV}\right)^2$, and forces us to realize a miraculous cancellation

$$m_{\text{bare}}^2 - \Delta m^2 = m_{\text{physical}}^2, \tag{1.1}$$

that is,

$$\text{O}\left(10^{38}\text{GeV}^2\right) - \text{O}\left(10^{38}\text{GeV}^2\right) = \text{O}\left(10^{4}\text{GeV}^2\right). \tag{1.2}$$

This problem originates from the separation between the electroweak scale 100GeV and the gravitational scale 10^{19}GeV, and thus it is called the hierarchy problem.

The most famous answer to this unnaturalness is the supersymmetry (SUSY) [2], a symmetry which transforms boson to fermion, or vice versa. In a supersymmetric theory, all particles accompany their supersymmetric partners, or "superpartners," and therefore if we extend the Standard Model with the SUSY, we have bosonic quarks, bosonic leptons, and fermionic gauge bosons, as the partners of the quarks, the leptons, and the gauge bosons. They are called "squarks," "sleptons," and "gauginos," respectively. They also contribute to the mass correction, and under the SUSY, the correction is calculated to be zero.[*1] Also, the SUSY is significant for grand unification theories (GUTs) and string theories.

[*1] For a more detailed discussion, See Ref. [3].

However, very sad to say, the minimal supersymmetric standard model (MSSM) [4, 5, 6], which is the minimal supersymmetric extension of the Standard Model, has a big *problem*, not unnaturalness. Under the MSSM, the lifetime of proton is naïvely estimated to be less than one second. If we would like to obtain the current experimental bounds 10^{29}yr [7], we must yet introduce an unnaturalness of order 10^{13}.

Why does this proton decay problem emerge? — In the Standard Model, the baryon number B and the lepton number L are accidentally conserved by the gauge symmetry. For the rigidity of the gauge symmetry and the skimpiness of the field content, we could not construct B- or L-violating operators. However, in the MSSM, the field content is doubly extended. Now B- and L-violating operators can be constructed, which invoke proton decay.

To solve this proton decay problem, usually we impose the R-parity [6], a Z_2 symmetry which forbids B- and L-violating operators again, on the MSSM. This seems a nice way, because we have never observed B- or L-violating events. Also, this "MSSM with R-parity" provides a very nice explanation of the dark matter problem. We know that our familiar matters, e.g., electron, proton, and neutron, account for about 4% of the substance of this universe. [8, 9] We consider that 21% of the substance is some other matter, called "dark matter," and the rest 75% is not even matter, which we call "dark energy." As we will discuss in this thesis (Chap. B), if the R-parity is conserved, the lightest supersymmetric particle (LSP) becomes stable in the MSSM scheme. Therefore, if the LSP has appropriate mass, it can be a good candidate of the dark matter.

As we have seen, the R-parity is a very attractive choice. It explains even the dark matter problem, as well as the proton decay problem. However, it is installed arbitrarily. We just imposed *by hand*. Therefore, fairly speaking, it is also unnatural. We should, to explore this mysterious universe, consider other choices than the R-parity, as well as the R-parity case.

Actually, we have other ways but the R-parity to circumvent the proton decay problem. If we impose the conservation of either *B or L*, proton decay does not occur. We call these models "SUSY without R-parity," or "R-parity violating SUSY," and in this thesis, we will explore the "SUSY without R-parity."

◆Why not *R*-parity?

But why do we abandon the very beautiful R-parity?

— Now we have just celebrated the rebirth of the LHC. In the LHC experiments, the discovery of the SUSY as well as the Higgs boson is expected. However, the studies on the detection of the SUSY are, almost all of them, with the assumption of the R-parity conservation.

If the R-parity conservation is realized in nature, we will discover the SUSY at the LHC soon, which will solve even the dark matter problem, and then we will come to develop deeper understanding of the universe. However, if not? Then, we might be unable to know the existence of the SUSY even if the SUSY is realized in nature. Also we will have no answer to the dark matter problem, and even be unable to reject the scenario that the dark matter is the LSP.

In the last decade, 2000's, We had long waited for the LHC. Now, at last, the time has come. We should exhaust the experimental results obtained at the LHC, and to this end, it is important to be free from any obsessions, as well as be stick to the beautiful, attractive scenario.

◆Outline of this thesis

This thesis focuses on the R-parity violating SUSY, and discuss its phenomenological aspects.

The author studied and wrote a paper [1], with two collaborators 遠藤 基 (Motoi ENDO) and 濱口 幸一 (Koichi HAMAGUCHI), about cosmological constraints on the magnitude of the R-parity violation. We will discuss what we presented in the paper in Chapter 4, with much verbosity.

As preparatory of the discussion, in Chapter 2, we review the R-parity violating SUSY, and some constraints obtained mainly from collider experiments. Also, in Chapter 3, we review the property of the universe before the electroweak phase transition (temperature $T \gtrsim 100\text{GeV}$), which we considered in the paper. The last part, Chapter 5, is devoted to conclusion and discussion.

Also we present several appendices. Appendix A is a brief review of the Standard Model. In Appendix B, the SUSY is reviewed, and we discuss the higher dimensional proton decay operators and the R-parity. We will see that the R-parity is not sufficient to prohibit the proton decay. Appendix C is a brief review of the cosmology, mainly on the Hubble expansion.

Chapter 2

SUSY and R-Parity

To begin with, we discuss the R-parity, its effect to the minimal supersymmetric standard model (MSSM), and the restrictions on the R-parity violating parameters.

Section 2.1 Review: the R-Parity

2.1.1 Proton Decay Problem

The superpotential of the MSSM is constructed as*1

$$\begin{aligned} W = \mu H_\mathrm{u} H_\mathrm{d} + y_{uij} H_\mathrm{u} Q_i \bar{U}_j + y_{dij} H_\mathrm{d} Q_i \bar{D}_j + y_{eij} H_\mathrm{d} L_i \bar{E}_j \\ + \kappa_i H_\mathrm{u} L_i + \frac{1}{2}\lambda_{ijk} L_i L_j \bar{E}_k + \lambda'_{ijk} L_i Q_j \bar{D}_k + \frac{1}{2}\lambda''_{ijk} \bar{U}_i \bar{D}_j \bar{D}_k. \end{aligned} \tag{2.1}$$

Here, the $\bar{U}\bar{D}\bar{D}$ term violates the baryon number B, and three operators $LL\bar{E}$, $LQ\bar{D}$, and $H_\mathrm{u}L$ violate the lepton number L. These terms cause a disastrous event, the decay of proton. The Feynman diagram of the decay is, for example, described as Fig. 2.1. Here, $\bar{U}_1\bar{D}_1\bar{D}_2$ ($\Delta B = \pm 1$) and $L_1 Q_1 \bar{D}_2$ ($\Delta L = \pm 1$) interactions invoke $p \to \pi e^+$ decay. The decay rate Γ is approximately

$$\Gamma \sim \left|\lambda'_{112}\lambda''_{112}\right|^2 \frac{m_\mathrm{proton}^5}{m_{\tilde{s}_\mathrm{R}}^4} = \frac{\left|\lambda'_{112}\lambda''_{112}\right|^2}{2.9\times 10^{-20}\mathrm{yr}} \left(\frac{1\mathrm{TeV}}{m_{\tilde{s}_\mathrm{R}}}\right)^4, \tag{2.2}$$

while the lifetime of proton according to this decay mode is measured as longer than 1.6×10^{33}yr (90% confidence level) [7]. Therefore those parameters are restricted as

$$\left|\lambda'_{112}\lambda''_{112}\right| \lesssim 10^{-27}\left(\frac{m_{\tilde{s}_\mathrm{R}}}{1\mathrm{TeV}}\right)^2, \tag{2.3}$$

which is *unnatural*. This is the proton decay problem.

*1 Here we use the convention $\lambda_{ijk} = -\lambda_{jik}$ and $\lambda''_{ijk} = -\lambda''_{ikj}$. For more detail information, see App. B.1.

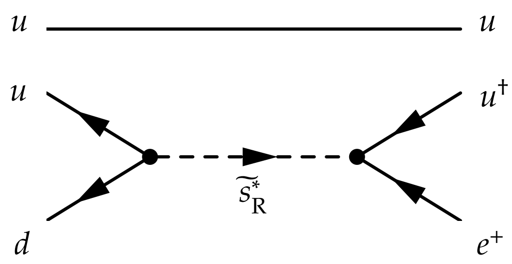

Fig. 2.1 Feynman diagram of the proton decay (with no suppression). The time goes from left to right, and the arrows denote the directions of the *left chirality*. For example, initial u and d must be right-handed, and the intermediate $\widetilde{s}_\mathrm{R}^*$ is left-handed, since it is an antiparticle of a right-handed particle.

Thus, in order to solve this problem, we usually install the conservation of the R-parity [6] into the MSSM. The R-parity is a discrete Z_2 symmetry defined as

$$P_R := (-1)^{3B-L+2s}, \tag{2.4}$$

where B, L and s are the baryon number, the lepton number and the spin of the particle, respectively.

The exact conservation of the R-parity restricts the superpotential as

$$W = W_\mathrm{RPC} := \mu H_\mathrm{u} H_\mathrm{d} + y_{\mathrm{u}_{ij}} H_\mathrm{u} Q_i \bar{U}_j + y_{\mathrm{d}_{ij}} H_\mathrm{d} Q_i \bar{D}_j + y_{\mathrm{e}_{ij}} H_\mathrm{d} L_i \bar{E}_j. \tag{2.5}$$

Note that the R-parity makes B and L again conserved in the MSSM, as they were in the Standard Model. Under this superpotential, the proton decay could not occur, and again the proton would be a stable particle.

* * *

Now we seem to have circumvented the proton decay problem. However, to be honest with nature, we have to consider higher-dimensional operators which can invoke the proton decay.

The discussion about the higher-dimensional proton decay is presented in App. B.2.1, and there we will conclude that we should use another symmetry, the proton hexality [10], than the R-parity. Though, the phenomenology under the proton hexality is almost the same as that under the R-parity, and in almost all cases, we need not look after the difference between the R-parity and the proton hexality.

Therefore, here we do not pay attention to the higher-dimensional proton decay, and go forward with the R-parity.

2.1.2 R-Parity and Dark Matter

Now we have circumvented the proton decay problem for the sake of the R-parity. Actually, this R-parity conservation brings us another attractive feature. That is, a solution of the dark matter problem.

The definition of the R-parity is equivalent to the following one:

$$\begin{cases} P_R = +1 & \text{for Standard Model particles,} \\ P_R = -1 & \text{for superpartners.} \end{cases} \tag{2.6}$$

Consider the lightest particle among the superpartners (R-*odd* particles). This particle, which is called the lightest supersymmetric particle (LSP), cannot decay under the R-parity conservation, because lighter particles than the LSP are all *even* in the R-parity. Therefore, if the R-parity is conserved, the LSP is always stable, and would be an attractive candidate for the dark matter.

* * *

Actually, the R-parity conserving scenario is respected for this feature, that is, for *one* symmetry solves *two* problems. As we will see in the next section, resultingly we must impose a constraint to forbid proton decay even if we are away from the R-parity conserving models. When we come to impose a symmetry, we surely want to use the one which solves two problem. Therefore, we usually use the R-parity.

2.1.3 Other Choices than R-Parity

Here we shall mention an interesting properties of the proton decay. What we would like to say is, the proton decay does not occur if at least one of the following two properties is satisfied:

1. The baryon parity $(-1)^{3B}$ is conserved.
2. The lepton parity $(-1)^{L}$ is conserved and the LSP is heavier than proton ($m_{\text{LSP}} > m_{\text{proton}}$).

The first condition is obvious. If proton would decay, the final state must be $B = 0$, because proton is the lightest baryon, and thus the baryon parity must change. Therefore, if the baryon parity is conserved, proton would not decay. Note that the conservation of the baryon number is a sufficient condition for this case.

On the other hand, the second one is a bit complicated and needs some explanation. Assume that the lepton parity is conserved. As we have just seen, the decay process must be $\Delta B = -1$, and thus $(-1)^{3B-L} = -1$, by the assumption. This means the R-parity of the final state is odd, so we need one superparticle in the final state. Therefore, if the lepton parity is conserved, the LSP must be lighter than proton for the proton decay to be invoked.

The conservation of the R-parity saturates the first condition as long as we consider only 4-dimensional operators of the MSSM, and therefore the proton decay is circumvented.

* * *

Now we can see that we have actually three choices to forbid the proton decay.

(i) Forbid both B- and L-violation: The first way is to impose the R-parity, or other symmetries, so that the superpotential is restricted as

$$W = W_{\mathrm{RPC}} := \mu H_{\mathrm{u}} H_{\mathrm{d}} + y_{\mathrm{u}_{ij}} H_{\mathrm{u}} Q_i \bar{U}_j + y_{\mathrm{d}_{ij}} H_{\mathrm{d}} Q_i \bar{D}_j + y_{\mathrm{e}_{ij}} H_{\mathrm{d}} L_i \bar{E}_j. \tag{2.7}$$

In this case, B and L are conserved as the Standard Model. Moreover, with great pleasure, the LSP becomes stable, and can be a candidate for the dark matter.

Note that we consider only the MSSM scheme. If we introduced other particles to extend the MSSM, the R-parity might not forbid B- or L-violation. (Imagine a superfield which carries $3B = L = 1$.)

(ii) Forbid B-violation: The second way is to forbid B-violating interactions and restrict the superpotential as

$$W = W_{\mathrm{RPC}} + \kappa_i H_{\mathrm{u}} L_i + \frac{1}{2}\lambda_{ijk} L_i L_j \bar{E}_k + \lambda'_{ijk} L_i Q_j \bar{D}_k \tag{2.8}$$

with imposing some symmetry. In this case the LSP is not responsible for the dark matter.

(iii) Forbid L-violation: The last way is to forbid L-violating interactions with some symmetry. The superpotential would be

$$W = W_{\mathrm{RPC}} + \frac{1}{2}\lambda''_{ijk} \bar{U}_i \bar{D}_j \bar{D}_k, \tag{2.9}$$

and in this case the LSP must be heavier than proton for fear proton might decay. Also the LSP is not a dark matter candidate.

> Now we consider only the MSSM, and its 4-dimensional operators. In this scheme, the conservation of the lepton parity and the lepton number are equivalent.
>
> Also note that we distinguish these three patterns by the form of the superpotential, not the symmetry imposed on, because we are interested in phenomenology.
>
> The discussion on what happens when we consider higher dimensional operators, and that on the symmetries which we should impose in each case, are presented in App. B.3.

The first choice is widely discussed. In this thesis, we focus on the second and the third cases, the R-parity violating MSSM.

Section 2.2 Constraints on the Couplings

Now we have circumvented the proton decay problem. However, the R-parity violating couplings

$$\kappa_i, \quad \lambda_{ijk}, \quad \lambda'_{ijk}; \qquad \lambda''_{ijk} \tag{2.10}$$

have other constraints, mainly from collider experiments.

We can eliminate the couplings κ_i by redefining the fields H_d and L_i. Thus we use the following form as the superpotential:

$$W = W_\mathrm{RPC} + \frac{1}{2}\lambda_{ijk}L_iL_j\bar{E}_k + \lambda'_{ijk}L_iQ_j\bar{D}_k \tag{2.11}$$

for the L-violating scenario, and

$$W = W_\mathrm{RPC} + \frac{1}{2}\lambda''_{ijk}\bar{U}_i\bar{U}_j\bar{D}_k \tag{2.12}$$

for the B-violating scenario, and discuss the constraints and the bounds on the R-parity violation parameters.

We have 9 + 27 L-violating parameters and 9 B-violating parameters. For simplicity, we will focus only on the absolute value, that is, we will ignore complex phases.

◆Single-coupling bounds

We saw in the last section that the product of λ'' and λ' is restricted as

$$\left|\lambda'_{112}\lambda''_{112}\right| \lesssim 10^{-27}\left(\frac{m_{\tilde{s}_\mathrm{R}}}{1\mathrm{TeV}}\right)^2. \tag{2.13}$$

This is surely a constraint on the R-parity violating couplings, especially a constraint on a *product* of the couplings.

In the context of the constraints, however, usually "single-coupling bounds," which are the bounds on the indivisual R-parity violating couplings when only the particular coupling is non-zero, are discussed. This is mostly for simplicity, but not very unreasonable, because the bounds of products are generally much more severe than the single-coupling bounds, as we saw for the proton decay case.

In this thesis, we mainly focus on the single-coupling bounds.

* * *

However, we should be careful when we discuss single-coupling bounds, because several constraints are those on the *difference* of the R-parity violating couplings. We will see examples of such situations soon in the following discussion.

◆Preparation

To simplify expressions, we follow Refs. [11, 12] and define

$$r_{ijk}(X) := \frac{1}{4\sqrt{2}G_{\mathrm{F}}}\frac{\left|\lambda_{ijk}\right|^2}{m_X^2}, \qquad r'_{ijk}(X) := \frac{1}{4\sqrt{2}G_{\mathrm{F}}}\frac{\left|\lambda'_{ijk}\right|^2}{m_X^2}, \tag{2.14}$$

and in addition,

$$r_{ijk;lmn}(X) := \frac{1}{4\sqrt{2}G_{\mathrm{F}}}\frac{\Re\left(\lambda^*_{ijk}\lambda'_{lmn}\right)}{m_X^2}, \tag{2.15}$$

where m_X is the mass of a particle X and G_{F} is the Fermi constant, $1.116\times10^{-5}\mathrm{GeV}^{-2}$.

2.2.1 μ and τ Decay: for λ_{ijk} etc.

First we consider the leptonic decay of μ and τ, as discussed in Ref. [13], and previously in Refs. [11, 14] (charged current universality). This discussion yields the bounds on λ_{ijk}.

Here we consider the decay rate of the events

$$e_i \to \nu_i e_j \nu_j^\dagger, \tag{2.16}$$

and corresponding two values:

$$R_\tau = \frac{\Gamma(\tau\to\nu_\tau\, e\, \nu_e^\dagger)}{\Gamma(\tau\to\nu_\tau\, \mu\, \nu_\mu^\dagger)}, \qquad R_{\tau\mu} = \frac{\Gamma(\tau\to\nu_\tau\, \mu\, \nu_\mu^\dagger)}{\Gamma(\mu\to\nu_\mu\, e\, \nu_e^\dagger)}. \tag{2.17}$$

In the Standard Model, or the R-parity conserving MSSM (R_{p}-MSSM), these processes are invoked mainly by the gauge interaction mediated by W-boson. However, if we have

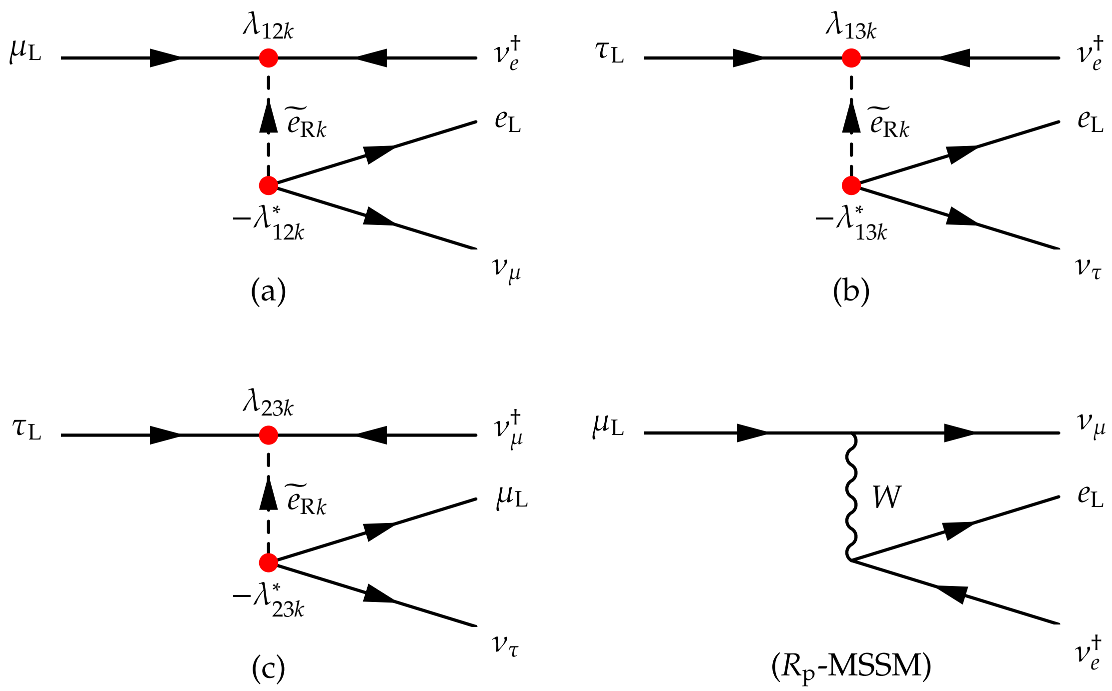

Fig. 2.2 Possible R-parity violating contributions to (a) $\mu \to e\nu_e^\dagger\nu_\mu$, (b) $\tau \to e\nu_e^\dagger\nu_\tau$, (c) $\tau \to \mu\nu_\mu^\dagger\nu_\tau$, and corresponding R_p-MSSM process.

$LL\bar{E}$ term, it also contributes to the process. See Fig. 2.2 for the Feymnan diagrams of the R_p-MSSM process and $LL\bar{E}$-induced case.

Therefore, the values R_τ and $R_{\tau\mu}$ are shifted by the R-parity violating processes. The shifts are calculated as

$$\frac{R_\tau}{(R_\tau)_{\text{SM}}} = 1 + 2\sum_k \left[r_{13k}(\widetilde{e}_{Rk}) - r_{23k}(\widetilde{e}_{Rk})\right], \tag{2.18}$$

$$\frac{R_{\tau\mu}}{(R_{\tau\mu})_{\text{SM}}} = 1 + 2\sum_k \left[r_{23k}(\widetilde{e}_{Rk}) - r_{12k}(\widetilde{e}_{Rk})\right], \tag{2.19}$$

which means that the difference between the experimental result and the Standard Model expected values of R_τ and $R_{\tau\mu}$ give us constraints on the R-parity violating couplings r_{ijk}, that is, λ_{ijk}.

The calculated results under the Standard Model v.s. the experimental results are

$$\frac{R_\tau}{(R_\tau)_{\text{SM}}} = \frac{1.028(4)}{1.028} \qquad \frac{R_{\tau\mu}}{(R_{\tau\mu})_{\text{SM}}} = \frac{1.312(6)\times 10^6}{1.309\times 10^6}, \tag{2.20}$$

where the Standard Model precision values (denominators) are obtained from Ref. [12], and the experimental results (numerators) are from Ref. [7]. Therefore we obtain the following $2\sigma = 95\%$ bounds:

$$-0.051^2 < \sum_k \left[|\lambda_{13k}|^2 - |\lambda_{23k}|^2\right]\left(\frac{100\text{GeV}}{m_{\tilde{e}_{\text{R}k}}}\right)^2 < 0.051^2 \qquad (R_{\tau\mu}) \tag{2.21}$$

$$-0.048^2 < \sum_k \left[|\lambda_{23k}|^2 - |\lambda_{12k}|^2\right]\left(\frac{100\text{GeV}}{m_{\tilde{e}_{\text{R}k}}}\right)^2 < 0.062^2. \qquad (R_{\tau}) \tag{2.22}$$

* * *

If we see these bounds from the viewpoint of single-coupling bounds, they are

$$\lambda_{13k} < 0.051, \qquad \lambda_{23k} < 0.051, \qquad \lambda_{12k} < 0.048, \tag{2.23}$$

for $m = 100$GeV. However, we can see easily that, e.g.,

$$\lambda_{131} = \lambda_{231} = \lambda_{121} = 0.3, \qquad \text{others} = 0 \tag{2.24}$$

is allowed within these constraints. As you can see, we had better be aware that the single-couplings are not the true bounds of the parameters, and we should review the constraining equations, e.g., Eqs. (2.21) and (2.22), even if we overlook the bounds on the products of the couplings.*2

2.2.2 π AND τ DECAY: FOR λ'_{i1k} ETC.

Next, in order to constrain λ'_{i1k}, we consider the values

$$R_\pi = \frac{\Gamma(\pi^- \to e\,\nu_e^\dagger)}{\Gamma(\pi^- \to \mu\,\nu_\mu^\dagger)}, \qquad R_{\tau\pi} = \frac{\Gamma(\tau \to \pi^-\,\nu_\tau^\dagger)}{\Gamma(\pi^- \to \mu\,\nu_\mu^\dagger)}. \tag{2.25}$$

This discussion is also from Ref. [13] and Refs. [11, 14]. The processes induced by the R-parity violating terms, and that of the R_{p}-MSSM, are in Fig. 2.3.

The decay rates of the events are shifted by the R-parity violating processes as [15, 16]*3

$$\frac{\Gamma(\pi \to e_i \nu_i^\dagger)}{\Gamma_{\text{SM}}(\pi \to e_i \nu_i^\dagger)} = 1 + \frac{2}{|V_{ud}|}\sum_k \left[r'_{i1k}(\tilde{d}_{\text{R}k}) - \frac{2m_\pi^2}{m_{e_i}(m_u + m_d)} r_{iki;k11}(\tilde{e}_{\text{L}k})\right], \tag{2.26}$$

$$\frac{\Gamma(\tau \to \pi^- \nu_\tau)}{\Gamma_{\text{SM}}(\tau \to \pi^- \nu_\tau)} = 1 + \frac{2}{|V_{ud}|}\sum_k \left[r'_{31k}(\tilde{d}_{\text{R}k}) - \frac{134.2\text{MeV}}{m_u + m_d} r_{k33;k11}(\tilde{e}_{\text{L}k})\right], \tag{2.27}$$

---

*2 We will see in the next discussion that overlooking the bounds on the products is not so bad because generally the products are severely constrained.

*3 We calculate Eq. (2.27) by ourselves in App. 2.i, while Eq. (2.26) is obtained from Ref. [16].

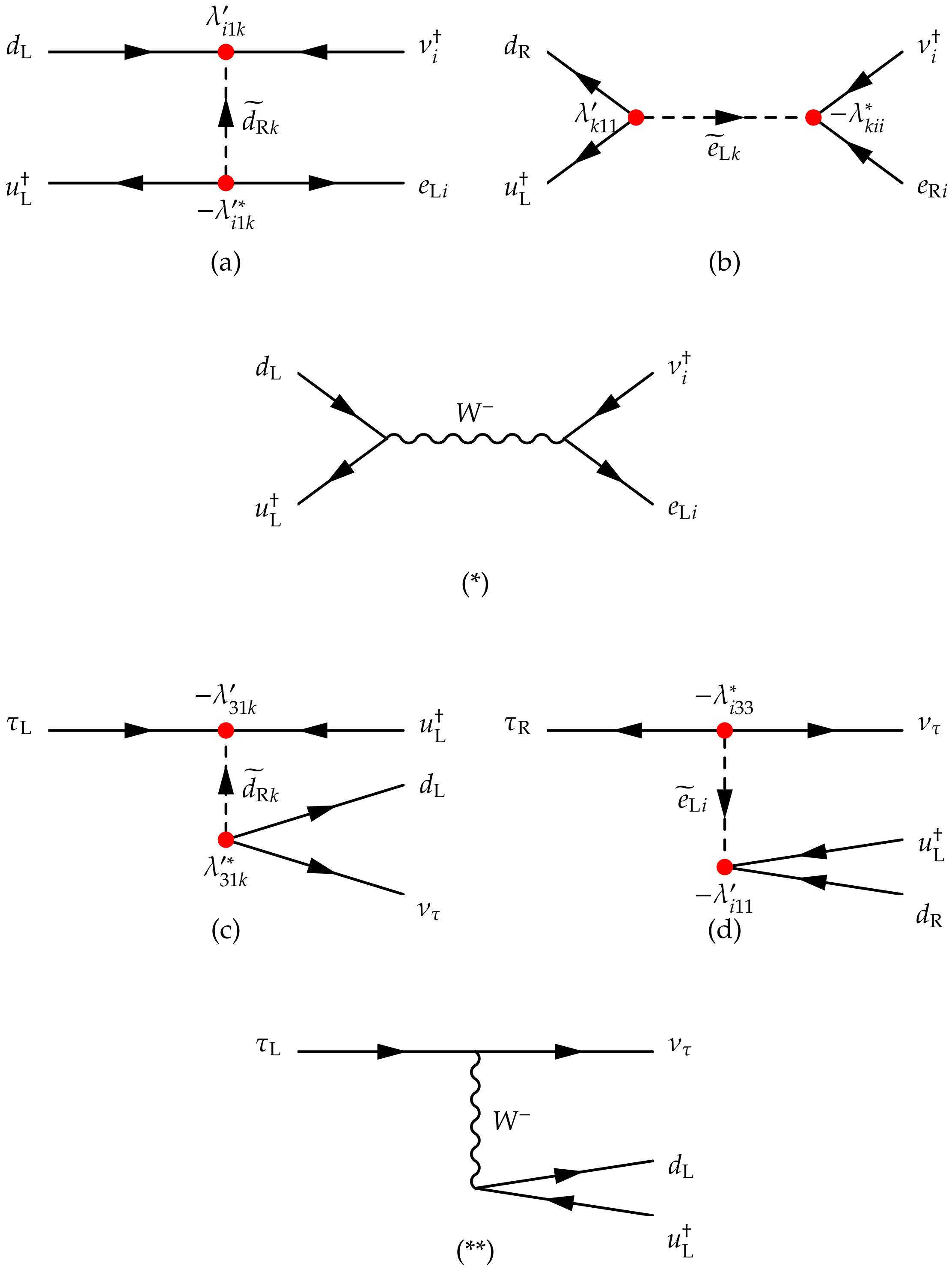

Fig. 2.3 Possible R-parity violating contributions to (a,b) $\pi^- \to e_i \nu_j^\dagger$, (c,d) $\tau \to \pi^- \nu_\tau$, and (*,**) corresponding R_p-MSSM processes. Note that $i \neq k$ in (b), and $i \neq 3$ in (d).

and thus the values R_π and $R_{\tau\pi}$ are also shifted, as

$$\frac{R_\pi}{(R_\pi)_{\rm SM}} = 1 + \frac{2}{|V_{ud}|}\sum_k \Big[r'_{11k}(\widetilde{d}_{{\rm R}k}) - r'_{21k}(\widetilde{d}_{{\rm R}k}) - \frac{7.624\times 10^4 {\rm MeV}}{m_u + m_d} r_{1k1;k11}(\widetilde{e}_{{\rm L}k}) + \frac{368.7{\rm MeV}}{m_u+m_d} r_{2k2;k11}(\widetilde{e}_{{\rm L}k}) \Big] \quad (2.28)$$

$$\frac{R_{\tau\pi}}{(R_{\tau\pi})_{\rm SM}} = 1 + \frac{2}{|V_{ud}|}\sum_k \Big[r'_{31k}(\widetilde{d}_{{\rm R}k}) - r'_{21k}(\widetilde{d}_{{\rm R}k}) - \frac{134.2{\rm MeV}}{m_u + m_d} r_{k33;k11}(\widetilde{e}_{{\rm L}k}) + \frac{368.7{\rm MeV}}{m_u+m_d} r_{2k2;k11}(\widetilde{e}_{{\rm L}k}) \Big] \quad (2.29)$$

Meanwhile, the Standard Model calculated results (denominators) and the experimental values (numerators) are [7, 12]

$$\frac{R_\pi}{(R_\pi)_{\rm SM}} = \frac{1.230(4)\times 10^{-4}}{1.235\times 10^{-4}} \qquad \frac{R_{\tau\mu}}{(R_{\tau\mu})_{\rm SM}} = \frac{9.775(71)\times 10^3}{9.771^{+0.009}_{-0.013}\times 10^3}. \quad (2.30)$$

Therefore we obtain the following $2\sigma = 95\%$ bounds:

$$-0.058^2 < \sum_k \left\{ \left[|\lambda'_{11k}|^2 - |\lambda'_{21k}|^2 \right] \left(\frac{100{\rm GeV}}{m_{\widetilde{d}_{{\rm R}k}}} \right)^2 - \Re\left[\alpha_1 \lambda^*_{1k1}\lambda'_{k11} - \alpha_2 \lambda^*_{2k2}\lambda'_{k11} \right] \left(\frac{100{\rm GeV}}{m_{\widetilde{e}_{{\rm L}k}}} \right)^2 \right\} < 0.028^2, \qquad (R_\pi) \quad (2.31)$$

$$-0.072^2 < \sum_k \left\{ \left[|\lambda'_{31k}|^2 - |\lambda'_{21k}|^2 \right] \left(\frac{100{\rm GeV}}{m_{\widetilde{d}_{{\rm R}k}}} \right)^2 - \Re\left[\alpha_3 \lambda^*_{k33}\lambda'_{k11} - \alpha_2 \lambda^*_{2k2}\lambda'_{k11} \right] \left(\frac{100{\rm GeV}}{m_{\widetilde{e}_{{\rm L}k}}} \right)^2 \right\} < 0.075^2, \qquad (R_{\tau\pi}) \quad (2.32)$$

where

$$\alpha_1 = \frac{7.624\times 10^4{\rm MeV}}{m_u+m_d}, \qquad \alpha_2 = \frac{368.7{\rm MeV}}{m_u+m_d}, \qquad \alpha_3 = \frac{134.2{\rm MeV}}{m_u+m_d}. \quad (2.33)$$

* * *

These are, in the terms of the single-coupling bounds,

$$\lambda'_{11k} < 0.028, \qquad \lambda'_{21k} < 0.058, \qquad \lambda'_{31k} < 0.075, \quad (2.34)$$

for $m = 100{\rm GeV}$.[*4] We can see that the products of two (or more) coupling constants are severely restricted. This is because they generally invoke exotic events not included in the $R_{\rm p}$-MSSM.

[*4] Actually our calculated result is a bit different from the original one of Ref. [12], which gives $\lambda'_{31k} < 0.06$.

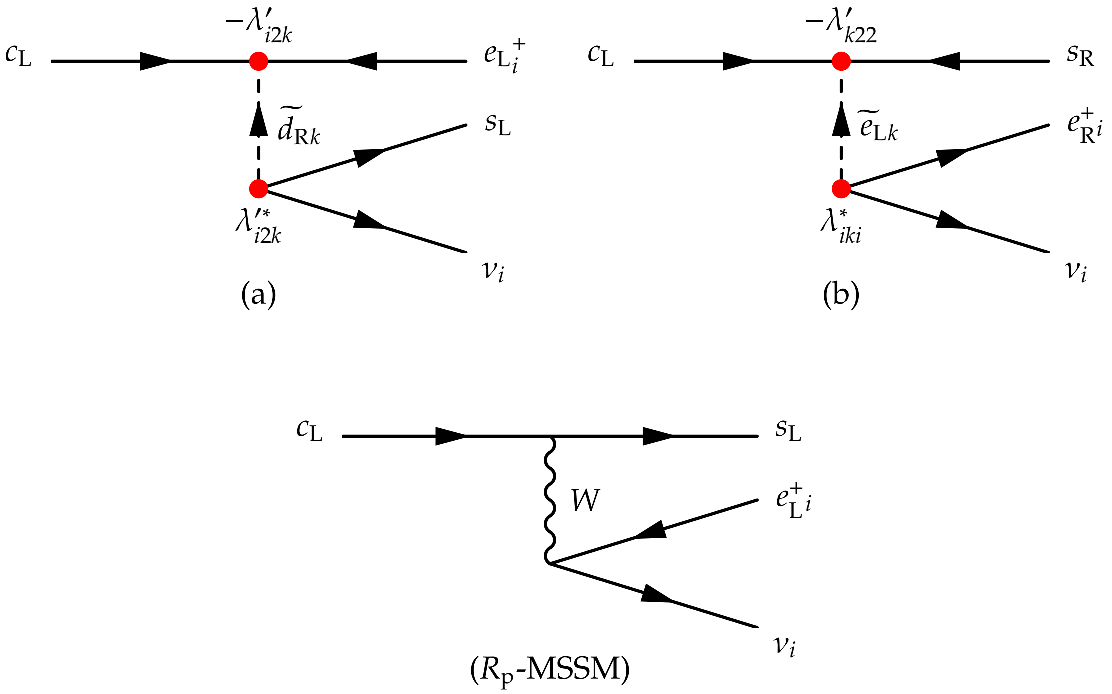

Fig. 2.4 Possible *R*-parity violating contributions to (a,b) $D^0 \to K^- e_i^+ \nu_i$, and corresponding R_p-MSSM processes. Note that $i \neq k$ in (b).

2.2.3 Semileptonic D and Leptonic D_s Decay: for λ'_{i2k}

Now the turn for λ'_{i2k}. The discussion similar to what we did for the π-decay in the previous section yields some other bounds [13]. Now our targets are

$$R_{D^0} := \frac{\Gamma(D^0 \to \mu^+\nu_\mu K^-)}{\Gamma(D^0 \to e^+\nu_e K^-)}, \tag{2.35}$$

$$R_{D^+} := \frac{\Gamma(D^+ \to \mu^+\nu_\mu \overline{K^0})}{\Gamma(D^+ \to e^+\nu_e \overline{K^0})}, \tag{2.36}$$

$$R^*_{D^+} := \frac{\Gamma\left(D^+ \to \mu^+\nu_\mu \overline{K^*}(892)^0\right)}{\Gamma\left(D^+ \to e^+\nu_e \overline{K^*}(892)^0\right)}, \tag{2.37}$$

and

$$R_{D_s}(\tau\mu) := \frac{\Gamma(D_s^+ \to \tau^+\nu_\tau)}{\Gamma(D_s^+ \to \mu^+\nu_\mu)}. \tag{2.38}$$

Note that the mesons are:

$$D^0 \equiv (c\bar{u}), \qquad D^+ \equiv (c\bar{d}), \qquad D_s \equiv (c\bar{s}); \qquad K^- \equiv (s\bar{u}), \qquad \overline{K^0} \equiv (s\bar{d}). \tag{2.39}$$

Thus the contribution of the R-parity violating interactions in R_{D^0}, R_{D^+} and $R^*_{D^+}$ are as Fig. 2.4, and in $R_{D_s}(\tau\mu)$ are the ones similar to (a), (b) and (*) of Fig. 2.3.

Here we ignore the bounds on the products for simplicity, that is, ignore (b) of both Figs. 2.4 and 2.3. Then, the shifts are calculated as

$$\frac{R_{D^0}}{(R_{D^0})_{\mathrm{SM}}} = \frac{R_{D^+}}{(R_{D^+})_{\mathrm{SM}}} = \frac{R^*_{D^+}}{\left(R^*_{D^+}\right)_{\mathrm{SM}}} = 1 + \frac{2}{|V_{cs}|}\left[r'_{22k}(\widetilde{d}_{\mathrm{R}k}) - r'_{12k}(\widetilde{d}_{\mathrm{R}k})\right] \tag{2.40}$$

and

$$\frac{R_{D_s}(\tau\mu)}{(R_{D_s}(\tau\mu))_{\mathrm{SM}}} = 1 + \frac{2}{|V_{cs}|}\left[r'_{32k}(\widetilde{d}_{\mathrm{R}k}) - r'_{22k}(\widetilde{d}_{\mathrm{R}k})\right]. \tag{2.41}$$

Considering the experimental values and the Standard Model calculated results, we can obtain

$$-0.26^2 < \sum_k \left[\left|\lambda'_{22k}\right|^2 - \left|\lambda'_{12k}\right|^2 \right] \left(\frac{100\text{GeV}}{m_{\tilde{d}_{\text{R}k}}} \right)^2 < 0.15^2 \qquad (R_{D^0}) \tag{2.42}$$

$$-0.24^2 < \sum_k \left[\left|\lambda'_{22k}\right|^2 - \left|\lambda'_{12k}\right|^2 \right] \left(\frac{100\text{GeV}}{m_{\tilde{d}_{\text{R}k}}} \right)^2 < 0.37^2 \qquad (R_{D^+}) \tag{2.43}$$

$$-0.25^2 < \sum_k \left[\left|\lambda'_{22k}\right|^2 - \left|\lambda'_{12k}\right|^2 \right] \left(\frac{100\text{GeV}}{m_{\tilde{d}_{\text{R}k}}} \right)^2 < 0.30^2 \qquad (R^*_{D^+}) \tag{2.44}$$

$$-0.24^2 < \sum_k \left[\left|\lambda'_{32k}\right|^2 - \left|\lambda'_{22k}\right|^2 \right] \left(\frac{100\text{GeV}}{m_{\tilde{e}_{\text{R}k}}} \right)^2 < 0.33^2 \qquad (R_{D_s}(\tau\mu)), \tag{2.45}$$

as the 2σ-bounds, and for the single-coupling bounds,

$$|\lambda'_{22k}| < 0.15, \qquad |\lambda'_{12k}| < 0.24, \qquad |\lambda'_{32k}| < 0.33. \tag{2.46}$$

2.2.4 Then How are λ'_{i3k}?

Now it is the time when we should discuss λ'_{i3k}. However, we cannot apply the above discussions here, because these terms correspond to the bottom and the top quark. Thus we must find another way.

To this end, it is appropriate to see the contribution to the one-loop correction of $Ze_i^+e_i^-$ vertex, but since this is much more complicated than the other discussions, we do not discuss in detail. The corresponding diagrams are as Fig. 2.5, and this yields the following 2σ-bounds:

$$\lambda'_{13k} < 0.47, \qquad \lambda'_{23k} < 0.45, \qquad \lambda'_{33k} < 0.58. \tag{2.47}$$

These are much looser bounds than we had obtained in the previous discussions.

2.2.5 Constraints on B-Violating terms: λ''_{ijk}

It is very difficult to obtain the constraints on the B-violating terms from collider experiments, because in the discussion we have to examine the inner structure of baryons or mesons, and to take the QCD effects into consideration. Thus we do not present the procedure of obtaining the constraints here.

Actually, B-violating terms are severely constrained from cosmology, which we will discuss in Chap. 4. In the discussion, we will see that the B-violating terms are constrained as

$$\lambda''_{ijk} \lesssim 10^{-6} \left(\frac{m_{\tilde{q}}}{100\text{GeV}} \right)^{1/2}, \tag{2.48}$$

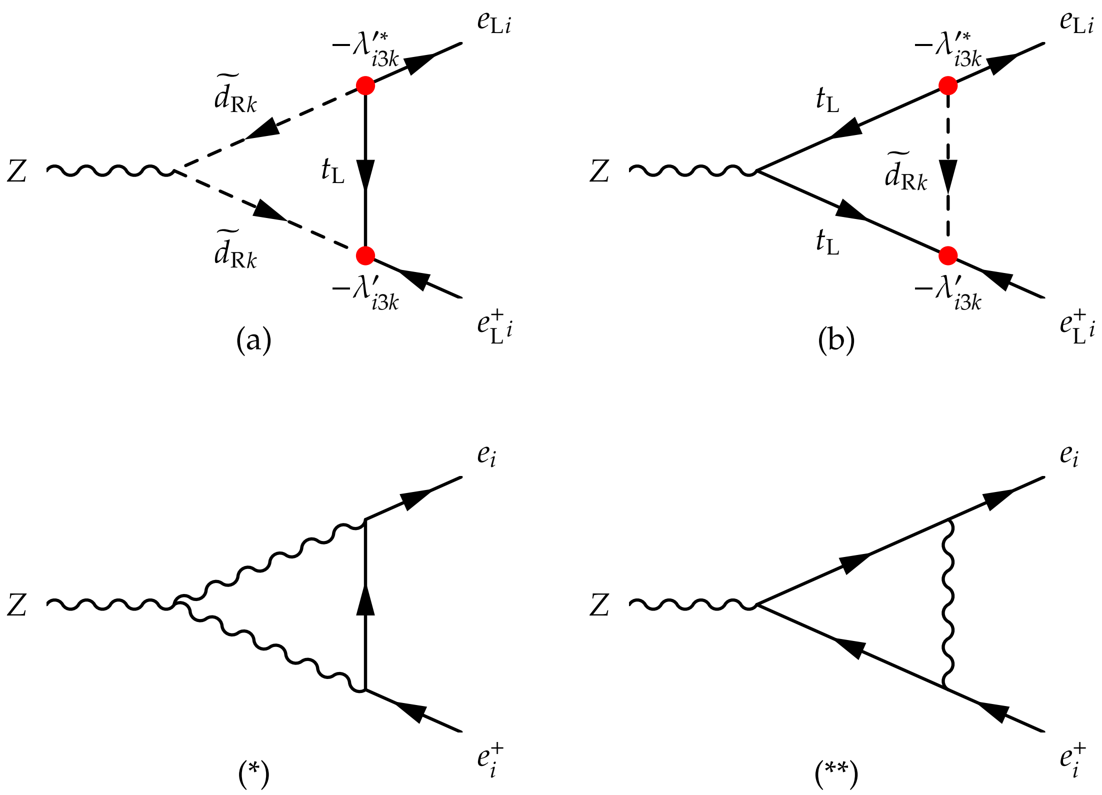

Fig. 2.5 The contribution to the one-loop correction of $Ze_i^+e_i^-$ vertex. The upper figures are from the R-parity violating interactions. The lower ones are of the Standard Model, from which we can obtain the R_p-MSSM contribution by changing two of the three intermediate particles to their superpartners.

which is rough estimation of Eq. (4.2), for all i, j, and k, or

$$\sqrt{\sum_{ijk} \left|\lambda''_{ijk}\right|^2} \lesssim (4\text{–}5) \times 10^{-7}, \tag{2.49}$$

which is our more precise analysis, Eq. (4.44).

2.2.6 A List of Single-Coupling Bounds

Here as the conclusion of this chapter we present a list of the current single-coupling bounds, Table 2.1.

Here, we do not use the results derived from the neutrino mass bounds in this table except λ'_{133}, for which no bounds are known, because the mass of neutrino largely depends on other structures. Also we do not use the gravitino- or axino-corresponding events as sources.

Our discussion covers the starred values. The sources of the other values are: (†) λ_{12k} are from CKM unitarity, (‡) λ'_{111} from the neutrino-lees double beta decay, (§) λ'_{121} and λ'_{131} from atomic physics parity violation, (¶) λ'_{132} from forward-backward asymmetry, (#) λ'_{23k} and λ'_{33k} from the contribution of the loop effect to Z-boson decay, (♭) λ''_{11k}, λ''_{312} and λ''_{313} from neutron-antineutron oscillation, and (‖) λ''_{2jk} and λ''_{i23} from the renormalization group. We do not discuss these sources here.

We can see that almost all constraints are of order 10^{-1}–10^{-3}. We will see in Chap. 4 that we can give much more stringent constraints on not only B-violating couplings (as we mentioned above) but also L-violating couplings if the lepton flavor is mixed enough in the early universe.

$\lambda_{ijk}L_iL_jL_k$		$\lambda'_{ijk}L_iQ_j\bar{D}_k$								$\lambda''_{ijk}\bar{U}_i\bar{D}_j\bar{D}_k$	
121	$0.03^{(a)\dagger}$	111	$0.0007^{(b)\ddagger}$	211	$0.06^{(a)\star}$	311	$0.06^{(a)\star}$	112	$\sim 10^{-7(c)\flat}$
122	$0.03^{(a)\dagger}$	112	$0.03^{(a)\star}$	212	$0.06^{(a)\star}$	312	$0.06^{(a)\star}$	113	$\sim 10^{-7(c)\flat}$
123	$0.03^{(a)\dagger}$	113	$0.03^{(a)\star}$	213	$0.06^{(a)\star}$	313	$0.06^{(a)\star}$	123	$1.25^{(c)\|}$
131	$0.05^{(a)\star}$	121	$0.03^{(a)\S}$	221	$0.1^{(a)\star}$	321	$0.3^{(a)\star}$	212	$1.25^{(c)\|}$
132	$0.05^{(a)\star}$	122	$0.2^{(a)\star}$	222	$0.1^{(a)\star}$	322	$0.3^{(a)\star}$	213	$1.25^{(c)\|}$
133	$0.05^{(a)\star}$	123	$0.2^{(a)\star}$	223	$0.1^{(a)\star}$	323	$0.3^{(a)\star}$	223	$1.25^{(c)\|}$
231	$0.05^{(a)\star}$	131	$0.03^{(a)\S}$	231	$0.45^{(b)\#}$	331	$0.58^{(b)\#}$	312	$0.002^{(c)\flat}$
232	$0.05^{(a)\star}$	132	$0.28^{(b)\P}$	232	$0.45^{(b)\#}$	332	$0.58^{(b)\#}$	313	$0.003^{(c)\flat}$
233	$0.05^{(a)\star}$	133	$(0.0004)^{(c)}$	233	$0.45^{(b)\#}$	333	$0.58^{(b)\#}$	323	$1.12^{(c)\|}$

Table 2.1 A list of the current single-coupling bounds when the mass of all the super-particles are 100GeV. The data are obtained from (a) Ref. [12], (b) Ref. [13], (c) Ref. [11]. See the text for details.

Appendix 2.i The RPV Contribution to $\tau \to \pi^- \nu_\tau$

In this section we will calculate Eq. (2.27), the contribution of the *R*-parity violating terms to the R_p-MSSM result. The notations and conventions are all from Peskin's book [17] (See: App. A.1 for example).

This discussion is along Ref. [16].

The operators which induce the processes are

$$\mathcal{O}_{\mathrm{SM}} = -\left(\frac{4G_{\mathrm{F}}V^*_{ud}}{\sqrt{2}}\right)\left(\overline{d}\gamma^\mu P_{\mathrm{L}} u\right)\left(\overline{\nu_\tau}\gamma_\mu P_{\mathrm{L}}\tau\right) \tag{2.50}$$

$$\mathcal{O}_{(\mathrm{c})} = -\sum_k \frac{\left|\lambda'_{31k}\right|^2}{m^2_{\tilde{d}_{\mathrm{R}k}}}\left(\overline{d}P_{\mathrm{L}}\tau\right)\left(\overline{\nu_\tau}P_{\mathrm{L}}u\right) \tag{2.51}$$

$$= -\sum_k \frac{\left|\lambda'_{31k}\right|^2}{2m^2_{\tilde{d}_{\mathrm{R}k}}}\left(\overline{d}\gamma^\mu P_{\mathrm{L}} u\right)\left(\overline{\nu_\tau}\gamma_\mu P_{\mathrm{L}}\tau\right) \tag{2.52}$$

$$\mathcal{O}_{(\mathrm{d})} = \sum_k \frac{\lambda^*_{k33}\lambda'_{k11}}{m^2_{\tilde{e}_{\mathrm{L}k}}}\left(\overline{\nu_\tau}P_{\mathrm{R}}\tau\right)\left(\overline{d}P_{\mathrm{L}}u\right). \tag{2.53}$$

Here we employed the Fierz identity in the derivation of the second operator. Then we convert the u and d quarks to the pion, using [16]

$$\langle 0|\,\overline{u}(x)\gamma^\mu P^{\mathrm{L}}_{\mathrm{R}} d(x)\left|\pi(p)\right\rangle = \pm\frac{f_\pi}{\sqrt{2}}\cdot p^\mu \mathrm{e}^{-\mathrm{i}px} \tag{2.54}$$

$$\langle 0|\,\overline{u}(x) P^{\mathrm{L}}_{\mathrm{R}} d(x)\left|\pi(p)\right\rangle = \mp\frac{f_\pi}{\sqrt{2}}\cdot \frac{m_\pi^2}{m_u+m_d}\mathrm{e}^{-\mathrm{i}px}, \tag{2.55}$$

where f_π is the pion decay constant: $f_\pi \simeq 92.4\mathrm{MeV}$. m (and p in the following equations) denotes the mass (and the momentum) of each particle (pion, τ, ν_τ, and up/down quark).

Now the amplitude (matrix element) $\mathcal{M}$ is calculated as

$$\begin{aligned}\mathcal{M}_{\mathrm{SM}} &= \left(-\frac{4G_{\mathrm{F}}V^*_{ud}}{\sqrt{2}}\right)\cdot\left(-\frac{f_\pi}{\sqrt{2}}p^\mu\right)\cdot\overline{u}^s(p_\nu)\left[\gamma_\mu P_{\mathrm{R}}\right]u^t(p_\tau)\\ &= \frac{f_\pi}{\sqrt{2}}\cdot\frac{4G_{\mathrm{F}}V^*_{ud}}{\sqrt{2}}\overline{u}^s(p_\nu)\left[\not{p}_\pi P_{\mathrm{R}}\right]u^t(p_\tau)\end{aligned} \tag{2.56}$$

$$\begin{aligned}\mathcal{M}_{\mathrm{total}} &= M_{\mathrm{SM}} + \frac{f_\pi}{\sqrt{2}}\cdot\sum_k\frac{\left|\lambda'_{31k}\right|^2}{2m^2_{\tilde{d}_{\mathrm{R}k}}}\overline{u}^s(p_\nu)\left[\not{p}_\pi P_{\mathrm{R}}\right]u^t(p_\tau)\\ &\quad - \frac{f_\pi}{\sqrt{2}}\sum_k\frac{\lambda^*_{k33}\lambda'_{k11}}{m^2_{\tilde{e}_{\mathrm{L}k}}}\frac{m_\pi^2}{m_u+m_d}\overline{u}^s(p_\nu)P_{\mathrm{R}}u^t(p_\tau).\end{aligned} \tag{2.57}$$

Here note that the symbol $u^{s/t}(p)$ denotes not the up quark (as previous equations) but the Fourier transformation of the particle, as Peskin's book [17, Chap. 3].

Therefore, using the approximation

$$|\mathcal{M}_{\text{total}}|^2 \simeq |\mathcal{M}_{\text{SM}}|^2 + 2\Re\left(\mathcal{M}_{\text{SM}}^* \mathcal{M}_{\text{RPV}}\right), \tag{2.58}$$

we obtain the following result:

$$\mathcal{M}_{\text{SM}} = C \cdot \left[4(p_\pi \cdot p_\nu)(p_\pi \cdot p_\tau) - 2m_\pi^2 (p_\nu \cdot p_\tau)\right] \tag{2.59}$$

$$\Re\left(\mathcal{M}_{\text{SM}}^* \mathcal{M}_{\text{RPV}}\right) = C \cdot \left[\frac{r'_{31k}(\widetilde{d}_{\text{R}k})}{|V_{ud}|}\left[4(p_\pi \cdot p_\nu)(p_\pi \cdot p_\tau) - 2m_\pi^2 (p_\nu \cdot p_\tau)\right] - \frac{2r_{k33;k11}(\widetilde{e}_{\text{L}k})}{|V_{ud}|}\frac{m_\pi^2}{m_u + m_d} m_\tau (p_\nu \cdot p_\pi)\right]. \tag{2.60}$$

Here $C = \left|2G_{\text{F}} V_{ud} f_\pi\right|^2$, but we do not interested in this value. We have neglected the phase of V_{ud} for simplicity.

Since this is a 2-body decay process, taking τ's rest frame, we can determine the momenta as

$$p_\tau = \begin{pmatrix} m_\tau \\ \mathbf{0} \end{pmatrix}, \qquad p_\pi = \begin{pmatrix} \sqrt{m_\pi^2 + p^2} \\ \boldsymbol{p} \end{pmatrix}, \qquad p_\nu = \begin{pmatrix} \|\boldsymbol{p}\| \\ \boldsymbol{p} \end{pmatrix}; \qquad \|\boldsymbol{p}\| = \sqrt{m_\tau^2 + m_\pi^2} - m_\tau, \tag{2.61}$$

and obtain the following result:

$$\frac{\Gamma(\tau \to \pi^- \nu_\tau)}{\Gamma_{\text{SM}}(\tau \to \pi^- \nu_\tau)} = 1 + \frac{2}{|V_{ud}|^2}\left[r'_{31k}(\widetilde{d}_{\text{R}k}) - \frac{134.4\text{MeV}}{m_u + m_d} \cdot r_{k33;k11}(\widetilde{e}_{\text{L}k})\right]. \tag{2.62}$$

We use the numerical value 134.4MeV instead of the symbolic notation for simplicity.

Chapter 3

Universe before EWPT

In this chapter we discuss the properties of the universe between the SUSY breaking and the electroweak phase transition (EWPT). We consider the era when the temperature T of the universe is $10\text{TeV} \gtrsim T \gtrsim 100\text{GeV}$. Here we concentrate on the minimal supersymmetric standard model (MSSM).

Section 3.1 Preliminaries

3.1.1 Mass Structure of Higgs Sector

To begin with, we shall consider the property of the Higgs sector before the EWPT.

The mass terms of the Higgs bosons under the MSSM are given as, if the R-parity is conserved,[*1]

$$V = {m_1}^2 \left|H_\mathrm{u}^0\right|^2 + {m_2}^2 \left|H_\mathrm{d}^0\right|^2 - \left(bH_\mathrm{u}^0 H_\mathrm{d}^0 + \text{H. c.}\right) + \frac{g_1^2 + g_2^2}{8}\left(\left|H_\mathrm{u}^0\right|^2 - \left|H_\mathrm{d}^0\right|^2\right)^2 \\ \text{where} \quad {m_1}^2 := \left|\mu\right|^2 + m_{H_\mathrm{d}}^2, \qquad {m_2}^2 := \left|\mu\right|^2 + m_{H_\mathrm{u}}^2, \tag{3.1}$$

and in order to invoke the electroweak symmetry breaking, the parameters must be so that the minimum is not at $H_\mathrm{u}^0 = H_\mathrm{d}^0 = 0$. (See Eqs. (B.2) and (B.4) for the meaning of each parameters.)

However, in the early universe where its temperature T is above $\gtrsim 100\text{GeV}$, thermal effects "hold up" the potential and thus the minimum would be $H_\mathrm{u}^0 = H_\mathrm{d}^0 = 0$. Therefore, in this era, the electroweak symmetry $\mathrm{SU}(2)_\mathrm{weak} \times \mathrm{U}(1)_Y$ is (still) alive. In this thesis we do not discuss the details of the thermal effects, and go on with regarding the electroweak symmetry as unbroken.

[*1] This potential is also discussed in App. B.i with the R-parity violating terms. Or as a nice review, see Ref. [3, Sec. 7].

Meanwhile, higgsinos, four Weyl fermions, form two Dirac fermions whose mass are both $|\mu|$:

$$
\begin{aligned}
& W \supset \mu H_{\mathrm{u}} H_{\mathrm{d}} + \text{H. c.} && (3.2)\\
\longrightarrow\ & \mathcal{L} \supset \mu(\widetilde{H_{\mathrm{u}}^{+}}\widetilde{H_{\mathrm{d}}^{-}} - \widetilde{H_{\mathrm{u}}^{0}}\widetilde{H_{\mathrm{d}}^{0}}) + \text{H. c.} \\
& = -|\mu| \begin{pmatrix} \eta^* H_{\mathrm{d}}^{-} & H_{\mathrm{u}}^{+\dagger} \end{pmatrix} \begin{pmatrix} H_{\mathrm{u}}^{+} \\ \eta H_{\mathrm{d}}^{-\dagger} \end{pmatrix} - |\mu| \begin{pmatrix} \eta^* H_{\mathrm{d}}^{0} & H_{\mathrm{u}}^{0\dagger} \end{pmatrix} \begin{pmatrix} H_{\mathrm{u}}^{0} \\ \eta H_{\mathrm{d}}^{0\dagger} \end{pmatrix} \\
& =: -|\mu| \overline{\Psi_{\mathrm{D}}^{+}} \Psi_{\mathrm{D}}^{+} - |\mu| \overline{\Psi_{\mathrm{D}}^{0}} \Psi_{\mathrm{D}}^{0}. && (3.3)
\end{aligned}
$$

Here, Ψ_{D}^{+} and Ψ_{D}^{0} are Dirac fermions, and η is a phase defined as $-\eta^* := \mu/|\mu|$, which is with no importance.

Note that these higgsinos do not mix with gauginos (or leptons) to form neutralinos or chaginos before the EWPT.

3.1.2 Sphaleron

In the Standard Model, we have an anomalous interaction, called the "sphaleron"*2process.[18, 19, 20] This is a 12-fermion interaction which is symbolically illustrated as

$$
\mathcal{O} = \prod_{i=1\ldots3} \left(q_i^{\mathrm{R}} q_i^{\mathrm{G}} q_i^{\mathrm{B}} l_i \right), \tag{3.4}
$$

where i is the generation index, and R, G and B denote the $\mathrm{SU}(3)_{\mathrm{strong}}$ color. For example, we have

$$
c^\dagger s^\dagger \longrightarrow u\, d\, d\, s\, t\, b\, b\, \nu_e\, \nu_\mu\, \nu_\tau \tag{3.5}
$$

interaction. What is important is that this process *violates* the baryon and lepton number B and L, but does not violate $B-L$. Now in *all* the interactions, $B-L$ is *conserved*.

This process originates the anomaly of the baryon- and lepton-current: J_μ^B and J_μ^L:

$$
J_\mu^B := \frac{1}{3} \sum_{\text{generation}} \sum_{\text{color}} \overline{q_{\mathrm{L}}} \gamma_\mu q_{\mathrm{L}}, \qquad J_\mu^L := \sum_{\text{generation}} \overline{l_{\mathrm{L}}} \gamma_\mu l_{\mathrm{L}}; \tag{3.6}
$$

$$
\begin{aligned}
\partial_\mu J^{B\mu} = \partial_\mu J^{L\mu} &= \frac{-3{g_2}^2}{16\pi^2} \frac{1}{2} \operatorname{Tr} \left(\epsilon^{\mu\nu\rho\sigma} W_{\mu\nu} W_{\rho\sigma} \right) && (3.7)\\
&= \partial_\mu \left[\frac{-3{g_2}^2}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} \operatorname{Tr} \left(W_{\nu\rho} W_\sigma + \frac{2}{3} \mathrm{i} g W_\nu W_\rho W_\sigma \right) \right]. && (3.8)
\end{aligned}
$$

*2 σφαλερον, δεριε from σφαλερός.

(See App. A for notation. Especially note that W_μ is the W-boson field and $W_{\mu\nu}$ is its field strength.) We can see that the B and L are violated here, by the instanton effect, that is, the sphaleron process can be regarded as the transition between vacua, B and L of which are different by the same number. Here also note that $B - L$ is conserved.

As this process is the transition between vacua, between which an energy barrier stands, its probability is suppressed by the factor

$$\exp\left(-\frac{8\pi^2}{{g_2}^2}\right) \approx 10^{-81}. \tag{3.9}$$

However, Kuzmin, Rubakov, and Shaposhnikov [21] found that in early universe *before the EWPT* this process is enhanced by thermal effects, and even exceeds the Hubble expansion rate. Also Ringwald calculated [22] the rate to find that the process would achieve equilibrium.

Their study is based on the Standard Model, not the MSSM, but can be applied to our MSSM case. In the MSSM, we have another SU(2) fermion, the higgsino. Therefore, the interaction is illustrated as[*3]

$$O = \widetilde{h}_{\mathrm{u}}\widetilde{h}_{\mathrm{d}} \prod_{i=1\ldots3} \left(q_i^{\mathrm{R}} q_i^{\mathrm{G}} q_i^{\mathrm{B}} l_i\right). \tag{3.10}$$

In summary: there is the sphaleron process in the early universe before the EWPT, and it is strong enough to achieve equilibrium. It conserves $B - L$, but violates B and L.

3.1.3 Relation between Number and Chemical Potential

Next we derive the relation between the number density of a particle and its chemical potential.

Here we define the "yield" N of a particle by the number density n as

$$N := \frac{n}{T^3}, \tag{3.11}$$

where T is the temperature of the universe. This is expressed by the distribution function f and the degree of freedom g of the particle as

$$N = \frac{n}{T^3} = \frac{g}{T^3} \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} f(\boldsymbol{k}). \tag{3.12}$$

$f(\boldsymbol{k})$ is the Maxwell–Boltzmann, the Fermi–Dirac or the Bose–Einstein distribution

$$f_{\mathrm{MB}}(\boldsymbol{k}) = \frac{1}{\mathrm{e}^{(E-\mu)/T}}, \qquad f_{\mathrm{BE}}(\boldsymbol{k}) = \frac{1}{\mathrm{e}^{(E-\mu)/T} - 1}, \qquad f_{\mathrm{FD}}(\boldsymbol{k}) = \frac{1}{\mathrm{e}^{(E-\mu)/T} + 1}. \tag{3.13}$$

[*3] Note that these higgsinos form Dirac fermion, as we discussed just above.

E is the energy, which is given by $\sqrt{m^2+\|\boldsymbol{k}\|^2}$, and m and μ are the mass and the chemical potential of the particle, respectively.

However, number density cannot be calculated analytically, as long as we use the Bose–Einstein or the Fermi–Dirac distribution.[*4] To obtain an analytical expression, we ought to use the approximation that the particle obeys the Maxwell–Boltzmann distribution. Then we have the following expression:

$$N_{\mathrm{MB}} = \frac{g}{\pi^2}F_2\left(\frac{m}{T}\right)\exp\left(\frac{\mu}{T}\right), \qquad \text{where} \quad F_i(x) := x^2K_i(x). \tag{3.14}$$

Here, $K_i(x)$ is the modified Bessel function of the second kind. If the particle is massless, we use $F_2(0) = 2$.[*5]

In the next section, we will calculate relations between chemical potentials in the early universe. Actually, during the calculation, we will come down to use this approximated expression.

Section 3.2 Relations between Chemical Potentials

◆Gauge bosons and gauginos

Start from the gauge bosons. We have the gauge self couplings W^0–W^0–W^+–W^- and W^0–W^+–W^-. This means that we have both $X \to YW^0W^0$ and $X \to YW^0$ events, and therefore $X \equiv Y + 2W^0$ and $X \equiv Y + W^0$. (The symbol $\equiv$ denotes "equal in terms of the chemical potentials" in this section.) Thus we know that $W^0 \equiv 0$ and $W^+ + W^- \equiv 0$. Also this fact tells us that the sum of the chemical potential of a particle and its antiparticle is zero: $\overline{X} \equiv -X$, as well as that of B-boson is zero: $B \equiv 0$.

Here, the Majorana mass of $\widetilde{W}$ and $\widetilde{B}$ allows us to have the process

$$e_{\mathrm{R}} + \widetilde{e}_{\mathrm{R}}^* \to \widetilde{B} \to \widetilde{B}^\dagger \to e_{\mathrm{R}}^\dagger + \widetilde{e}_{\mathrm{R}} \tag{3.15}$$

etc. as Fig. 3.1. Thus we know $\widetilde{W} \equiv \widetilde{B} \equiv 0$, and the chemical potentials of a particle and its superpartner are the same: $\widetilde{X} \equiv X$. Also we have 3-point interaction $\widetilde{W}$–$\widetilde{W}^+$–$\widetilde{W}^-$, which tells us $\widetilde{W}^+ + \widetilde{W}^- \equiv 0$. Considering gaugino–gaugino–gauge-boson interactions, we conclude

$$B \equiv W \equiv \widetilde{B} \equiv \widetilde{W}^0 \equiv 0, \qquad W^+ \equiv \widetilde{W}^+ \equiv -W^- \equiv -\widetilde{W}^-, \qquad X \equiv -\overline{X} \equiv \widetilde{X} \equiv -\widetilde{\overline{X}}. \tag{3.16}$$

[*4] Incidentally, we present more detailed discussion on these distribution functions and the numerically calculated results in App. C.1.2.

[*5] The expression (3.12) results in $F_2(0) = 2$ if we use $f = f_{\mathrm{MB}}$ and $m = 0$, while the limit of the modified Bessel function gives the same result: $\lim_{x\to 0} x^2K_i(x) = 2$.

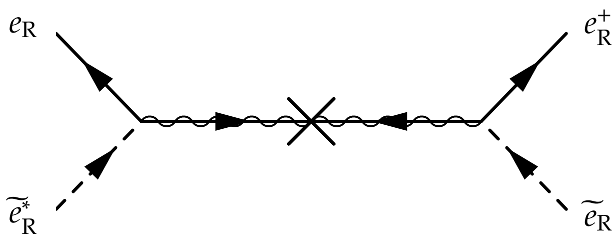

Fig. 3.1 In the presence of Majorana mass, this kind of processes occurs, which guarantees that the chemical potential of the particle is zero. See the text for details.

We will not discuss the relations for gluon and gluino, since they are much more complicated. Actually, most of these discussions which we have done for gauge bosons and gauginos will be spoiled by the (strong) presumption which we will introduce later.

◆Matter and Higgs sector

Let us go on the quark sector. Since we have the CKM mixings which are strong enough to achieve equilibria, the chemical potentials of quarks are flavor-independent. Therefore

$$u_{\mathrm{L}} \equiv c_{\mathrm{L}} \equiv t_{\mathrm{L}} \equiv d_{\mathrm{L}} + W^{+} \equiv s_{\mathrm{L}} + W^{+} \equiv b_{\mathrm{L}} + W^{+}, \tag{3.17}$$

$$u_{\mathrm{R}} \equiv c_{\mathrm{R}} \equiv t_{\mathrm{R}}, \tag{3.18}$$

$$d_{\mathrm{R}} \equiv s_{\mathrm{R}} \equiv b_{\mathrm{R}}. \tag{3.19}$$

The lepton sector actually has mixings, PMNS mixings, but here we *leave* the chemical potentials as flavor-*dependent*. We will discuss whether the mixings in lepton sector achieve equilibria or not in the next chapter. Now we take only

$$\nu_i \equiv e_{\mathrm{L}i} + W^{+} \qquad \text{where} \quad i = 1, 2, 3 \text{ (flavor index)}, \tag{3.20}$$

into consideration.

For the Higgs sector, we know

$$H^{+}_{\mathrm{u}} \equiv H^{0}_{\mathrm{u}} + W^{+}, \qquad\qquad H^{0}_{\mathrm{d}} \equiv H^{-}_{\mathrm{d}} + W^{+}. \tag{3.21}$$

That is all for the quark, lepton, and Higgs sector.[*6]

[*6] Note that we have already derived the relationship of the chemical potential of antiparticles and superpartners at Eq. (3.16).

Usually when we discuss whether the process is strong enough to achieve equilibrium or not, we compare the process rate versus the Hubble parameter, as we will see in Sec. 4.1.2. The Hubble parameter H is, as is discussed in App. C.1.3, roughly given as

$$H \sim \frac{25T^2}{M_{\rm pl}}. \tag{3.22}$$

On the other hand, considering quark scattering mediated by W-boson as the CKM mixing process, the rate of the process under thermal effects is approximated as (See: Ref. [23] or Eq. (4.3))

$$\Gamma \sim \frac{(g_2\theta)^2}{4\pi}T, \tag{3.23}$$

where g_2 is the gauge coupling ≈ 0.65 and θ is the CKM mixing angle. The CKM mixing angle is ≈ 0.2 for Q_1–Q_2 mixing and ≈ 0.008 for Q_1–Q_3 mixing [7]. Therefore,

$$\frac{\Gamma}{H} \sim \theta^2 \cdot 10^{14}\left(\frac{T}{100\text{GeV}}\right)^{-1} \gg 1, \tag{3.24}$$

and thus CKM mixing is considered to be achieve equilibria.

* * *

From this assumption, usually it is said that

> if $\theta \gtrsim 10^{-7}$, the process is equilibrated, and if not, the equilibrium is not achieved.

However, to discuss precisely, we have to solve the Boltzmann equation, which describes the time evolution of a value. This is what we will do in the next chapter.

◆A strong presumption

Before discussing the 湯川 interactions, we introduce an above-mentioned strong presumption. That is, *all the particles must have been generated with gauge-invariance.* For example, the number of electrons is the same as that of electron-neutrinos, and the number of red-colored quarks is the same as blue- and green-colored quarks. Since the masses of the particles in a gauge multiplet is the same before the EWPT, these equalities in numbers yield equalities in chemical potentials

$$u_{\rm L\,red} \equiv u_{\rm L\,blue} \equiv u_{\rm L\,green}, \qquad e_{\rm L} \equiv \nu_e, \tag{3.25}$$

and so on. Now we know

$$W^+ \equiv \widetilde{W^+} \equiv g \equiv \widetilde{g} \equiv 0. \tag{3.26}$$

◆Superpotential and sphaleron process

Now we have only 12 independent chemical potentials, those of the following particles:

$$H_{\mathrm{u}}^0, \quad H_{\mathrm{d}}^0, \quad u_{\mathrm{L}}, \quad u_{\mathrm{R}}, \quad d_{\mathrm{R}}, \quad l_{i\mathrm{L}}, \quad l_{i\mathrm{R}}. \tag{3.27}$$

This fact tells us that "the particles in a supermultiplet have the same chemical potentials." Therefore, from now, we use names of the supermultiplets instead of particles to express relationships between chemical potentials:

$$H_{\mathrm{u}}, \quad H_{\mathrm{d}}, \quad Q, \quad \bar{U}, \quad \bar{D}, \quad L_i, \quad \bar{E}_i. \tag{3.28}$$

(Be careful that, as the right-handed fermion are barred, $\bar{U} \equiv -u_{\mathrm{R}}$, and so on.)

Anyway, we have other three types of constraints, those which come from the superpotential, the sphaleron process, and the conservation of the hypercharge.

The constraints from the 湯川 interactions are expressed as

$$\mu_Q + \mu_{\bar{D}} + \mu_{H_{\mathrm{u}}} = 0, \qquad \mu_Q + \mu_{\bar{D}} + \mu_{H_{\mathrm{d}}} = 0, \qquad \mu_{L_i} + \mu_{\bar{E}_i} + \mu_{H_{\mathrm{d}}} = 0. \tag{3.29}$$

Similarly, the μ-term gives us a relation

$$\mu_{H_{\mathrm{u}}} + \mu_{H_{\mathrm{d}}} = 0. \tag{3.30}$$

The sphaleron process results in the following relation, as we discussed in Sec. 3.1.2:

$$9\mu_Q + \sum_i \mu_{L_i} + \mu_{H_{\mathrm{u}}} + \mu_{H_{\mathrm{d}}} = 0. \tag{3.31}$$

◆Hypercharge conservation

The last constraint, the conservation of the hypercharge, is a bit complicated. As we are under the "generated with gauge invariance" presumption, the sum of the hypercharge over all the particles in the whole universe is zero. Therefore

$$\sum_{i=\text{generations}} \left(\frac{1}{6} n_{[Q_i]} - \frac{2}{3} n_{[\bar{U}_i]} + \frac{1}{3} n_{[\bar{D}_i]} - \frac{1}{2} n_{[L_i]} + n_{[\bar{E}_i]} \right) + \frac{1}{2} n_{[H_{\mathrm{u}}]} - \frac{1}{2} n_{[H_{\mathrm{d}}]} = 0, \tag{3.32}$$

where $n_{[X]}$ denotes the whole effective number of the particles which belong to the supermultiplet X. For example,

$$\begin{aligned} n_{[\bar{U}]} &:= \sum_i n_{[\bar{U}_i]} \\ &= \sum_{\text{generation}} \sum_{\text{color}} \left(-n_{u_\mathrm{R}} - n_{\widetilde{u}_\mathrm{R}} + n_{u_\mathrm{R}^\dagger} + n_{\widetilde{u}_\mathrm{R}^*}\right) \\ &= 9 \cdot \frac{T^3}{\pi^2}\left[F_2\left(\frac{m_{u_\mathrm{R}}}{T}\right) + F_2\left(\frac{m_{\widetilde{u}_\mathrm{R}}}{T}\right)\right]\left(2\sinh\frac{\mu_{\bar{U}}}{T}\right) \\ &\simeq 9 \cdot \frac{2T^3}{\pi^2}\left[2 + F_2\left(\frac{m_{\widetilde{u}_\mathrm{R}}}{T}\right)\right]\frac{\mu_{\bar{U}}}{T} \\ &=: 9 \cdot \frac{2T^3}{\pi^2} g_\mathrm{eff}\left(\frac{m_{\widetilde{u}_\mathrm{R}}}{T}\right)\frac{\mu_{\bar{U}}}{T}. \end{aligned} \tag{3.33}$$

We have used Eq. (3.14) with approximating the distribution functions as the Maxwell–Boltzmann type, the fact that quarks are massless before the EWPT, the approximation that $\sinh(\mu/T) \simeq \mu/T$, and $g_{u_\mathrm{R}} = g_{\widetilde{u}_\mathrm{R}} = g_{u_\mathrm{R}^\dagger} = g_{\widetilde{u}_\mathrm{R}^*} = 1$. Note that $n_{[\bar{U}]}$ denotes the effective number of *anti*particles, as well as $\mu_{\bar{U}}$. We have also defined

$$g_\mathrm{eff}(x) := 2 + F_2(x) = 2 + x^2 K_2(x). \tag{3.34}$$

Next we do the following approximations to simplify Eq. (3.32):

- all the squarks have the same mass $m_{\widetilde{q}}$,
- all the leptons have the same mass $m_{\widetilde{l}}$,
- all the Higgs bosons are massless.

Then we obtain

$$\begin{aligned} 0 &= g_\mathrm{eff}\left(\frac{m_{\widetilde{q}}}{T}\right)\frac{\frac{1}{6}\cdot 18\mu_Q - \frac{2}{3}\cdot 9\mu_{\bar{U}} + \frac{1}{3}\cdot 9\mu_{\bar{D}}}{T} \\ &\quad + \sum_i g_\mathrm{eff}\left(\frac{m_{\widetilde{l}}}{T}\right)\frac{-\frac{1}{2}\cdot 2\mu_{L_i} + 1\cdot\mu_{\bar{E}_i}}{T} \\ &\quad + g_\mathrm{eff}\left(\frac{m_{\widetilde{H}}}{T}\right)\frac{\frac{1}{2}\cdot 2\mu_{H_\mathrm{u}} - \frac{1}{2}\cdot 2\mu_{H_\mathrm{d}}}{T} \end{aligned} \tag{3.35}$$

$$\begin{aligned} &= \frac{2}{3}\left[g_\mathrm{eff}\left(\frac{m_{\widetilde{q}}}{T}\right) - g_\mathrm{eff}\left(\frac{m_{\widetilde{l}}}{T}\right)\right]\frac{\mu_{L_i}}{T} \\ &\quad - \left[9g_\mathrm{eff}\left(\frac{m_{\widetilde{q}}}{T}\right) + 3g_\mathrm{eff}\left(\frac{m_{\widetilde{l}}}{T}\right) + 2g_\mathrm{eff}\left(\frac{m_{\widetilde{H}}}{T}\right)\right]\frac{\mu_{H_\mathrm{d}}}{T}. \end{aligned} \tag{3.36}$$

◆Conclusion

As a result, we can express the chemical potentials of the MSSM particles by only four parameters μ_{L_i} and μ_{H_d} as

$$\mu_Q = -\frac{1}{3}\overline{\mu_L}, \qquad \mu_{\bar{U}} = \frac{1}{3}\overline{\mu_L} + \mu_{H_\mathrm{d}}, \qquad \mu_{\bar{D}} = \frac{1}{3}\overline{\mu_L} - \mu_{H_\mathrm{d}},$$
$$\mu_{\bar{E}_i} = -\mu_{L_i} - \mu_{H_\mathrm{d}}, \qquad \mu_{H_\mathrm{u}} = -\mu_{H_\mathrm{d}}, \tag{3.37}$$

where

$$\overline{\mu_L} := \frac{1}{3}\sum_i \mu_{L_i}. \tag{3.38}$$

Also $\overline{\mu_L}$ and μ_{H_d} are related as follows:

$$\mu_{H_\mathrm{d}} = -C_{H_\mathrm{d}}(T)\,\overline{\mu_L}, \tag{3.39}$$

where

$$C_{H_\mathrm{d}}(T) := \frac{2g_\mathrm{eff}\left(m_{\tilde{q}}/T\right) + 6g_\mathrm{eff}\left(m_{\tilde{l}}/T\right)}{9g_\mathrm{eff}\left(m_{\tilde{q}}/T\right) + 3g_\mathrm{eff}\left(m_{\tilde{l}}/T\right) + 2g_\mathrm{eff}\left(m_{\tilde{H}}/T\right)}. \tag{3.40}$$

When we assume that $g_\mathrm{eff} = 4$ for all the particles, which means all the particles are massless or the temperature is extremely high, we obtain

$$C_{H_\mathrm{d}} = \frac{4}{7}, \tag{3.41}$$

$$\mu_\mathrm{baryon} = \frac{1}{3}\left(18\mu_Q - 9\mu_{\bar{U}} - 9\mu_{\bar{D}}\right) = -4\overline{\mu_L} \tag{3.42}$$

$$\mu_\mathrm{lepton} = \sum_i \left(2\mu_{L_i} - \mu_{\bar{E}_i}\right) = \frac{51}{7}, \tag{3.43}$$

and the well-known result reappears:

$$\mu_\mathrm{baryon} = \frac{28}{79}\left(\mu_\mathrm{baryon} - \mu_\mathrm{lepton}\right). \tag{3.44}$$

Chapter 4

Cosmological Limit to RPV Parameters

Now we are ready to discuss the strong constraints on the R-parity violating couplings which come from cosmology. This chapter is devoted to the constraints.

Section 4.1 Introduction

4.1.1 Overview

There the baryon is, though the antibaryon is not. We can create antibaryon only in colliders, and even when we create antibaryon, they immediately annihilate.

Why does this universe have baryon, and no antibaryon? Our Standard Model contains the baryon–antibaryon symmetry, but the universe does not, and the baryon number B is positive: $B > 0$. How come the symmetry broke up in the early universe? How was this baryon–antibaryon *asymmetry* brought to us?

This is one of the biggest problem in cosmology. Even in 1967, when we did not know τ-lepton or charm quark, Sakharov [24] put forward the famous three conditions. Also, many models to achieve the asymmetry are proposed. In this thesis we do not discuss the models, and focus on the following important fact:

> large B-violation might wash out the asymmetry.

Especially the $\bar{U}\bar{D}\bar{D}$ coupling in the MSSM *do* wash out, and thus critical.

However, the story does not end here. Before the electroweak phase transition (EWPT) of the universe, there is the sphaleron process, as we discussed in Sec. 3.1.2. The sphaleron process transforms baryon into lepton, or vise versa, and thus

> violation of *lepton* number L might also wash out the *baryon* asymmetry.

Though a good loophole to avoid this L-violation constraint was found. Note that the sphaleron process preserves not only $B - L$ but also

$$\frac{1}{3}B - L_e \qquad \frac{1}{3}B - L_\mu \qquad \frac{1}{3}B - L_\tau \tag{4.1}$$

respectively. Therefore, supposing that one of the lepton number, say L_3 (or L_τ), is exactly conserved, sphaleron could not erase B even when the other lepton numbers are completely violated. $B/3$ which corresponds to L_3 would survive in this case.

However, this loophole can be covered. Davidson pointed out [25] that *lepton flavor violation* (LFV) would spoil the separated $B/3 - L_i$ conservation. Think again. If we have LFV processes, and they mix all the three generations e, μ and τ, then the separated lepton number L_i are not "conserved number," and therefore *any L-violation process* must be small lest the baryon asymmetry should be washed out.

* * *

This is our story. Endo, Hamaguchi, and the author found [1] that such LFV processes which can be strong enough to wash out the baryon asymmetry are *naturally expected* in typical SUSY GUT models, and calculated the bounds on the R-parity violating parameters. In this chapter, we review the work more verbosely.

But before reviewing the work, we will move back to past discussions as an introduction.

4.1.2 B-violation Erases Baryon–Antibaryon Asymmetry

As we mentioned, B-violating processes spoils the baryon asymmetry. Bouquet and Salati estimated [23] these bounds. They assume that the existing baryon asymmetry was produced in $T \gtrsim 100\text{GeV}$ era, and under this assumption concluded that the B-violating coupling λ'' must satisfy

$$\lambda'' < 10^{-6}\left(\frac{m_{\tilde{q}}}{100\text{GeV}}\right)^{1/2} \tag{4.2}$$

lest the asymmetry should be washed out. This constraint is much stricter than those which we obtained in Sec. 2.2.

The procedure of their estimation is as follows.

They considered $qq \to \widetilde{q\gamma}$ as a annihilation process, and estimate the annihilation rate Γ and the Hubble expansion rate H (See: App. C.1.3) as

$$\Gamma \approx \frac{\alpha\lambda''^2}{4\pi}\frac{T^5}{(T^2+m_{\tilde{q}}^2)^2}, \qquad H \approx 20\frac{T^2}{M_{\rm pl}}, \tag{4.3}$$

and also consider the baryon-washout process would be out of equilibrium if

$$H \gg \Gamma \qquad \text{for} \quad T \gtrsim m_{\tilde{q}}. \tag{4.4}$$

These conditions result in the bound (4.2).

4.1.3 L-VIOLATION MAY ALSO BE HARMFUL

Also L-violating processes would, in presence of the sphaleron effects, wash out the baryon asymmetry. This feature was first pointed out by Kuzmin, Rubakov and Shaposhnikov [21] in 1985, and Campbell, Davidson, Ellis and Olive applied [26] this effect to the constraints on the R-parity violating couplings. The bounds are first estimated [27, 28] as

$$\lambda, \lambda', \lambda'' \lesssim 10^{-7}\left(\frac{m_{\rm SUSY}}{1{\rm TeV}}\right)^{1/2} \tag{4.5}$$

by discussions similar to what we explained just above, and subsequent works [29, 30] support this estimation.

It was pointed out in Ref. [31] that the wash out can be avoided if at least one of $B/3 - L_i$ is conserved. Dreiner and Ross [32] applied this fact to the R-parity violating couplings. Davidson mentioned [25] that the lepton flavors are unlikely to be conserved in supersymmetric standard models, and also noted that LFV effect would cover the loophole. Then the constraints on the L-violationg couplings reappear. In the paper, the following bounds are estimated:

$$\lambda, \lambda' < 10^{-7}, \qquad \frac{(m_L^2)_{ij}}{(m_L^2)_{ii}} \lesssim 5\times 10^{-2}, \tag{4.6}$$

where the former is the baryon wash out bounds for the *lepton-number* violating couplings, and the latter one denotes how small should the lepton flavor violation be in order to realize the "separatedly conserved" loophole. Here, $(m_L^2)_{ij}$ is the slepton mass matrix in the MSSM ~~SUSY~~ part (B.4).

* * *

Now what we will discuss is much more nice calculation of these bounds, which Endo, Hamaguchi, and the author presented in Ref. [1].

4.1.4 Clarification

Here we clarify the condition: we cannot obtain the current universe (the presence of the baryon asymmetry) if

- the *present* baryon asymmetry was created before EWPT, and
- the B-violating processes are strong enough to wash out the baryon asymmetry,

or

- the *present* baryon asymmetry was created before EWPT,
- there is no source of baryon asymmetry after the EWPT,
- the L-violating processes are strong enough to wash out the baryon asymmetry, and
- these L-violating processes invade to all the lepton generations, i.e., for all L_i's, at least one of the following conditions is satisfied:
  - it is directly attacked, e.g. we have $W \supset H_\mathrm{u}L_i$, $L_iL\bar{E}$, or $L_iQ\bar{D}$ interactions, or
  - it is mixed by LFV processes with another generation L_j, and L_j is attacked.

Section 4.2 Lepton Flavor Violation Bounds

First, we discuss how fast the lepton flavor is mixed under LFV processes, in other words, how large violation is necessary to mix the flavors. Here, we do not introduce any R-parity violations to concentrate on the effect of the LFV.

In the MSSM with the R-parity, we have the following lepton interaction terms:

- gauge interactions,
- 湯川 interactions,
- $\not{SUSY}$ slepton mass terms m_L^2 and $m_{\bar{E}}^2$,
- a $\not{SUSY}$ trilinear term a_e (which we ignore here).

Usually, to discuss the low energy phenomenology of the LFV, we use the basis where the 湯川 matrix is diagonal as well as the gauge interactions. In this basis, we have a term

$$W \supset (y_e)_{ii}L_i\bar{E}_iH_\mathrm{d}, \tag{4.7}$$

which has no mixing, in the superpotential, and soft slepton masses with lepton flavor violation

$$-\mathcal{L} \supset (m_L^2)_{ij}\widetilde{L}_i^*\widetilde{L}_j + (m_{\bar{E}}^2)_{ij}\widetilde{\bar{e}}_i\widetilde{\bar{e}}_j \tag{4.8}$$

in the Lagrangian.

To discuss the LFV effects in the early universe, however, it is more appropriate to take the basis where the slepton mass matrices are diagonal [25], as well as the gauge interactions. Note that the gauge interaction is more suitable to be diagonalized than the 湯川 interactions, because the gaugino–slepton–lepton interactions are stronger than the 湯川, and therefore we cannot let the gauge coupling not diagonal with making the 湯川 matrices diagonal. Thus, starting from the basis where the 湯川 matrix is diagonal, we rotate the leptons and the sleptons by the same matrix, which guarantees that the gauge interactions remain diagonal, so that the mass matrices should be diagonalized. Assuming that the mixing angles are small, we can express these rotations as

$$L_i \to L_i + \sum_{i\neq j} \theta_{ij}^L L_j, \qquad \bar{E}_i \to \bar{E}_i + \sum_{i\neq j} \theta_{ij}^{\bar{E}} \bar{E}_j, \tag{4.9}$$

where $\theta_{ij} \simeq -\theta_{ji}$ are the mixing angles.

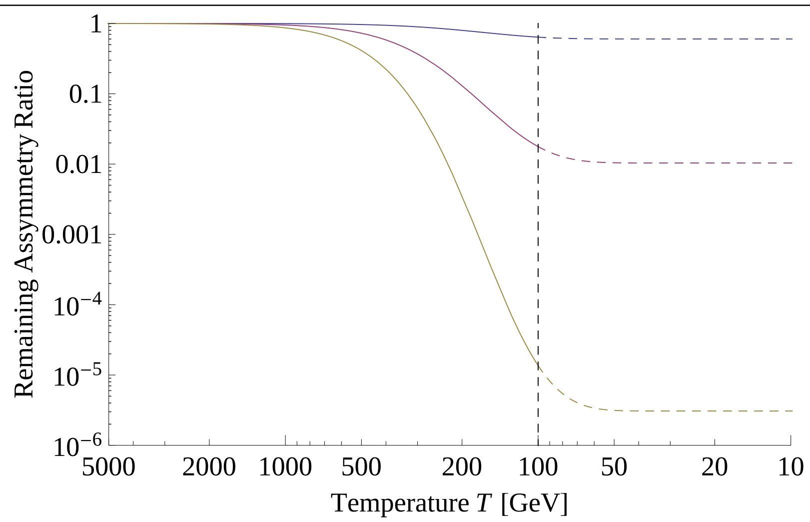

Fig. 4.1 Time evolution of $N_{[L_2-L_3]}$ for slepton mixing angles $\theta_{23}^{L/\bar{E}} = 1\times10^{-6}, 3\times10^{-6}$, and 5×10^{-6}, from the top to the bottom, for $m_{\widetilde{H}} = 600$GeV, $m_{\widetilde{l}} = 200$GeV, and $\tan\beta = 10$. The vertical dashed line denotes the sphaleron decoupling temperature $T_* \simeq 100$GeV. The normalization is arbitrary. The time evolution of $N_{[L_1-L_3]}$ for $\theta_{13}^{L/\bar{E}} = (1-5)\times10^{-6}$ is almost the same.

Note that those mixing angles are different from the dimensionless parameters

$$(\delta_L)_{ij} := \frac{(m^2_L)_{ij}}{(m^2_L)_{ii}}, \qquad (\delta_{\bar{E}})_{ij} := \frac{(m^2_{\bar{E}})_{ij}}{(m^2_{\bar{E}})_{ii}}, \tag{4.10}$$

which are familiar in the context of the LFV rare processes. They are related as

$$\theta^L_{ij} \simeq \left(\frac{m^2_L}{\Delta m^2_L}\right)(\delta_L)_{ij}, \qquad \theta^{\bar{E}}_{ij} \simeq \left(\frac{m^2_{\bar{E}}}{\Delta m^2_{\bar{E}}}\right)(\delta_{\bar{E}})_{ij}. \tag{4.11}$$

In this new basis, the LFV effects appear only in the 湯川 couplings, which are given by

$$W_{\rm LFV} \supset \sum_{i\neq j} h_{ij} L_i \bar{E}_j H_{\rm d} \qquad \text{where} \quad h_{ij} := (y_e)_{ii}\theta^{\bar{E}}_{ij} + (y_e)_{jj}\theta^L_{ji}. \tag{4.12}$$

For instance,

$$\begin{aligned} h_{23} &:= h_2\theta^{\bar{E}}_{23} + h_3\theta^L_{32} \\ &\simeq \left(0.0061\cdot\theta^{\bar{E}}_{23} + 0.10\cdot\theta^L_{32}\right)\left(\frac{\tan\beta}{10}\right). \end{aligned} \tag{4.13}$$

We now estimate how much the lepton flavor asymmetry $L_i - L_j$ is erased due to the above LFV interactions. To this end, we solve the Boltzmann equation for the evolution of $L_i - L_j$. Here, for simplicity, we include only the effect of the higgsino decay and its inverse process, $\widetilde{H} \rightleftarrows \widetilde{L}_i\bar{E}_j$ and $\widetilde{H} \rightleftarrows L_i\widetilde{\bar{E}}_j$, assuming that the higgsino is heavier than the sleptons. Other processes such as $2 \to 2$ scatterings and those with Higgs bosons may be comparably important, but it is expected that the bounds on the mixing angles will change only by order one factors. Note that these additional effects only strengthen the erasure effect, and therefore the bounds which we will derive should be regarded as conservative ones.

For an introduction of the Boltzmann equation, see App. 4.ii. In the appendix, we also derive the Boltzmann equation which describes the lepton difference $L_i - L_j$ as

$$T\frac{\mathrm{d}}{\mathrm{d}T}N_{[L_i-L_j]} = \frac{16(\Gamma_{ij}+\Gamma_{ji})}{3H}\frac{F_1\left(m_{\widetilde{H}}/T\right)}{F_2\left(m_{\widetilde{l}}/T\right)+2}N_{[L_i-L_j]}, \tag{4.14}$$

where T is the temperature of the universe, H is the Hubble parameter, $F_i(x) := x^2K_i(x)$

with $K_i(x)$ being the modified Bessel functions of the second kind. $N_{[L_i-L_j]}$ is defined as

$$N_{[L_i-L_j]} := N_{[L_i]} - N_{[L_j]} \tag{4.15}$$

$$:= \frac{n_{[L_i]} - n_{[L_j]}}{T^3}, \tag{4.16}$$

and here $N_{[L_i]}$, $n_{[L_i]}$, etc. denote the "effective yield," or the effective number density, of lepton in i-th generation (See: Eq. (3.33) as an example). The partial decay rate Γ_{ij} is given by

$$\Gamma_{ij} = \frac{|h_{ij}|^2}{32\pi} m_{\widetilde{H}} \left(1 - \frac{m_{\widetilde{l}}^2}{m_{\widetilde{H}}^2}\right)^2, \tag{4.17}$$

where $m_{\widetilde{H}}$ and $m_{\widetilde{l}}$ are the masses of higgsino and sleptons, respectively. We assume that the slepton masses are approximately the same, as the end of Sec. 3.2. Note that the Boltzmann equation (4.14) is symmetric under the exchange of the left-handed and right-handed slepton mixings, $\theta^L_{ij} \leftrightarrow \theta^{\bar{E}}_{ij}$, i.e., they give the same effect on the evolution of $N_{[L_i-L_j]}$.

In Fig. 4.1, the time evolution of $N_{L_2-L_3}$ is shown for $\theta^{L/\bar{E}}_{23} \simeq (1-5)\times 10^{-6}$, for $m_{\widetilde{H}} = 600$ GeV, $m_{\widetilde{l}} = 200$ GeV, and $\tan\beta = 10$. One can see that the flavor asymmetry is rapidly decreased for $T \lesssim m_{\widetilde{H}}$, and almost washed out for $\theta^{L/\bar{E}}_{23} \gtrsim 3\times 10^{-6}$. The time evolution of $N_{L_1-L_3}$ for $\theta^{L/\bar{E}}_{13} \simeq (1-5)\times 10^{-6}$ is essentially the same.

In Fig. 4.2 and Fig. 4.3, we show the dilution factors

$$D_{[L_i-L_j]} := \frac{N_{[L_i-L_j]}(T_*)}{N_{[L_i-L_j]}(T \gg T_*)} \tag{4.18}$$

as functions of the mixing angles $\theta^{L/\bar{E}}_{ij}$, where $T_* \sim 100$ GeV is the temperature when the sphaleron process is decoupled. In the numerical calculations, we take $T_* = 100$ GeV, $m_{\widetilde{H}} = 200$, 600 and 1200 GeV, $m_{\widetilde{l}}/m_{\widetilde{H}} = 0.4$ and 0.8, and $\tan\beta = 10$. Note that the dilution effect is weaker for $m_{\widetilde{H}} = 200$GeV than for 600GeV. This is because for $m_{\widetilde{H}} = 200$GeV the duration of the $L_i - L_j$ erasure is shorter than for $m_{\widetilde{H}} = 600$GeV.

One can see that the lepton flavor asymmetries $L_2 - L_3$, $L_1 - L_3$, and $L_1 - L_2$ are washed away for

$$\theta^{L/\bar{E}}_{23} \gtrsim (0.3-1.0)\times 10^{-5} \cdot \left(\frac{\tan\beta}{10}\right)^{-1}, \tag{4.19}$$

$$\theta^{L/\bar{E}}_{13} \gtrsim (0.3-1.0)\times 10^{-5} \cdot \left(\frac{\tan\beta}{10}\right)^{-1}, \tag{4.20}$$

$$\theta^{L/\bar{E}}_{12} \gtrsim (0.6-1.6)\times 10^{-4} \cdot \left(\frac{\tan\beta}{10}\right)^{-1}, \tag{4.21}$$

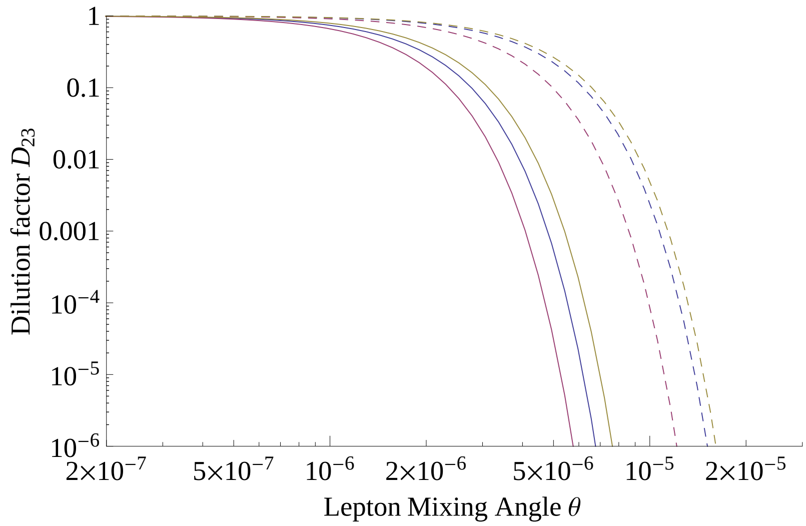

Fig. 4.2 The dilution factor $D_{[L_2-L_3]}$ ($D_{[L_1-L_3]}$) as a function of the slepton mixing angle θ^{L}_{23} (θ^{L}_{13}) or $\theta^{\bar{E}}_{23}$ ($\theta^{\bar{E}}_{13}$), for $m_{\widetilde{H}}$ = 600, 200 and 1200 GeV, from the left to the right. The slepton mass $m_{\widetilde{l}}$ is $0.4 m_{\widetilde{H}}$ for the solid lines and $0.8 m_{\widetilde{H}}$ for the dashed lines. We took $T_* = 100$GeV and $\tan\beta = 10$.

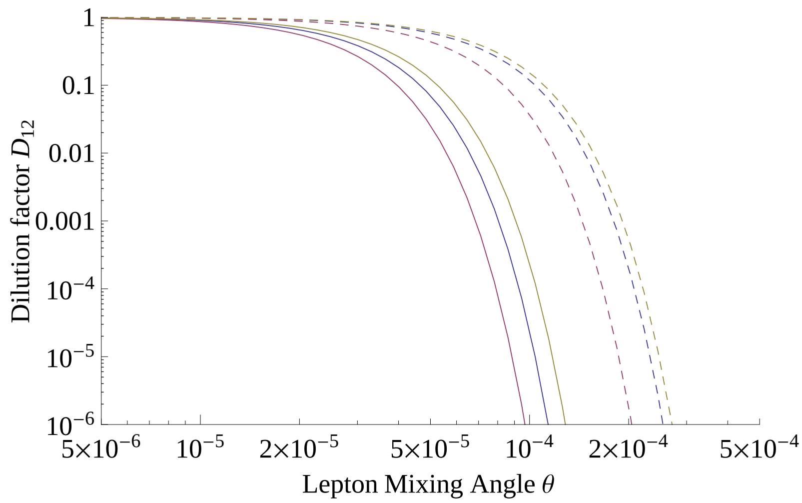

Fig. 4.3 The same as Fig. 4.2 but for the dilution factor $D_{[L_1-L_2]}$ as a function of the slepton mixing angle $\theta^{L/\bar{E}}_{12}$.

respectively. (We take the value where the dilution factor becomes $D_{L_i-L_j} \simeq 0.01$.) If any two of these inequalities are simultaneously satisfied, all lepton flavor numbers become essentially the same: $L_1 = L_2 = L_3$, and hence $B - L_1/3 = B - L_2/3 = B - L_3/3$.

4.2.1 NOTE: SUCH LFVS ARE NATURALLY EXPECTED!

Here we will present that these (enough large) LFVs are naturally expected from the viewpoint of higher energy theories.

◆m_L^2 and the right-handed neutrinos

First, let us see that m_L^2 would be an actual source of the LFVs in the presence of the right-handed neutrinos. This discussion is along Ref. [33]. We describe the right-handed neutrinos in the superfield notation as $\bar{N}_i$, like the right-handed electrons $\bar{E}_i$. They are singlets of both SU(2) and SU(3), and have no hypercharge. Thus the superpotential is modified as

$$W_{\mathrm{RPC}} \mathrel{+}= (y_\nu)_{ij} H_{\mathrm{u}} \bar{N}_i L_j + (\mu_N)_{ij} \bar{N}_i \bar{N}_j, \tag{4.22}$$

and also the $\not{\text{SUSY}}$ part is modified as

$$L_{\not{\text{SUSY}}} \mathrel{+}= -\left(m_\nu^2\right)_{ij} \widetilde{\bar{\nu}}_i^* \widetilde{\bar{\nu}}_j - \left((b_N)_{ij} \widetilde{\bar{\nu}}_i \widetilde{\bar{\nu}}_j + (a_\nu)_{ij} H_{\mathrm{u}} \widetilde{\bar{\nu}}_i \widetilde{L}_j + \text{H. c.}\right) \tag{4.23}$$

Here, for simplicity, we consider only the R-parity conserving terms.[*1] This superpotential W_{RPC} explains the experimental fact that the (left-handed) neutrino mass are extremely small, with the see-saw mechanism [34, 35]. Here, μ_N is assumed to be extremely large.

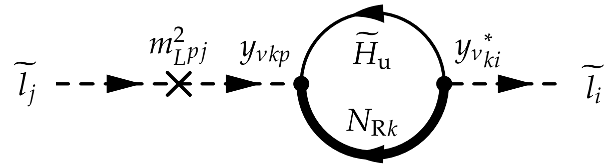

Fig. 4.4 One of the new contributions to the renormalization group equation of m_L^2 under the right-handed neutrino $\bar{N}_i$.

*1 For your information: $W_{\mathrm{RPV}} \mathrel{+}= b_i N_i + y'_i H_{\mathrm{u}} H_{\mathrm{d}} \bar{N}_i + y''_{ijk} \bar{N}_i \bar{N}_j \bar{N}_k$.

Note that we cannot diagonalize both y_e and y_ν without disturbing the SU(2) gauge symmetry.[*2] Here we have diagonalized y_e, and thus y_ν is not diagonal. Then, the right-handed neutrinos contribute to the renormalization group equation. Fig. 4.4 is one of such contributions, and the equation is modified as

$$\frac{\mathrm{d}}{\mathrm{d}\log E}\left(m_L^2\right)_{ij} = \left[\frac{\mathrm{d}}{\mathrm{d}\log E}(m_L^2)_{ij}\right]_{\mathrm{MSSM}} + \frac{1}{16\pi^2}\left[\left(y_\nu^\dagger y_\nu m_L^2 + m_L^2 y_\nu^\dagger y_\nu + 2y_\nu^\dagger m_L^2 y_\nu\right)_{ij} + 2m_{H_\mathrm{u}}^2\left(y_\nu^\dagger y_\nu\right)_{ij} + 2\left(a_\nu^\dagger a_\nu\right)_{ij}\right]. \tag{4.24}$$

Here, we assume that the ~~SUSY~~ parameters are unified at the unification scale M_{pl} as

$$\left(m_L^2\right)_{ij} = m_0^2\delta_{ij}, \qquad m_{H_\mathrm{u}}^2 = m_0^2; \qquad (a_\nu)_{ij} = a_0^2. \tag{4.25}$$

Then we obtain an approximate solution for the additional contributions to the mass terms:

$$\left(\Delta m_L^2\right)_{ij} \approx -\frac{1}{16\pi^2}(y_\nu^\dagger y_\nu)_{ij}\left(6m_0^2 + 2a_0^2\right)\log\frac{M_{\mathrm{pl}}}{M_{\mathrm{R}}}, \tag{4.26}$$

where M_{R} is the mass of the right-handed neutrino. (We assume that the masses are nearly independent of the flavor index.)

Finally we obtain

$$\left(m_L^2\right)_{ij} \approx \begin{cases} m_0^2 & (i=j), \\ -\dfrac{3m_0^2 + a_0^2}{8\pi^2}\,(y_\nu)^*_{ki}\,(y_\nu)_{kj}\log\dfrac{M_{\mathrm{pl}}}{M_{\mathrm{R}}} & (i\neq j). \end{cases} \tag{4.27}$$

Thus δ_L, which is defined as Eq. (4.10), is now

$$(\delta_L)_{ij} \approx \frac{3+a^2}{8\pi^2}\,(y_\nu)^*_{ki}\,(y_\nu)_{kj}\log\frac{M_{\mathrm{pl}}}{M_{\mathrm{R}}} \approx 0.1\cdot(y_\nu)^*_{ki}\,(y_\nu)_{kj}\,, \tag{4.28}$$

where $a^2 := a_0^2/m_0^2$. Note that the rotation angle θ^L is much larger than δ_L, as we saw in (4.11). Therefore we can conclude that our results Eqs. (4.19)–(4.21) are naturally expected.

◆$m_{\bar{E}}^2$ and GUTs

Next, we discuss to conclude that $m_{\bar{E}}^2$ can also be expected to be large enough to mix the lepton flavors when we consider SU(5) grand unified theories (GUTs).

[*2] In the Standard Model, we diagonalize both of them, which results in flavor violating $W^\pm$–q–q interactions. See App. A.2.3.

It is expected that our $U(1)_Y$, $SU(2)_{weak}$ and $SU(3)_{color}$ gauge symmetries are unified at some very high energy, and many models are proposed as such unified theories. Here we consider SU(5) GUTs, where our three gauge symmetries are unified to one SU(5) gauge symmetry at some high energy scale M_{GUT}.

> To be honest, it is a bit difficult to embed the R-parity violating SUSY models into SU(5) GUTs, because we need B- or L-parity conservation instead of that of R-parity, and they draw a sharp contrast between baryon and lepton.
> You can see this feature by considering the interactions, which we will present in Eq. (4.32). The R-parity violating interactions $\bar{U}\bar{U}\bar{D}$, $LQ\bar{D}$ and $LL\bar{E}$ appear all together.
> However we do not discuss those matters in this thesis.

Our superfields are embedded in SU(5) representations as follows:

$$\mathbf{10}_i \ni Q_i, \bar{U}_i, \bar{E}_i, \qquad \bar{\mathbf{5}}_i \ni \bar{D}_i, L_i, \qquad \mathbf{5}_H \ni H_u^C, H_u, \qquad \bar{\mathbf{5}}_H \ni H_d^C, H_d. \tag{4.29}$$

Here, H_u^C and H_d^C are "colored Higgs" of up-type and of down type.

> The colored Higgs particles are considered to be extremely heavy, lest the proton should decay via the interactions $\bar{U}\bar{D}H_d^C$ and QLH_d^C (See Eq. (4.32)) with a process like Fig. 2.1). The decay rate is
>
> $$\Gamma \sim \left|(y_C)_{11}\right|^4 \frac{m_{\text{proton}}^5}{M^4} \times 3 = \frac{1}{1\times 10^{32}\text{yr}} \left(\frac{\left|(y_C)_{11}\right|}{10^{-6}}\right)^4 \left(\frac{10^{10}\text{GeV}}{M}\right)^4, \tag{4.30}$$
>
> where M is the mass of the colored Higgs. Thus the mass must be heavier than (at least) 10^{10}GeV, and usually considered as $\sim M_{GUT}$.

We can form the following gauge singlets:

$$\mathbf{10}\,\mathbf{10}\,\mathbf{5} \qquad \mathbf{10}\,\bar{\mathbf{5}}\,\bar{\mathbf{5}} \qquad \mathbf{5}\,\bar{\mathbf{5}}, \tag{4.31}$$

Table 4.1 The property of the heavy particles which we introduce in this section. See Tab. B.1 for the MSSM particles.

Neutrino (chiral multiplet)			
	SU(3)	SU(2)	U(1)
$\bar{N}_i$	**1**	**1**	0

Colored Higgs (chiral multiplet)			
	SU(3)	SU(2)	U(1)
H_u^C	**3**	**1**	−1/3
H_d^C	$\bar{\mathbf{3}}$	**1**	1/3

and thus the following terms in the superpotential can be obtained:

$$\begin{aligned}
(y_A)_{ij}\ \mathbf{10}_i\mathbf{10}_j\mathbf{5}_H &\to -4\,(y_A)_{ij}\left[Q_iQ_jH_\mathrm{u}^\mathrm{C}+\left(\bar U_iQ_j+\bar U_jQ_i\right)H_\mathrm{u}+\left(\bar U_i\bar E_j+\bar U_j\bar E_i\right)H_\mathrm{u}^\mathrm{C}\right],\\
(y_B)_{ijk}\ \mathbf{10}_i\bar{\mathbf{5}}_j\bar{\mathbf{5}}_k &\to (y_B)_{ijk}\left[\bar U_i\bar D_j\bar D_k-\bar E_iL_jL_k-Q_i\left(L_j\bar D_k-L_k\bar D_j\right)\right],\\
(y_C)_{ij}\ \mathbf{10}_i\bar{\mathbf{5}}_j\bar{\mathbf{5}}_H &\to (y_C)_{ij}\left(\bar U_i\bar D_jH_\mathrm{d}^\mathrm{C}-\bar E_iL_jH_\mathrm{d}-Q_i\bar D_jH_\mathrm{d}+Q_iL_jH_\mathrm{d}^\mathrm{C}\right),\\
\mu\ \mathbf{5}_H\bar{\mathbf{5}}_H &\to \mu\left(H_\mathrm{u}^\mathrm{C}H_\mathrm{d}^\mathrm{C}+H_\mathrm{u}H_\mathrm{d}\right),\\
\mu_i\ \mathbf{5}_H\bar{\mathbf{5}}_i &\to \mu_i\left(H_\mathrm{u}^\mathrm{C}\bar D_i+H_\mathrm{u}L_i\right).
\end{aligned}\tag{4.32}$$

Now consider that we are above GUT scale M_GUT, and let us do the mass-diagonalization procedure as we did in the Standard Model (See: App. A.2.3). In other words, we will write down Eq. (4.32) in the basis which we usually use in the Standard Model. We have the following terms in the superpotential:

$$W\supset (y_u)_{ij}\bar U_iH_\mathrm{u}Q_j+(y_u)_{ij}H_\mathrm{u}^\mathrm{C}\bar U_i\bar E_j-(y_d)_{ij}\bar D_iH_\mathrm{d}Q_j-(y_d)_{ij}\bar E_iH_\mathrm{d}L_j,\tag{4.33}$$

where y_u is symmetric (at least at the GUT scale), but not normal[*3], so we have to use the singular value decomposition method. The procedure is similar to what we will present in App. A.2.3, but here, as $y_d=y_e$, the rotations are

$$Q^1\mapsto\Psi_u^\dagger Q^1,\quad Q^2\mapsto\Psi_d^\dagger Q^2,\quad L\mapsto\Psi_d^\dagger L,\qquad U\mapsto\Phi_u^\dagger U,\quad D\mapsto\Phi_d^\dagger D,\quad E\mapsto\Phi_d^\dagger E,\tag{4.34}$$

and the superpotential would be diagonalized:

$$W\supset (y'_u)_{ii}\bar U_iH_\mathrm{u}Q_i+(y'_u)_{ii}H_\mathrm{u}^\mathrm{C}\bar U_i\left(\Psi_u\Phi_d^\dagger\right)_{ij}\bar E_j-(y'_d)_{ii}\bar D_iH_\mathrm{d}Q_i-(y'_d)_{ii}\bar E_iH_\mathrm{d}L_i.\tag{4.35}$$

(Note that this equation is written in mass eigenstates, and y'_u and y'_d are diagonal.) What is important is the non-diagonal term. This non-diagonal matrix $\Psi_u\Phi_d^\dagger$ is the very Cabibbo–小林–益川 matrix V_CKM, and therefore we have

$$W\supset (y'_u)_iH_\mathrm{u}^\mathrm{C}\bar U'_i\,(V_\mathrm{CKM})_{ij}\,\bar E'_j\tag{4.36}$$

in the superpotential. This term would contribute to the renormalization group equation of $m_{\bar E}^2$ as Fig. 4.5.

Defining

$$X_{ij}:=(y'_u)_i\,(V_\mathrm{CKM})_{ij}\,,\tag{4.37}$$

we can approximately write down the contribution to $m_{\bar E}^2$ as

$$\left(\Delta m_{\bar E}^2\right)_{ij}\approx-\frac{3}{16\pi^2}(X^\dagger X)_{ij}\left(6m_0^2+2a_0^2\right)\log\frac{M_\mathrm{pl}}{M_\mathrm{GUT}}\tag{4.38}$$

$$\simeq-\frac{3}{16\pi^2}(y'_u)_{33}^2(V_\mathrm{CKM})_{3i}^*(V_\mathrm{CKM})_{3j}\left(6m_0^2+2a_0^2\right)\log\frac{M_\mathrm{pl}}{M_\mathrm{GUT}}.\tag{4.39}$$

[*3] A matrix A is "normal" when it satisfies $AA^\dagger=A^\dagger A$.

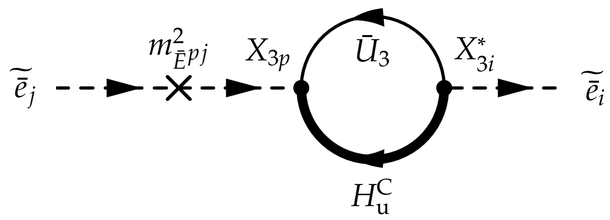

Fig. 4.5 One of the new contribution to the renormalization group equation of $m^2_{\bar{E}}$ under the colored higgs. Here $X_{ij} := (y'_u)_i (V_{\rm CKM})_{ij}$

Here we assume that the mass of the colored Higgs is $\sim M_{\rm GUT}$, and the coefficient 3 comes from the $SU(3)_{\rm color}$ symmetry.

Therefore $\delta_{\bar{E}}$ is

$$(\delta_{\bar{E}})_{ij} \approx \frac{3(3+a^2)}{8\pi^2} (y'_u)^2_{33} (V_{\rm CKM})^*_{3i} (V_{\rm CKM})_{3j} \log \frac{M_{\rm pl}}{M_{\rm GUT}} \tag{4.40}$$

$$\sim \begin{cases} 10^{-4} & \text{for 1–2 mixing,} \\ 10^{-2} & \text{for 2–3 mixing,} \\ 10^{-3} & \text{for 1–3 mixing.} \end{cases} \tag{4.41}$$

Therefore $\theta^{\bar{E}}$ is also expected to be large enough to mix all the lepton flavors, and it is natural for us to assume that all the lepton flavor asymmetries are equilibrated.

Section 4.3 Implications for the R-Parity Violation

In the last section Sec. 4.2, we showed that, under the large slepton mixing angles which satisfy at least two of Eqs. (4.19)–(4.21), all the lepton flavor asymmetries are equilibrated, i.e.,

$$L_1 = L_2 = L_3. \tag{4.42}$$

Also we have shown just above that such large slepton mixings are expected in a wide class of SUSY models. Therefore, not only the B-violating coupling but also L-violating ones are expected to be constrained by the cosmological constraints.

In this section, we discuss the bounds on the R-parity violating couplings, assuming that we have such large lepton flavor violations.

4.3.1 Cosmological Bounds on the R-Parity Violation in the presence of Slepton Mixings

We assume that at least two of Eqs. (4.19)–(4.21) are satisfied, and hence all $B - L_i/3$ are equilibrated. Then, in order to avoid the baryon erasure, any of $B - L_i/3$ violating processes should not become effective before the electroweak phase transition.

We calculate the dilution factor

$$D_{B-L} = \frac{N_{B-L}(T_*)}{N_{B-L}(T \gg T_*)} \tag{4.43}$$

as functions of the R-parity violating couplings λ_{ijk}, λ'_{ijk}, λ''_{ijk}, and κ_i.[*4] The corresponding Boltzmann equations are shown in Appendix 4.ii.3. The results are shown in Figs. 4.6–4.9. Here, for simplicity, we have assumed that all sleptons and all squarks have the same masses $m_{\tilde{l}}$ and $m_{\tilde{q}}$, respectively.

From the figures, one can see that the couplings should satisfy

$$\sqrt{\sum_{ijk} \left|\lambda''_{ijk}\right|^2} \lesssim (4\text{–}5) \times 10^{-7}, \tag{4.44}$$

$$\sqrt{\sum_{ijk} \left|\lambda'_{ijk}\right|^2} \lesssim (3\text{–}6) \times 10^{-7}, \tag{4.45}$$

$$\sqrt{\sum_{ijk} \left|\lambda_{ijk}\right|^2} \lesssim (0.6\text{–}1) \times 10^{-6}, \tag{4.46}$$

$$\sqrt{\sum_{i} \left|\frac{\kappa_i}{\mu}\right|^2} \lesssim (1\text{–}2) \times 10^{-6} \left(\frac{\tan\beta}{10}\right)^{-1}, \tag{4.47}$$

for $m_{\tilde{q}} \simeq 200 - 1200\text{GeV}$ and $m_{\tilde{l}} \simeq 100 - 400\text{GeV}$. (Again, we took the value where the dilution of the $B - L$ becomes $D_{B-L} \simeq 0.01$.) We should note that the bounds on the $\bar{U}\bar{D}\bar{D}$ coupling λ''_{ijk} in Eq. (4.44) apply even without the lepton flavor violation.

These are *our* cosmological constraints on the R-parity violating couplings. As you can see, these are much severer than what we obtained in Chap. 2. Also these are very important for collider phenomenology, which we will discuss in the next chapter.

[*4] We do not discuss the bounds on the R-parity violating soft terms, for simplicity.

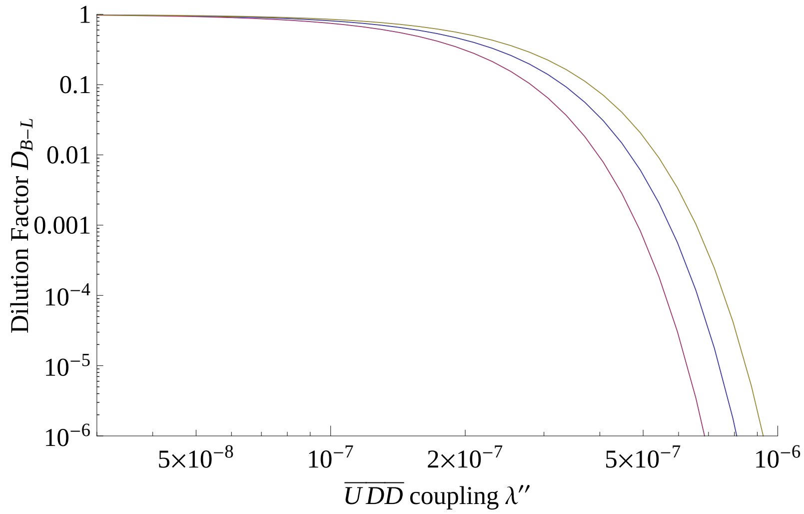

Fig. 4.6 The dilution factor D_{B-L} in the presence of an R-parity violating term $\lambda''\bar{U}_i\bar{U}_j\bar{D}_k$ for $m_{\tilde{q}}$ = 600, 200 and 1200 GeV, from the left to the right. We took $m_{\tilde{l}}$ = 100GeV and $m_{\tilde{H}}$ = 300GeV, but this result is nearly independent of these masses. Other parameters are: $\tan\beta = 10$ and $T_* = 100$GeV.

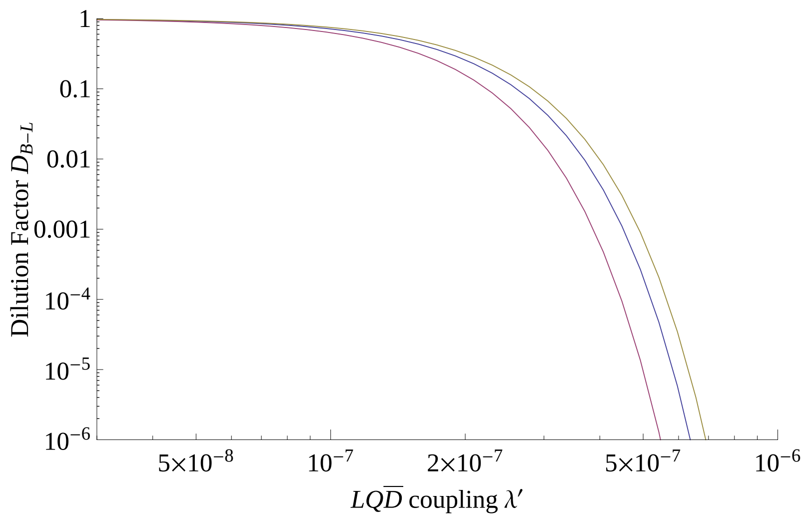

Fig. 4.7 The same as Fig. 4.6 for $\lambda' L_i Q_j \bar{D}_k$ interaction. Parameters are the same as Fig. 4.6. $m_{\tilde{l}}$ and $m_{\tilde{H}}$ hardly affect the result again.

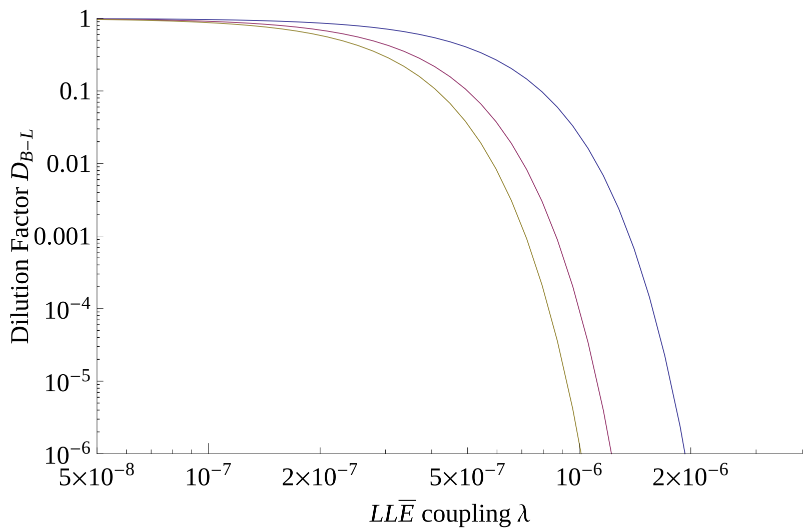

Fig. 4.8 The same as Fig. 4.6 for $\lambda L_i L_j \bar{E}_k$ interaction. $m_{\tilde{l}} = 400$, 200 and 100 GeV, from the left to the right, $m_{\tilde{q}} = 600$GeV and $m_{\tilde{H}} = 300$GeV. In this case the result depends on $m_{\tilde{l}}$, and is almost independent of the other masses.

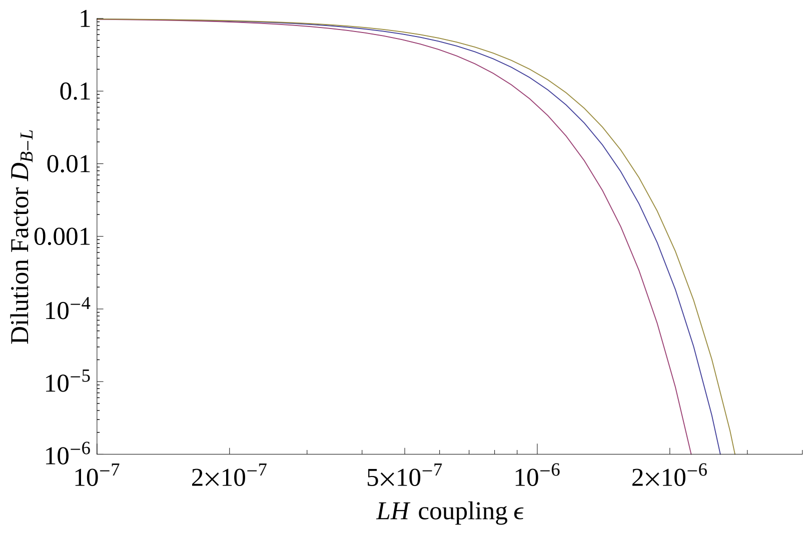

Fig. 4.9 The same as Fig. 4.6 in the presence of a bilinear R-parity violating term $\kappa_i L_i H_\mathrm{u}$ as a function of $\epsilon_i := \kappa_i/\mu$. The masses and the other parameters are the same as Fig. 4.6. The result is nearly independent of $m_{\tilde{l}}$, $m_{\tilde{H}}$, and the generation index i.

Appendix 4.i Decay and Inverse Decay in Details

If we have a decay process $X \to AY$ in the early universe, then we also have its inverse process $AY \to X$. This "inverse decay" process usually does not realized because its phase space is very limited, but if the particles are in a thermal bath, the thermal effects help the process to occur.

The rates are described as

$$R(X \to AY) = n_X \Big\langle \Gamma_{X\to AY} \Big\rangle, \qquad R(AY \to X) = n_A n_Y \Big\langle (\sigma v)_{AY\to X} \Big\rangle, \tag{4.48}$$

where n is the number density of the particle, v is the relative velocity between A and Y, and $\langle\ \rangle$ denotes the thermal average.

Since we have decay and inverse decay, the time evolution of a particle is governed not by the decay rate, but the "effective decay rate," which we will define as follows:[*5]

$$\begin{aligned} \Big\langle\!\Big\langle X \leftrightarrows AY \Big\rangle\!\Big\rangle &:= R(X \to AY) - R(AY \to X) \\ &= n_X \Big\langle \Gamma_{X\to AY} \Big\rangle - n_A n_Y \Big\langle (\sigma v)_{AY\to X} \Big\rangle. \end{aligned} \tag{4.49}$$

In this appendix, we will examine the relation between decay rates and inverse decay rates, and then obtain the useful expression of the effective decay rate.

◆The rate without thermal effects

At first, we calculate the decay rate and the crosssection of one particle without thermal effects. The decay rate in the rest frame is described as [17]

$$\Gamma^0_{X\to AY} = \frac{1}{2m_X} \int \mathrm{d}\Pi_A \mathrm{d}\Pi_Y (2\pi)^4 \delta^{(4)}(k_X - k_A - k_Y) \Big[\sum |\mathcal{M}|^2\Big]_{\mathrm{D}}, \tag{4.50}$$

where $\mathcal{M}$ is the invariant matrix element of the process, the summation symbol denotes sum over the final states, and $\mathrm{d}\Pi$ is the phase space integral which is defined as

$$\mathrm{d}\Pi := \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} \frac{1}{2E}. \tag{4.51}$$

In a similar manner, we can write down the crosssection of the inverse decay as

$$\sigma^0_{AY\to X} = \frac{1}{2E_A 2E_Y |v_A - v_Y|} \int \mathrm{d}\Pi_X (2\pi)^4 \delta^{(4)}(k_X - k_A - k_Y) \Big[\sum |\mathcal{M}'|^2\Big]_{\mathrm{ID}}, \tag{4.52}$$

[*5] In the next section (App. 4.ii), we will use this effective decay rate in the Boltzmann equation.

but this process does not occur without thermal effects, for the momentum conservation severely restricts the initial states.

Note that these matrix elements are related as follows:

$$\frac{1}{g_A g_Y}\left[\sum|\mathcal{M}|^2\right]_{\mathrm{D}} = \frac{1}{g_X}\left[\sum|\mathcal{M}'|^2\right]_{\mathrm{ID}}, \tag{4.53}$$

for we shall take the sum over the final states.

◆Momentum distribution and number density

Now let us introduce the temperature. We will consider the situation that all the particles are in a thermal bath whose temperature is T. Then, the momentum distribution $f(\boldsymbol{k})$ of each particle is given by the Maxwell–Boltzman, the Fermi–Dirac or the Bose–Einstein distribution

$$f_{\mathrm{MB}}(\boldsymbol{k}) = \frac{1}{\mathrm{e}^{(E-\mu)/T}}, \qquad f_{\mathrm{BE}}(\boldsymbol{k}) = \frac{1}{\mathrm{e}^{(E-\mu)/T}-1}, \qquad f_{\mathrm{FD}}(\boldsymbol{k}) = \frac{1}{\mathrm{e}^{(E-\mu)/T}+1}, \tag{4.54}$$

where E is the energy, which is given by $\sqrt{m^2+\|\boldsymbol{k}\|^2}$, and m and μ are the mass and the chemical potential of the particle, respectively. The number density n of a particle is expressed by its distribution function f and degree of freedom g as

$$n = g\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}f(\boldsymbol{k}). \tag{4.55}$$

◆Statistical effect on final states

To calculate the rates (4.48), we have to take care of one more thing, that is, the statistical effect on the final states. If a particle decays into a bosonic state, the decay rate is enhanced by the Bose–Einstein statistics, while if into a fermionic state, it is suppressed by the Fermi–Dirac one.

The decay rate at finite temperature, which includes the final state effect, is given by

$$\Gamma_{X\to AY} = \frac{1}{2E_X}\int\mathrm{d}\Pi_A\mathrm{d}\Pi_Y\,\phi_A\phi_Y(2\pi)^4\delta^{(4)}(k_X-k_A-k_Y)\left[\sum|\mathcal{M}|^2\right]_{\mathrm{D}}. \tag{4.56}$$

Here ϕ is the function of the final state effect, which is defined as

$$\phi_{\mathrm{MB}}(\boldsymbol{k}) := 1, \qquad \phi_{\mathrm{BE}}(\boldsymbol{k}) := 1+f_{\mathrm{BE}}(\boldsymbol{k}), \qquad \phi_{\mathrm{FD}}(\boldsymbol{k}) := 1-f_{\mathrm{FD}}(\boldsymbol{k}), \tag{4.57}$$

or more simply,[*6]

$$\phi(\boldsymbol{k}) := \mathrm{e}^{(E-\mu)/T}f(\boldsymbol{k}). \tag{4.58}$$

[*6] Though seems to be non-trivial, these definitions are equivalent.

We would mention that, when we consider the Bose–Einstein or the Fermi–Dirac distribution, this thermal decay rate cannot be transformed into that in the rest frame of the decaying particle because of the final state effects.

Similarly, we can obtain the cross section of the inverse decay at finite temperature as

$$\sigma_{AY\to X} = \frac{1}{2E_A 2E_Y\,|v_A - v_Y|}\int \mathrm{d}\Pi_X\,\phi_X (2\pi)^4\delta^{(4)}(k_X - k_A - k_Y)\left[\sum|\mathcal{M}'|^2\right]_{\mathrm{ID}}. \tag{4.59}$$

◆Calculate and calculate

Now we can calculate the rates (4.48):

$$\begin{aligned}
&R(X\to AY)\\
&\quad = n_X\left\langle\Gamma_{X\to AY}\right\rangle\\
&\quad = g_X\int\frac{\mathrm{d}^3\boldsymbol{k}_X}{(2\pi)^3}f_X\cdot\Gamma_{X\to AY}\\
&\quad = g_X\int \mathrm{d}\Pi_X\mathrm{d}\Pi_A\mathrm{d}\Pi_Y\,(f_X\phi_A\phi_Y)(2\pi)^4\delta^{(4)}(k_X-k_A-k_Y)\left[\sum|\mathcal{M}|^2\right]_{\mathrm{D}},
\end{aligned} \tag{4.60}$$

$$\begin{aligned}
&R(AY\to X)\\
&\quad = n_A n_Y\left\langle\sigma_{AY\to X}\cdot|v_A-v_Y|\right\rangle\\
&\quad = g_A g_Y\int \mathrm{d}\Pi_A\mathrm{d}\Pi_Y\mathrm{d}\Pi_X(f_A f_Y\phi_X)(2\pi)^4\delta^{(4)}(k_X-k_A-k_Y)\left[\sum|\mathcal{M}'|^2\right]_{\mathrm{ID}}\\
&\quad = g_X\int \mathrm{d}\Pi_A\mathrm{d}\Pi_Y\mathrm{d}\Pi_X(f_A f_Y\phi_X)(2\pi)^4\delta^{(4)}(k_X-k_A-k_Y)\left[\sum|\mathcal{M}|^2\right]_{\mathrm{D}}
\end{aligned} \tag{4.61}$$

Therefore, the effective decay rate, which is defined as Eq. (4.49), is

$$\begin{aligned}
\left\langle\!\left\langle X\rightleftarrows AY\right\rangle\!\right\rangle &= g_X\int \mathrm{d}\Pi_A\mathrm{d}\Pi_Y\mathrm{d}\Pi_X(f_X\phi_A\phi_Y - f_A f_Y\phi_X)\\
&\qquad\times(2\pi)^4\delta^{(4)}(k_X-k_A-k_Y)\left[\sum|\mathcal{M}|^2\right]_{\mathrm{D}}\\
&= g_X\int \mathrm{d}\Pi_A\mathrm{d}\Pi_Y\mathrm{d}\Pi_X\cdot f_X f_A f_Y\cdot \mathrm{e}^{E_X/T}\left[\mathrm{e}^{-(\mu_A+\mu_Y)/T} - \mathrm{e}^{-\mu_X/T}\right]\\
&\qquad\times(2\pi)^4\delta^{(4)}(k_X-k_A-k_Y)\left[\sum|\mathcal{M}|^2\right]_{\mathrm{D}}.
\end{aligned} \tag{4.62}$$

From this result, we can see that this process works to reduce the imbalance between $\mu_A + \mu_Y$ and μ_X, and finally achieves the equilibrium to satisfy the equality $\mu_X = \mu_A + \mu_Y$. Note that this fact holds regardless of the statistics and the degree of freedom of the particles.

◆Maxwell–Boltzmann approximation

Generally, these event rates are very difficult to transform into some useful expressions analytically. However, if we approximate the statistics of X, A and Y to the Maxwell–Boltzmann type, it can be expressed as

$$\begin{aligned}
&\int \mathrm{d}\Pi_A \mathrm{d}\Pi_Y \mathrm{d}\Pi_X \cdot f_X f_A f_Y \cdot \mathrm{e}^{E_X/T} (2\pi)^4 \delta^{(4)}(k_X - k_A - k_Y) \left[\sum |\mathcal{M}|^2\right]_\mathrm{D} \\
&\quad \approx \mathrm{e}^{\left(\mu_X + \mu_A + \mu_Y\right)/T} \int \mathrm{d}\Pi_A \mathrm{d}\Pi_Y \mathrm{d}\Pi_X \mathrm{e}^{-E_X/T} (2\pi)^4 \delta^{(4)}(k_X - k_A - k_Y) \left[\sum |\mathcal{M}|^2\right]_\mathrm{D} \\
&\quad = \mathrm{e}^{\left(\mu_X + \mu_A + \mu_Y\right)/T} \int \mathrm{d}\Pi_X \mathrm{e}^{-E_X/T} \cdot 2m_X \Gamma^0_{X\to AY} \\
&\quad = \mathrm{e}^{\left(\mu_X + \mu_A + \mu_Y\right)/T} \frac{T^3}{2\pi^2} F_1\left(\frac{m_X}{T}\right) \Gamma^0_{X\to AY},
\end{aligned} \tag{4.63}$$

where $F_i(x)$ is defined as

$$F_i(x) := x^2 K_i(x) \tag{4.64}$$

through the modified Bessel function of the second kind $K_i(x)$. Thus

$$\left\langle\!\left\langle X \rightleftarrows AY \right\rangle\!\right\rangle = g_X \left[\mathrm{e}^{\mu_X/T} - \mathrm{e}^{(\mu_A + \mu_Y)/T}\right] \frac{T^3}{2\pi^2} F_1\left(\frac{m_X}{T}\right) \Gamma^0_{X\to AY}. \tag{4.65}$$

We will use this result in the next section.

Appendix 4.ii Boltzmann Equations

In the section, we will derive the Boltzmann equations which are used in this chapter, that is, Eq. (4.14), etc.

4.ii.1 Boltzmann Equation

The time evolution of the number density n_A of a certain particle A obeys the Boltzmann equation. In the (expanding) universe, it is described as

$$\frac{\mathrm{d}}{\mathrm{d}t} n_A + 3Hn_A = (\text{Interacting terms}), \tag{4.66}$$

where H is the Hubble parameter.

This equation (4.66) can be easily obtained: the Hubble parameter is defined by the scale of universe a as

$$H = \frac{\dot{a}}{a}, \tag{4.67}$$

where the dot denotes the time derivative. Therefore, we can write down the equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(na^3\right) = \Delta\,(\text{the number of the particle}) = a^3\Delta n. \tag{4.68}$$

This is equivalent to the equation.

When we consider only the part of the time evolution induced by a process $X \rightleftarrows AY$, that is, the decay of some particle X and its inverse process, we can write down the Boltzmann equation with the effective decay rate, which we have defined in the last section, as

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}n_A + 3Hn_A\Big|_{X\rightleftarrows AY} &= n_X\Big\langle\Gamma_{X\to AY}\Big\rangle - n_A n_Y\Big\langle(\sigma v)_{AY\to X}\Big\rangle \\ &= \Big\langle\!\Big\langle X \leftrightarrows AY\Big\rangle\!\Big\rangle. \end{aligned} \tag{4.69}$$

Here we assume that X, A, and Y are all in a thermal bath, and discuss the effect of (very weak) $X \rightleftarrows AY$ process. Using the "yield" $N := n/T^3$ as a variable, Eq. (4.69) becomes*7

$$T\frac{\mathrm{d}}{\mathrm{d}T}N_A\Big|_{X\rightleftarrows AY} = -\frac{1}{HT^3}\Big\langle\!\Big\langle X \leftrightarrows AY\Big\rangle\!\Big\rangle. \tag{4.70}$$

We have obtained the analytical expression of this effective decay rate in the previous section, under the approximation that all the particles obey the Maxwell–Boltzmann distribution. Here also we use the approximation, and use the result

$$\Big\langle\!\Big\langle X \rightleftarrows AY\Big\rangle\!\Big\rangle = g_X\left[\mathrm{e}^{\mu_X/T} - \mathrm{e}^{(\mu_A+\mu_Y)/T}\right]\frac{T^3}{2\pi^2}F_1\left(\frac{m_X}{T}\right)\Gamma^0_{X\to AY}. \tag{4.71}$$

Moreover, as the chemical potential $\mu_{\bar{A}}$ of the antiparticle is equal to $-\mu_A$, the effective rate of the processes of antiparticles are

$$\Big\langle\!\Big\langle \bar{X} \rightleftarrows \bar{A}\,\bar{Y}\Big\rangle\!\Big\rangle = g_X\left[\mathrm{e}^{-\mu_X/T} - \mathrm{e}^{-(\mu_A+\mu_Y)/T}\right]\frac{T^3}{2\pi^2}F_1\left(\frac{m_X}{T}\right)\Gamma^0_{X\to AY}, \tag{4.72}$$

*7 We have used $dT/dt = -HT$, assuming for simplicity that the effective degrees of freedom $g_{*s}(T)$ are constant.

and therefore

$$T\frac{\mathrm{d}}{\mathrm{d}T}\Big(N_A - N_{\bar{A}}\Big)\Big|_{X\rightleftarrows AY} = -\frac{1}{HT^3}\left[\left\langle\!\left\langle X \rightleftarrows AY\right\rangle\!\right\rangle - \left\langle\!\left\langle \bar{X} \rightleftarrows \bar{A}\,\bar{Y}\right\rangle\!\right\rangle\right] \tag{4.73}$$

$$= -\frac{g_X}{\pi^2}\frac{\Gamma^0_{X\to AY}}{H}F_1\left(\frac{m_X}{T}\right)\left[\sinh\left(\frac{\mu_X}{T}\right) - \sinh\left(\frac{\mu_A+\mu_Y}{T}\right)\right]. \tag{4.74}$$

4.II.2 Lepton Flavor Violation

Here, we derive the Boltzmann equation for the LFV process in the early universe. As an example, we consider as LFV processes those induced by the following term in the superpotential:

$$W \supset h_{23}L_2\bar{E}_3H_{\mathrm{d}}\,. \tag{4.75}$$

For simplicity, we discuss only the decays and inverse decays of the higgsinos $\widetilde{H}$, and assume that all sleptons have the same mass $m_{\tilde{l}}\,(< m_{\widetilde{H}})$.

We define the asymmetry $N_{[A]}$ of a supermultiplet as, e.g. for μ_{L},

$$N_{[\mu_{\mathrm{L}}]} := \Big(N_{\mu_{\mathrm{L}}} - N_{\mu_{\mathrm{L}}^\dagger}\Big) + \Big(N_{\widetilde{\mu}_{\mathrm{L}}} - N_{\widetilde{\mu}_{\mathrm{L}}^*}\Big). \tag{4.76}$$

Since leptons are massless before the electroweak phase transition, the asymmetry is

$$\begin{aligned} N_{[\mu_{\mathrm{L}}]} &= \frac{g_{\widetilde{\mu}_{\mathrm{L}}}}{2\pi^2}F_2\left(\frac{m_{\tilde{l}}}{T}\right)\left[\exp\left(\frac{\mu_{L_2}}{T}\right) - \exp\left(\frac{-\mu_{L_2}}{T}\right)\right] + \frac{2g_{\mu_{\mathrm{L}}}}{2\pi^2}\left[\exp\left(\frac{\mu_{L_2}}{T}\right) - \exp\left(\frac{-\mu_{L_2}}{T}\right)\right] \\ &= \frac{1}{\pi^2}g_{\mathrm{eff}}\left(\frac{m_{\tilde{l}}}{T}\right)\sinh\left(\frac{\mu_{L_2}}{T}\right), \end{aligned} \tag{4.77}$$

as we have discussed in Sec. 3.1.3.[*8] Also its time evolution induced by LFV processes is described as

$$\begin{aligned} T\frac{\mathrm{d}}{\mathrm{d}T}N_{[\mu_{\mathrm{L}}]}\Big|_{\mathrm{LFV}} = -\frac{1}{HT^3}\Big[&\left\langle\!\left\langle \widetilde{H}^0 \rightleftarrows \mu_{\mathrm{L}}\widetilde{\tau}_{\mathrm{R}}^*\right\rangle\!\right\rangle + \left\langle\!\left\langle \widetilde{H}^0 \rightleftarrows \widetilde{\mu}_{\mathrm{L}}\tau_{\mathrm{R}}^\dagger\right\rangle\!\right\rangle \\ &-(\text{their antiparticles' processes})\Big] \end{aligned} \tag{4.78}$$

$$= -2\cdot\frac{2}{\pi^2}\frac{\Gamma}{H}F_1\left(\frac{m_{\widetilde{H}}}{T}\right)\left[\sinh\left(-\frac{\mu_{H_{\mathrm{d}}}}{T}\right) - \sinh\left(\frac{\mu_{L_2}+\mu_{\bar{E}_3}}{T}\right)\right], \tag{4.79}$$

where

$$\Gamma := \frac{|h_{23}|^2}{32\pi}m_{\widetilde{H}}\left(1 - \frac{m_{\tilde{l}}^2}{m_{\widetilde{H}}^2}\right)^2 \tag{4.80}$$

[*8] $g_{\mathrm{eff}}(x) := F_2(x) + 2$, which is the same as Sec. 3.1.3.

is the partial decay rate of each process, which is the same for all four processes. Similarly,

$$T\frac{\mathrm{d}}{\mathrm{d}T}N_{[\nu_\mu]}\bigg|_{\mathrm{LFV}} = -2\cdot\frac{2}{\pi^2}\frac{\Gamma}{H}F_1\left(\frac{m_{\widetilde{H}}}{T}\right)\left[\sinh\left(-\frac{\mu_{H_\mathrm{d}}}{T}\right)-\sinh\left(\frac{\mu_{L_2}+\mu_{\bar{E}_3}}{T}\right)\right], \tag{4.81}$$

$$T\frac{\mathrm{d}}{\mathrm{d}T}N_{[\tau_\mathrm{R}]}\bigg|_{\mathrm{LFV}} = -4\cdot\frac{2}{\pi^2}\frac{\Gamma}{H}F_1\left(\frac{m_{\widetilde{H}}}{T}\right)\left[\sinh\left(\frac{\mu_{H_\mathrm{d}}}{T}\right)-\sinh\left(\frac{-\mu_{L_2}-\mu_{\bar{E}_3}}{T}\right)\right]. \tag{4.82}$$

Now let us consider the asymmetry of each lepton flavor, which is defined as, for example,

$$\begin{aligned} N_2 &:= N_{[\mu_\mathrm{L}]}+N_{[\mu_\mathrm{R}]}+N_{[\nu_\mu]} \\ &\simeq \frac{g_\mathrm{eff}\left(m_{\tilde{l}}/T\right)}{\pi^2}\frac{2\mu_{L_i}-\mu_{\bar{E}_i}}{T}. \end{aligned} \tag{4.83}$$

Under the LFV interaction $L_2\bar{E}_3H_\mathrm{d}$, the time evolution of the difference $N_2 - N_3$ is given by

$$T\frac{\mathrm{d}}{\mathrm{d}T}\left(N_2-N_3\right) = \frac{\mathrm{d}}{\mathrm{d}T}\left(N_{[\mu_\mathrm{L}]}+N_{[\nu_\mu]}-N_{[\tau_\mathrm{R}]}\right) \tag{4.84}$$

$$= \frac{16}{\pi^2}\frac{\Gamma}{H}F_1\left(\frac{m_{\widetilde{H}}}{T}\right)\left[\sinh\left(\frac{\mu_{H_\mathrm{d}}}{T}\right)+\sinh\left(\frac{\mu_{L_2}+\mu_{\bar{E}_3}}{T}\right)\right] \tag{4.85}$$

$$\simeq \frac{16}{\pi^2}\frac{\Gamma}{H}F_1\left(\frac{m_{\widetilde{H}}}{T}\right)\left[\frac{\mu_{H_\mathrm{d}}+\mu_{L_2}+\mu_{\bar{E}_3}}{T}\right], \tag{4.86}$$

where we have used $\mu \ll T$. On the other hand, reactions mediated by the diagonal lepton 湯川 couplings are in thermal equilibria for $T \lesssim 10^5\mathrm{GeV}$, which leads to

$$\mu_{L_i}+\mu_{\bar{E}_i}+\mu_{H_\mathrm{d}} = 0\,, \tag{4.87}$$

and hence

$$T\frac{\mathrm{d}}{\mathrm{d}T}\left(N_2-N_3\right) \simeq \frac{16}{\pi^2}\frac{\Gamma}{H}F_1\left(\frac{m_{\widetilde{H}}}{T}\right)\left[\frac{(2\mu_{L_2}-\mu_{\bar{E}_2})-(2\mu_{L_3}-\mu_{\bar{E}_3})}{3T}\right]. \tag{4.88}$$

Therefore, from Eq. (4.83) we obtain

$$T\frac{\mathrm{d}}{\mathrm{d}T}\left(N_2-N_3\right) = \frac{16\Gamma}{3H}\frac{F_1\left(m_{\widetilde{H}}/T\right)}{g_\mathrm{eff}\left(m_{\tilde{l}}/T\right)}\left(N_2-N_3\right). \tag{4.89}$$

4.ii.3 *R*-Parity Violation

The evolution of the $B - L$ asymmetry under the *R*-parity violating interactions can be discussed in the way similar to Sec. 4.ii.2. In this section, we will derive the time evolution of the $B - L$ asymmetry under *R*-parity violating interactions.

◆$B-L$ asymmetry and chemical potentials

We define the $B-L$ asymmetry as

$$N_{B-L} := \frac{n_{\text{baryon}} - n_{\text{lepton}}}{T^3}. \tag{4.90}$$

Using the approximation that $\mu \ll T$, we can calculate this as

$$\begin{aligned} N_{B-L} &= \frac{1}{3}\left(N_{[Q]} - N_{[\bar{U}]} - N_{[\bar{D}]}\right) - \sum_i \left(N_{[L_i]} - N_{[\bar{E}_i]}\right) \\ &\approx \frac{g_{\text{eff}}\left(m_{\tilde{q}}/T\right)}{\pi^2} \frac{6\mu_Q - 3\mu_{\bar{U}} - 3\mu_{\bar{D}}}{T} - \frac{g_{\text{eff}}\left(m_{\tilde{l}}/T\right)}{\pi^2} \frac{\sum\left(2\mu_{L_i} - \mu_{\bar{E}_i}\right)}{T} \\ &= \frac{g_{\text{eff}}\left(m_{\tilde{q}}/T\right)}{\pi^2} \frac{-4\overline{\mu_L}}{T} - \frac{g_{\text{eff}}\left(m_{\tilde{l}}/T\right)}{\pi^2} \frac{9\overline{\mu_L} + 3\mu_{H_\text{d}}}{T} \\ &= -\frac{1}{\pi^2} C_{B-L}(T) \frac{\overline{\mu_L}}{T}, \end{aligned} \tag{4.91}$$

where

$$C_{B-L}(T) := 4 g_{\text{eff}}\left(\frac{m_{\tilde{q}}}{T}\right) + \left(9 - 3C_{H_\text{d}}(T)\right) g_{\text{eff}}\left(\frac{m_{\tilde{l}}}{T}\right). \tag{4.92}$$

For the definitions of $N_{[X]}$, $\overline{\mu_L}$, and $C_{H_\text{d}}(T)$, please go back to P. 39, Eq. (3.38) and Eq. (3.40), respectively.

◆$\bar{U}\bar{D}\bar{D}$ interaction

If in the superpotential we have $\bar{U}_i\bar{D}_j\bar{D}_k$ term, we have the following 18 (= 3 × color) decay processes,

$$\widetilde{u}_i^* \to d_j d_k, \qquad \widetilde{d}_j^* \to u_i d_k, \qquad \widetilde{d}_k^* \to u_i d_j, \tag{4.93}$$

and their antiparticles' processes. They are all governed by the same decay rate

$$\Gamma_{\bar{U}_i\bar{D}_j\bar{D}_k} = \frac{1}{16\pi} |\lambda''_{ijk}|^2 m_{\tilde{q}}, \tag{4.94}$$

for now quarks are still massless, and we assume that the mass of squarks are the same.

Therefore, in the presence of the superpotential

$$W = \frac{1}{2}\lambda''_{ijk}\bar{U}_i\bar{D}_j\bar{D}_k, \tag{4.95}$$

the time evolution of N_{B-L} is given by

$$T\frac{\mathrm{d}}{\mathrm{d}T}\left(N_{B-L}\right) \simeq -\frac{1}{\pi^2}\cdot\frac{9\sum_{ijk}\Gamma_{\bar{U}_i\bar{D}_j\bar{D}_k}}{H}F_1\left(\frac{m_{\widetilde{q}}}{T}\right)\frac{\mu_{\bar{U}}+2\mu_{\bar{D}}}{T} \tag{4.96}$$

$$= \frac{9}{H}\sum_{ijk}\Gamma_{\bar{U}_i\bar{D}_j\bar{D}_k}F_1\left(\frac{m_{\widetilde{q}}}{T}\right)\frac{1+C_{H_\mathrm{d}}(T)}{C_{B-L}(T)}\cdot N_{B-L}. \tag{4.97}$$

We should emphasize that Eq. (4.97) holds even in the absence of the lepton flavor violation.

◆$LL\bar{E}$ interaction

Here we assume that the lepton flavor asymmetries vanish because of the lepton flavor violation, that is, we use $\mu_{L_1}=\mu_{L_2}=\mu_{L_3}$ in addition to Eq. (4.91). Under this assumption, the time evolution of N_{B-L} under the superpotential

$$W = \frac{1}{2}\lambda_{ijk}L_iL_j\bar{E}_k \tag{4.98}$$

is described as

$$T\frac{\mathrm{d}}{\mathrm{d}T}N_{B-L} \simeq -\frac{1}{\pi^2}\frac{3\sum_{ijk}\Gamma_{L_iL_j\bar{E}_k}}{H}F_1\left(\frac{m_{\widetilde{l}}}{T}\right)\frac{2\mu_L+\mu_{\bar{E}}}{T} \tag{4.99}$$

$$= \frac{3}{H}\sum_{ijk}\Gamma_{L_iL_j\bar{E}_k}F_1\left(\frac{m_{\widetilde{l}}}{T}\right)\frac{1+C_{H_\mathrm{d}}(T)}{C_{B-L}(T)}\cdot N_{B-L}\,, \tag{4.100}$$

where

$$\Gamma_{L_iL_j\bar{E}_k} = \frac{1}{16\pi}|\lambda_{ijk}|^2 m_{\widetilde{\ell}}. \tag{4.101}$$

◆$LQ\bar{D}$ interaction

Here also we assume the vanishment of lepton flavor asymmetries. The time evolution of N_{B-L} under the superpotential

$$W = \lambda'_{ijk}L_iQ_j\bar{D}_k \tag{4.102}$$

is

$$T\frac{\mathrm{d}}{\mathrm{d}T}N_{B-L} \simeq -\frac{1}{\pi^2}\left[\frac{12\sum_{ijk}\Gamma_{\widetilde{q}:L_iQ_j\bar{D}_k}}{H}F_1\left(\frac{m_{\widetilde{q}}}{T}\right)+\frac{6\sum_{ijk}\Gamma_{\widetilde{\ell}:L_iQ_j\bar{D}_k}}{H}F_1\left(\frac{m_{\widetilde{l}}}{T}\right)\right]\frac{\mu_L+\mu_Q+\mu_{\bar{D}}}{T}$$

$$= \frac{1}{H}\sum_{ijk}\left[1\Gamma_{\widetilde{q}:L_iQ_j\bar{D}_k}F_1\left(\frac{m_{\widetilde{q}}}{T}\right)+6\Gamma_{\widetilde{\ell}:L_iQ_j\bar{D}_k}F_1\left(\frac{m_{\widetilde{l}}}{T}\right)\right]\frac{1+C_{H_\mathrm{d}}(T)}{C_{B-L}(T)}N_{B-L}\,, \tag{4.103}$$

where

$$\Gamma_{\widetilde{q}:L_iQ_j\bar{D}_k} = \frac{1}{16\pi}|\lambda'_{ijk}|^2 m_{\widetilde{q}}, \qquad \Gamma_{\widetilde{\ell}:L_iQ_j\bar{D}_k} = \frac{1}{16\pi}|\lambda'_{ijk}|^2 m_{\widetilde{\ell}}. \tag{4.104}$$

◆Bilinear R-parity violation

As we will discuss in App. B.i, or as we have done in Sec. 2.2, the bilinear R-parity violating term $\kappa_i L_i H_\mathrm{u}$ induces, through the L_i–H_d mixings, effective trilinear couplings λ_{ijk} and λ'_{ijk}. Then, the time evolution of $B - L$ can be discussed by using the Boltzmann equations of the $LL\bar{E}$ and the $LQ\bar{D}$ cases, which we have just discussed.

Chapter 5

Conclusion

Now let us conclude this thesis.

* * *

We have seen that

- the R-parity is not necessary for the MSSM,
- the R-parity violating parameters are constrained,
- the constraints is mainly obtained from collider experiments,
- cosmology would bring us to much severe constraints if the lepton flavor is violated enough,

in this thesis. Here, we will discuss the application of the last severe constraints, which we have obtained from cosmology, to the detection of the "SUSY without R-parity" in colliders (LHC, etc.), and present the outlook for the future.

* * *

In the presence of slepton mixings, all the R-parity violating couplings must satisfy Eqs. (4.44)–(4.47) in order to avoid the baryon erasure. Interestingly, this means that the LSP has a long decay length at the LHC. For instance, suppose that the LSP is the stau $\widetilde{\tau}$, mainly consisting of the right-handed one $\widetilde{\tau}_{\mathrm{R}}$. If the $LL\bar{E}$ couplings λ_{ij3} meet the cosmological bounds (4.46), the decay length of $\widetilde{\tau}$ becomes

$$c\tau_{\widetilde{\tau}} \sim 50\mu\mathrm{m}\left(\frac{\lambda_{ij3}}{10^{-6}}\right)^{-2}\left(\frac{m_{\widetilde{\tau}}}{100\mathrm{GeV}}\right)^{-1}. \tag{5.1}$$

This is comparable to the tau-lepton decay length ($c\tau_\tau \simeq 87\mu\mathrm{m}$), which can be probed at the LHC.

To our pleasure, this is the shortest possible decay length, and in general we can expect a much longer one. For example, if the dominant decay of $\widetilde{\tau}$ is caused by λ_{ijk} ($k \neq 3$) or the $LQ\bar{D}$ coupling λ'_{ijk}, the decay length becomes longer since the decay rate is suppressed by the left-right mixing of $\widetilde{\tau}$ and/or the flavor mixing.

Also, for other LSP cases, we can obtain similar results. The dominant decay mode of the LSP is dependent to a great extent on what the LSP is and the pattern of the R-parity breaking. If it is three- or four-body decay [11], the decay length becomes even much longer.

These features may be a great help for us to detect R-parity violating SUSY models, and therefore now, in the LHC era, it is important to study the LHC phenomenology of R-parity violating SUSY models under the cosmological bounds Eqs. (4.44)–(4.47). We should examine various LSP candidates and various patterns of the R-parity violating couplings, and we leave it for future works.

Appendix A

Standard Model

Section A.1 Notations in this Appendix

In this part of the thesis, we use following conventions.

All fermionic fields are expressed as Dirac spinors. Dirac's gamma matrices, which shall satisfy

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}; \quad \{\gamma^\mu, \gamma_5\} = 0, \quad (\gamma_5)^2 = 1 \tag{A.1}$$

are defined as "chiral notation" like Peskin [17], that is,

$$\gamma^\mu := \begin{pmatrix} 0 & \sigma^\mu \\ \bar\sigma^\mu & 0 \end{pmatrix}; \quad \gamma_5 = -\frac{\mathrm{i}}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma = \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{A.2}$$

where σ^μ and $\bar\sigma^\mu$ are extended Pauli matrices and $\epsilon_{\mu\nu\rho\sigma}$ is the totally antisymmetric Lorentz tensor:

$$\sigma^\mu = (1, \sigma^i), \quad \bar\sigma^\mu = (1, -\sigma^i); \quad \epsilon^{0123} = -\epsilon_{0123} = 1. \tag{A.3}$$

As we use Dirac spinors, not Weyl spinors, projection operators appear explicitly in the Lagrangian. They are defined as

$$P_\mathrm{L} := \frac{1-\gamma_5}{2}, \quad P_\mathrm{R} := \frac{1+\gamma_5}{2}. \tag{A.4}$$

For the gauge group of the Standard Model, we use following notations as their representation.

τ^α denotes Gell-Mann matrices, and T^a is equal to $\sigma^a/2$, where σ^a is Pauli matrices. That is,

$$\mathrm{SU}(3)_\mathrm{strong}: \quad \left[\tau^a, \tau^b\right] = \mathrm{i}f^{abc}\tau^c, \quad \mathrm{Tr}\left(\tau^a\tau^b\right) = \frac{1}{2}\delta^{ab},$$
$$\mathrm{SU}(2)_\mathrm{weak}: \quad \left[T^a, T^b\right] = \mathrm{i}\epsilon^{abc}T^c, \quad \mathrm{Tr}\left(T^a T^b\right) = \frac{1}{2}\delta^{ab},$$

Section A.2 Standard Model

The Standard Model is one of the greatest achievement of science in the last century. It describes almost all physics below O(100GeV), the electroweak scale.

In this section, we introduce the Lagrangian of the Standard Model, discuss its spontaneous symmetry breaking from $\mathrm{SU}(2)_{\mathrm{weak}} \times \mathrm{U}(1)_Y$ to $\mathrm{U}(1)_{\mathrm{EM}}$ (the Higgs mechanism), and write down the Lagrangian after the symmetry breaking.

A.2.1 First Lagrangian

The Standard Model is characterized by its gauge group and field content. The gauge group is $\mathrm{SU}(3)_{\mathrm{strong}} \times \mathrm{SU}(2)_{\mathrm{weak}} \times \mathrm{U}(1)_Y$, and the field content is as Table A.1. From these two features, we can specify the model and write down the (renormalizable) Lagrangian of the Standard Model as follows:

$$\mathcal{L} = \mathcal{L}_{\mathrm{gauge}} + \mathcal{L}_{\mathrm{Higgs}} + \mathcal{L}_{\mathrm{matter}} + \mathcal{L}_{湯川}, \tag{A.5}$$

$$\text{where} \quad \mathcal{L}_{\mathrm{gauge}} = -\frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{4}W^{a\mu\nu}W^a_{\mu\nu} - \frac{1}{4}G^{a\mu\nu}G^a_{\mu\nu} \tag{A.6}$$

$$\mathcal{L}_{\mathrm{Higgs}} = \left|\left(\partial_\mu - \mathrm{i}g_2 W_\mu - \frac{1}{2}\mathrm{i}g_1 B_\mu\right)H\right|^2 - V(H), \tag{A.7}$$

$$\begin{aligned}
\mathcal{L}_{\mathrm{matter}} = {} & \overline{Q}_i \mathrm{i}\gamma^\mu \left(\partial_\mu - \mathrm{i}g_3 G_\mu - \mathrm{i}g_2 W_\mu - \frac{1}{6}\mathrm{i}g_1 B_\mu\right) P_{\mathrm{L}} Q_i \\
& + \overline{U}_i \mathrm{i}\gamma^\mu \left(\partial_\mu - \mathrm{i}g_3 G_\mu - \frac{2}{3}\mathrm{i}g_1 B_\mu\right) P_{\mathrm{R}} U_i \\
& + \overline{D}_i \mathrm{i}\gamma^\mu \left(\partial_\mu - \mathrm{i}g_3 G_\mu + \frac{1}{3}\mathrm{i}g_1 B_\mu\right) P_{\mathrm{R}} D_i \\
& + \overline{L}_i \mathrm{i}\gamma^\mu \left(\partial_\mu - \mathrm{i}g_2 W_\mu + \frac{1}{2}\mathrm{i}g_1 B_\mu\right) P_{\mathrm{L}} L_i \\
& + \overline{E}_i \mathrm{i}\gamma^\mu \left(\partial_\mu + \mathrm{i}g_1 B_\mu\right) P_{\mathrm{R}} E_i,
\end{aligned} \tag{A.8}$$

$$\mathcal{L}_{湯川} = -\overline{U}_i (y_u)_{ij} H P_{\mathrm{L}} Q_j + \overline{D}_i (y_d)_{ij} H^\dagger P_{\mathrm{L}} Q_j + \overline{E}_i (y_e)_{ij} H^\dagger P_{\mathrm{L}} L_j + \mathrm{H.\,c.} \tag{A.9}$$

Here, $V(H)$ is the Higgs potential, which we will discuss later. Also note that the field strengths are defined as, e.g. for SU(2) gauge fields,

$$W^a_{\mu\nu} := \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc} W^b_\mu W^c_\nu, \tag{A.10}$$

or with a notation $W := W^a T^a$,

$$W_{\mu\nu} := \partial_\mu W_\nu - \partial_\nu W_\mu - \mathrm{i}g\left[W_\mu, W_\nu\right]. \tag{A.11}$$

Table A.1 The field content of the Standard Model: here we omit the gauge indices, and subscripts i denote "generation indices", which run 1–3.

	$SU(3)_{strong}$	$SU(2)_{weak}$	$U(1)_Y$
Matter Fields (Fermionic / Lorentz Spinor)			
Q_i : Left-handed quarks	**3**	**2**	1/6
U_i : Right-handed up-type quarks	**3**	**1**	2/3
D_i : Right-handed down-type quarks	**3**	**1**	−1/3
L_i : Left-handed leptons	**1**	**2**	−1/2
E_i : Right-handed leptons	**1**	**1**	−1
Higgs Field (Bosonic / Lorentz Scalar)			
H : Higgs	**1**	**2**	1/2
Gauge Fields (Bosonic / Lorentz Vector)			
G : Gluons	**8**	**1**	0
W : Weak bosons	**1**	**3**	0
B : B boson	**1**	**1**	0

A.2.2 Higgs Mechanism

In the above discussion we did not write down the explicit form of the Higgs potential. Now let us discuss the Higgs sector.

The (renormalizable) Higgs potential must be, in order not to violate the gauge symmetry, as follows:

$$V(H) = -\mu^2(H^\dagger H) + \lambda \left(H^\dagger H\right)^2 . \tag{A.12}$$

Here μ^2 and λ are arbitrary real parameters, and $\lambda > 0$ in order not to run away the vacuum expectation values (VEVs) of the Higgs. Note that this μ is the only parameter which has non-zero mass dimension in the Standard Model.

If μ^2 were negative, the potential has a minimum at $|H| = 0$, and everything would be as it was. However we set $\mu^2 > 0$ here. Then the potential has minima at $|H|^2 = \mu^2/2\lambda$, which means the Higgs fields have non-zero VEVs, and the symmetry $SU(2)_{weak} \times U(1)_Y$ is broken.

To discuss this clearly, let us *redefine* the Higgs field so that the VEV is

$$\langle H\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}, \qquad \text{where} \quad v = \sqrt{\frac{\mu^2}{\lambda}}, \tag{A.13}$$

and parameterize fluctuations around the VEV as

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1 + \mathrm{i}\phi_2\\ v + (h + \mathrm{i}\phi_3)\end{pmatrix}. \tag{A.14}$$

Here h and ϕ_i are real scalar fields. h is known as "Higgs boson," and ϕ_i are 南部–Goldstone bosons according to the symmetry breaking, which the weak bosons "eat" to be massive.

The Higgs potential becomes

$$V(h) = \frac{\mu^2}{4v^2}h^4 + \frac{\mu^2}{v}h^3 + \mu^2h^2, \tag{A.15}$$

and now we know the Higgs boson has acquired mass $m_h = \sqrt{2}\mu$.

Accordingly, the kinetic term of the Higgs fields in $\mathcal{L}_{\text{Higgs}}$ turns into

$$\left|\left(\partial_\mu - \mathrm{i}g_2W_\mu - \frac{1}{2}\mathrm{i}g_1B_\mu\right)H\right|^2 = \frac{1}{2}(\partial_\mu h)^2 + \frac{(v+h)^2}{8}\left[{g_2}^2{W_1}^2 + {g_2}^2{W_2}^2 + (g_1B - g_2W_3)^2\right]. \tag{A.16}$$

Thus, we redefine the gauge fields with taking care of the norm of fields as follows:

$$W^\pm_\mu := \frac{1}{\sqrt{2}}(W^1_\mu \mp \mathrm{i}W^2_\mu), \qquad \begin{pmatrix}Z_\mu\\ A_\mu\end{pmatrix} := \begin{pmatrix}\cos\theta_{\mathrm{w}} & -\sin\theta_{\mathrm{w}}\\ \sin\theta_{\mathrm{w}} & \cos\theta_{\mathrm{w}}\end{pmatrix}\begin{pmatrix}W^3_\mu\\ B_\mu\end{pmatrix}, \tag{A.17}$$

where θ_{w} is the Weinberg angle, and e is the elementary electric charge, defined as

$$\tan\theta_{\mathrm{w}} := \frac{g_1}{g_2}, \quad e := -\frac{g_1g_2}{\sqrt{{g_1}^2 + {g_2}^2}}; \qquad g_1 = \frac{|e|}{\cos\theta_{\mathrm{w}}}, \quad g_2 = \frac{|e|}{\sin\theta_{\mathrm{w}}}. \tag{A.18}$$

We obtain the following terms in $\mathcal{L}_{\text{Higgs}}$:

$$\mathcal{L}_{\text{Higgs}} \supset \frac{1}{2}(\partial_\mu h)^2 + \frac{(v+h)^2}{8}\left[2{g_2}^2W^{+\mu}W^-_\mu + ({g_1}^2 + {g_2}^2)Z^2\right]. \tag{A.19}$$

Here we have omitted the 南部–Goldstone bosons.

> Here we present another form:
>
> $$g_1B_\mu = |e|A_\mu - \tan\theta_{\mathrm{w}}Z_\mu, \tag{A.20}$$
>
> $$g_2W_\mu = \frac{g_2}{\sqrt{2}}\left(W^+_\mu T^+ + W^-_\mu T^-\right) + \left(\frac{|e|}{\tan\theta_{\mathrm{w}}}Z_\mu + |e|A_\mu\right)T^3, \tag{A.21}$$
>
> where $W_\mu := W^a_\mu T^a$ as is already defined, and $T^\pm = T^1 \pm \mathrm{i}T^2$.

Note that the gauge bosons acquired the following masses:

$$m_A = 0, \qquad m_W = \frac{g_2}{2}v, \qquad m_Z = \frac{\sqrt{{g_1}^2 + {g_2}^2}}{2}v. \tag{A.22}$$

A.2.3 Mass of Fermions

Now let us move on to the 湯川 terms. The 湯川 interaction is, we writing down SU(2)- and generation-indices,

$$\begin{aligned}\mathcal{L}_{湯川} &= -\epsilon^{\alpha\beta}\overline{U}_i(y_u)_{ij}H^\alpha P_\mathrm{L}Q_j^\beta + \overline{D}_i(y_d)_{ij}\left(H^\dagger\right)^\alpha P_\mathrm{L}Q_j^\alpha + \overline{E}_i(y_e)_{ij}\left(H^\dagger\right)^\alpha P_\mathrm{L}L_j^\alpha + \text{H. c.}\\ &\supset \frac{1}{\sqrt{2}}(v+h)\left[(y_u)_{ij}\overline{U}_iP_\mathrm{L}Q_j^1 + (y_d)_{ij}\overline{D}_iP_\mathrm{L}Q_j^2 + (y_e)_{ij}\overline{E}_iP_\mathrm{L}L_j^2 + \text{H. c.}\right].\end{aligned} \tag{A.23}$$

We can see that these terms give masses to the fermions, and invoke fermion–fermion–Higgs interactions. However, the 湯川 matrices are not diagonal. Here we will diagonalize the matrices to obtain mass eigenstates.

We use the singular value decomposition method to mass matrices $Y_\bullet := vy_\bullet/\sqrt{2}$. Generally, any matrices can be transformed with two unitary matrices Ψ and Φ as

$$Y = \Phi^\dagger\begin{pmatrix}m_1 & 0 & 0\\ 0 & m_2 & 0\\ 0 & 0 & m_3\end{pmatrix}\Psi =: \Phi^\dagger M\Psi \qquad (m_i \ge 0). \tag{A.24}$$

Using this Ψ and Φ, we can rotate the basis as

$$Q^1 \mapsto \Psi_u^\dagger Q^1, \quad Q^2 \mapsto \Psi_d^\dagger Q^2, \quad L \mapsto \Psi_e^\dagger L, \qquad U \mapsto \Phi_u^\dagger U, \quad D \mapsto \Phi_d^\dagger D, \quad E \mapsto \Phi_e^\dagger E \tag{A.25}$$

to obtain mass eigenstates[*1]

Then the 湯川 terms are

$$\mathcal{L}_{湯川} = \left(1 + \frac{1}{v}h\right)\left[(m_u)_i\overline{U}_iP_\mathrm{L}Q_i^1 + (m_d)_i\overline{D}_iP_\mathrm{L}Q_i^2 + (m_e)_i\overline{E}_iP_\mathrm{L}L_i^2 + \text{H. c.}\right]. \tag{A.26}$$

in mass eigenstates.

In the transformation from the gauge eigenstates to the mass eigenstates, almost all the terms in the Lagrangian are not modified. However, only the terms of quark–quark–W

[*1] FYI: The fields in the left hand sides of (A.25) are in gauge eigenstates, as well as those in all the equations before (A.25). Meanwhile, in the right hand sides are in mass eigenstates.

interactions do change drastically, as

$$\mathcal{L} \supset \overline{Q} \mathrm{i}\gamma^\mu \left(-\mathrm{i}g_2 W_\mu - \frac{1}{6}\mathrm{i}g_1 B_\mu\right) P_\mathrm{L} Q \tag{A.27}$$

$$= \overline{Q} \frac{g_2}{\sqrt{2}} \left(\not{W}^+ T^+ + \not{W}^- T^-\right) P_\mathrm{L} Q \quad + \quad \text{(interaction terms with } Z \text{ and } A) \tag{A.28}$$

$$\mapsto \frac{g_2}{\sqrt{2}} \begin{pmatrix} \overline{Q}^1 \Psi_u & \overline{Q}^2 \Psi_d \end{pmatrix} \begin{pmatrix} 0 & \not{W}^+ \\ \not{W}^- & 0 \end{pmatrix} P_\mathrm{L} \begin{pmatrix} \Psi_u^\dagger Q^1 \\ \Psi_d^\dagger Q^2 \end{pmatrix} + (\quad \cdots \quad) \tag{A.29}$$

$$= \frac{g_2}{\sqrt{2}} \left[\overline{Q}^2 \not{W}^- X P_\mathrm{L} Q^1 + \overline{Q}^1 \not{W}^+ X^\dagger P_\mathrm{L} Q^2\right] + (\quad \cdots \quad), \tag{A.30}$$

where $X := \Psi_d \Psi_u^\dagger$ is a matrix, so-called the Cabibbo–小林–益川 (CKM) matrix, which is *not* diagonal, and *not* real, generally. These terms violate the flavor symmetry of quarks, and even the *CP*-symmetry.

In our notation, *CP*-transformation of a spinor is described as

$$\mathscr{C}\mathscr{P}(\psi) = -\mathrm{i}\eta^*(\overline{\psi}\gamma^2)^\mathrm{T}, \quad \mathscr{C}\mathscr{P}\left(\overline{\psi}\right) = \mathrm{i}\eta(\gamma^2\psi)^\mathrm{T}, \tag{A.31}$$

where η is a complex phase ($|\eta| = 1$). Under this transformation, those terms are transformed as, e.g.,

$$\begin{aligned} \mathscr{C}\mathscr{P}\left(\overline{Q}^2 \not{W}^- X P_\mathrm{L} Q^1\right) &= (\gamma^2 Q^2)^\mathrm{T} \mathscr{P}(-\not{W}^+) X P_\mathrm{L} (\overline{Q}^1 \gamma^2)^\mathrm{T} \\ &= -W_\mu^{+P} (\gamma^2 Q^2)^\mathrm{T} (\overline{Q}^1 X^\mathrm{T} \gamma^2 P_\mathrm{L} \gamma^{\mu\mathrm{T}})^\mathrm{T} \\ &= (\overline{Q}^1 \not{W}^+ X^\mathrm{T} P_\mathrm{L} Q^2). \end{aligned} \tag{A.32}$$

Therefore, we can see that the *CP*-symmetry is maintained if and only if $X^\mathrm{T} = X^\dagger$, that is, if and only if X is a real matrix.

A.2.4 Full Lagrangian after the Symmetry Breaking

After all, we obtain the following Lagrangian.

$$\mathcal{L} = \mathcal{L}_{\text{g.-int.}} + \mathcal{L}_{\text{g.-mass}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{matter(1)}} + \mathcal{L}_{\text{matter(2)}} + \mathcal{L}_{\text{湯川}}; \tag{A.33}$$

$$\begin{aligned}
\mathcal{L}_{\text{g.-int.}} &:= -\frac{1}{4}\left[G^{a\mu\nu}G^{a}_{\mu\nu} + (\partial Z)^{\mu\nu}(\partial Z)_{\mu\nu} + (\partial A)^{\mu\nu}(\partial A)_{\mu\nu} + 2(\partial W^{+})^{\mu\nu}(\partial W^{-})_{\mu\nu}\right] \\
&\quad + \frac{\mathrm{i}|e|}{\tan\theta_{\mathrm{w}}}\Big[(\partial W^{+})^{\mu\nu}W^{-}_{\mu}Z_{\nu} + (\partial W^{-})^{\mu\nu}Z_{\mu}W^{+}_{\nu} + (\partial Z)^{\mu\nu}W^{+}_{\mu}W^{-}_{\nu}\Big] \\
&\quad + \mathrm{i}|e|\Big[(\partial W^{+})^{\mu\nu}W^{-}_{\mu}A_{\nu} + (\partial W^{-})^{\mu\nu}A_{\mu}W^{+}_{\nu} + (\partial A)^{\mu\nu}W^{+}_{\mu}W^{-}_{\nu}\Big] \\
&\quad + (\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma})\left[\frac{|e|^2}{2\sin^2\theta_{\mathrm{w}}}W^{+}_{\mu}W^{+}_{\nu}W^{-}_{\rho}W^{-}_{\sigma} + \frac{|e|^2}{\tan^2\theta_{\mathrm{w}}}W^{+}_{\mu}Z_{\nu}W^{-}_{\rho}Z_{\sigma}\right. \\
&\qquad\left. + \frac{|e|^2}{\tan\theta_{\mathrm{w}}}\left(W^{+}_{\mu}Z_{\nu}W^{-}_{\rho}A_{\sigma} + W^{+}_{\mu}A_{\nu}W^{-}_{\rho}Z_{\sigma}\right) + |e|^2 W^{+}_{\mu}A_{\nu}W^{-}_{\rho}A_{\sigma}\right],
\end{aligned}$$

$$\mathcal{L}_{\text{g.-mass}} := {m_W}^2 W^{+\mu}W^{-}_{\mu} + \frac{{m_Z}^2}{2}Z^{\mu}Z_{\mu},$$

$$\begin{aligned}
\mathcal{L}_{\text{Higgs}} &:= \frac{1}{2}(\partial_\mu h)^2 - \frac{{m_h}^2}{8v^2}h^4 - \frac{{m_h}^2}{2v}h^3 - \frac{1}{2}{m_h}^2h^2 \\
&\quad + \frac{2{m_W}^2}{v}W^{+}W^{-}h + \frac{{m_Z}^2}{v}Z^2h + \frac{{m_W}^2}{v^2}W^{+}W^{-}h^2 + \frac{{m_Z}^2}{2v^2}Z^2h^2 \\
&\quad + \left(\frac{(m_u)_i}{v}\overline{U}_iP_{\mathrm{L}}Q^1_ih + \frac{(m_d)_i}{v}\overline{D}_iP_{\mathrm{L}}Q^2_ih + \frac{(m_e)_i}{v}\overline{E}_iP_{\mathrm{L}}L^2_ih + \text{H. c.}\right),
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{\text{matter(1)}} &:= \overline{Q}\,(\mathrm{i}\not{\partial} + g_3\not{G})\,P_{\mathrm{L}}Q + \overline{U}\,(\mathrm{i}\not{\partial} + g_3\not{G})\,P_{\mathrm{R}}U + \overline{D}\,(\mathrm{i}\not{\partial} + g_3\not{G})\,P_{\mathrm{R}}D \\
&\quad + \overline{L}\,(\mathrm{i}\not{\partial})\,P_{\mathrm{L}}L + \overline{E}\,(\mathrm{i}\not{\partial})\,P_{\mathrm{R}}E,
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{\text{matter(2)}} &:= \frac{g_2}{\sqrt{2}}\left[\overline{Q}^2\not{W}^{-}XP_{\mathrm{L}}Q^1 + \overline{Q}^1\not{W}^{+}X^{\dagger}P_{\mathrm{L}}Q^2\right] + \overline{L}\frac{g_2}{\sqrt{2}}\left(\not{W}^{+}T^{+} + \not{W}^{-}T^{-}\right)P_{\mathrm{L}}L \\
&\quad + |e|\cdot\overline{Q}\left[\left(T^3 + \frac{1}{6}\right)\not{A} + \left(\frac{T^3}{\tan\theta_{\mathrm{w}}} - \frac{\tan\theta_{\mathrm{w}}}{6}\right)\not{Z}^0\right]P_{\mathrm{L}}Q \\
&\quad + \frac{2}{3}|e|\cdot\overline{U}\,(\not{A} - \not{Z}\tan\theta_{\mathrm{w}})\,P_{\mathrm{R}}U \\
&\quad - \frac{1}{3}|e|\cdot\overline{D}\,(\not{A} - \not{Z}\tan\theta_{\mathrm{w}})\,P_{\mathrm{R}}D \\
&\quad + |e|\cdot\overline{L}\left[\left(T^3 - \frac{1}{2}\right)\not{A} + \left(\frac{T^3}{\tan\theta_{\mathrm{w}}} + \frac{\tan\theta_{\mathrm{w}}}{2}\right)\not{Z}^0\right]P_{\mathrm{L}}L \\
&\quad - |e|\cdot\overline{E}\,(\not{A} - \not{Z}\tan\theta_{\mathrm{w}})\,P_{\mathrm{R}}E,
\end{aligned}$$

$$\mathcal{L}_{\text{湯川}} := (m_u)_i\overline{U}_iP_{\mathrm{L}}Q^1_i + (m_d)_i\overline{D}_iP_{\mathrm{L}}Q^2_i + (m_e)_i\overline{E}_iP_{\mathrm{L}}L^2_i + \text{H. c.}$$

We have used an abridged notation

$$(\partial X)_{\mu\nu} := \partial_\mu X_\nu - \partial_\nu X_\mu. \tag{A.34}$$

A.2.5 Values of SM Parameters

Here we present the experimental values of the Standard Model, taken from the "Review of Particle Physics" [7].

■Parameters in low energy

$$\alpha_{\rm EM} = 1/137.035999679(94) \qquad G_{\rm F} = 1.166367(5)\times 10^{-5}{\rm GeV}^{-2}$$

■In the electroweak scale
These values are all in $\overline{\rm MS}$ scheme.

$$\alpha_{\rm EM}^{-1}(m_Z) = 127.925(16) \qquad m_W(m_W) = 80.398(25){\rm GeV}$$
$$\alpha_{\rm EM}^{-1}(m_\tau) = 133.452(16) \qquad m_Z(m_Z) = 91.1876(21){\rm GeV}$$
$$\alpha_{\rm s}(m_Z) = 0.1176(20) \qquad \sin^2\theta_{\rm W}(m_Z) = 0.23119(14)$$

■Mass of fundamental particles
These are PDG value. Here we ignore the renormalization effects.

e : 0.510998910(13)MeV	u : 1.5 to 3.3MeV	d : 3.5 to 6.0MeV
μ : 105.658367(4)MeV	c : $1.27^{+0.07}_{-0.11}$GeV	s : 104^{+26}_{-34}MeV
τ : 1.77784(17)GeV	t : $171.2_{\pm 2.1}$GeV	b : $4.20^{+0.17}_{-0.07}$GeV

■Estimation of Standard Model parameters
For the electroweak scale, we can roughly estimate the values as

$$e \sim 0.313, \qquad g_1 \sim 0.358, \qquad g_2 \sim 0.651; \qquad v = \sqrt{\frac{\mu^2}{\lambda}} \sim 246{\rm GeV}$$

Therefore 湯川 matrices are (after diagonalization)

$$y_u \approx \begin{pmatrix} 10^{-5} & 0 & 0 \\ 0 & 0.007 & 0 \\ 0 & 0 & 0.98 \end{pmatrix}, \qquad y_d \approx \begin{pmatrix} 3\times 10^{-5} & 0 & 0 \\ 0 & 0.0006 & 0 \\ 0 & 0 & 0.02 \end{pmatrix},$$
$$y_e \approx \begin{pmatrix} 3\times 10^{-6} & 0 & 0 \\ 0 & 0.0006 & 0 \\ 0 & 0 & 0.01 \end{pmatrix}.$$

Also, for $m_h \sim 120$GeV, we can estimate the parameters of the Higgs potential as $\mu \sim 85$GeV and $\lambda \sim 0.12$.

Appendix B
SUSY

Section B.1 MSSM

B.1.1 Gauge Group and Field Content

The minimal supersymmetric standard model (MSSM) [4, 5, 6] is the minimal supersymmetric extension of the Standard Model. Its gauge group is

$$\mathrm{SU}(3)_{\mathrm{color}} \times \mathrm{SU}(2)_{\mathrm{weak}} \times \mathrm{U}(1)_Y, \tag{B.1}$$

which is the same as the Standard Model,

The field content is as Table B.1. Note that we need two Higgs fields H_u and H_d to describe the 湯川 interactions, which is also good since we have no gauge anomaly with the two Higgs doublets.

This field content leads us to the following (general) superpotential of the MSSM as

$$\begin{aligned} W = \mu H_\mathrm{u} H_\mathrm{d} &+ (y_u)_{ij} H_\mathrm{u} Q_i \bar{U}_j + (y_d)_{ij} H_\mathrm{d} Q_i \bar{D}_j + (y_e)_{ij} H_\mathrm{d} L_i \bar{E}_j \\ &+ \kappa_i H_\mathrm{u} L_i + \frac{1}{2}\lambda_{ijk} L_i L_j \bar{E}_k + \lambda'_{ijk} L_i Q_j \bar{D}_k + \frac{1}{2}\lambda''_{ijk} \bar{U}_i \bar{D}_j \bar{D}_k. \end{aligned} \tag{B.2}$$

Note that the terms in the second line of Eq. (B.2) violate the baryon number B or the lepton number L, while those in the first line do not. These B- or L-violating terms cause the proton decay problem, and thus we usually impose the conservation of the R-parity [6]. These matters are discussed in Sec. 2.1.

We use the convention that $\lambda_{ijk} = -\lambda_{jik}$ and $\lambda''_{ijk} = -\lambda''_{ikj}$, for the superpotential (B.2) has the following asymmetry:

$$L_i L_j \bar{E}_k = -L_j L_i \bar{E}_k, \qquad \bar{U}_i \bar{D}_j \bar{D}_k = -\bar{U}_i \bar{D}_k \bar{D}_j. \tag{B.3}$$

B.1.2 SUSY Breaking Terms

From the field content and the superpotential, we can write down the Lagrangian of the MSSM[*1], which respects the supersymmetry. However, this Lagrangian is not what governs our current universe, because we know the universe is not supersymmetric. We have electron, whose mass is 0.511eV, but do not have such light bosons. (If the universe were supersymmetric, we had 0.511eV bosons.)

Thus we consider the supersymmetry is already broken so that the mass of superpartners becomes much heavier. In these twenty years, various models are proposed to achieve this feature, the SUSY-breaking (~~SUSY~~). To discuss such models is a very interesting theme, but in this thesis we do not focus on it. Instead, we give the general

Table B.1 The field content of the MSSM: here we omit the gauge indices, and subscripts i denote "generation indices", which run 1–3.

Matter and Higgs fields (chiral multiplet)					
	SU(3)	SU(2)	U(1)	spin 0	spin 1/2
Q_i	**3**	**2**	1/6	$(\widetilde{u}_{\rm L}, \widetilde{d}_{\rm L})$	$(u_{\rm L}, d_{\rm L})$
$\bar{U}_i$	$\bar{\mathbf{3}}$	**1**	−2/3	$\widetilde{u}^*_{\rm R}$	$u^\dagger_{\rm R}$
$\bar{D}_i$	$\bar{\mathbf{3}}$	**1**	1/3	$\widetilde{d}^*_{\rm R}$	$d^\dagger_{\rm R}$
L_i	**1**	**2**	−1/2	$(\widetilde{\nu}, \widetilde{e}_{\rm L})$	$(\nu, e_{\rm L})$
$\bar{E}_i$	**1**	**1**	1	$\widetilde{e}^*_{\rm R}$	$e^\dagger_{\rm R}$
$H_{\rm u}$	**1**	**2**	1/2	$(H^+_{\rm u}, H^0_{\rm u})$	$(\widetilde{H^+_{\rm u}}, \widetilde{H^0_{\rm u}})$
$H_{\rm d}$	**1**	**2**	−1/2	$(H^0_{\rm d}, H^-_{\rm d})$	$(\widetilde{H^0_{\rm d}}, \widetilde{H^-_{\rm d}})$
Gauge fields (vector multiplet)					
	SU(3)	SU(2)	U(1)	spin 1/2	spin 1
G	**8**	**1**	0	$\widetilde{g}$	g
W	**1**	**3**	0	$\widetilde{W}$	W
B	**1**	**1**	0	$\widetilde{B}$	B

[*1] We do not present the procedure here, for it is a bit long travel. If you want to follow the way, see Ref. [36, Secs.3–7], etc.

form of the $\not{\mathrm{SUSY}}$ effects, which appears in the Lagrangian.

The general form of $\not{\mathrm{SUSY}}$ terms is

$$
\begin{aligned}
\mathcal{L}_{\not{\mathrm{SUSY}}} = &-\frac{1}{2}\left(M_3\widetilde{g}\widetilde{g} + M_2\widetilde{W}\widetilde{W} + M_1\widetilde{B}\widetilde{B} + \text{H. c.}\right)\\
&-\left[(a_u)_{ij}H_\mathrm{u}\widetilde{Q}_i\widetilde{\bar{u}}_j - (a_d)_{ij}H_\mathrm{d}\widetilde{Q}_i\widetilde{\bar{d}}_j - (a_e)_{ij}H_\mathrm{d}\widetilde{L}_i\widetilde{\bar{e}}_j + \text{H. c.}\right]\\
&-\left[\left(m_Q^2\right)_{ij}\widetilde{Q}_i^*\widetilde{Q}_j + \left(m_L^2\right)_{ij}\widetilde{L}_i^*\widetilde{L}_j + \left(m_{\bar{U}}^2\right)_{ij}\widetilde{\bar{u}}_i^*\widetilde{\bar{u}}_j + \left(m_{\bar{D}}^2\right)_{ij}\widetilde{\bar{d}}_i^*\widetilde{\bar{d}}_j + \left(m_{\bar{E}}^2\right)_{ij}\widetilde{\bar{e}}_i^*\widetilde{\bar{e}}_j\right]\\
&-\left[m_{H_\mathrm{u}}^2H_\mathrm{u}^*H_\mathrm{u} + m_{H_\mathrm{d}}^2H_\mathrm{d}^*H_\mathrm{d} + (bH_\mathrm{u}H_\mathrm{d} + \text{H. c.})\right]\\
&-\left[\frac{1}{2}\xi_{ijk}\widetilde{L}_i\widetilde{L}_j\widetilde{\bar{e}}_k + \xi'_{ijk}\widetilde{L}_i\widetilde{Q}_j\widetilde{\bar{d}}_k + \frac{1}{2}\xi''_{ijk}\widetilde{\bar{u}}_i\widetilde{\bar{d}}_j\widetilde{\bar{d}}_k + \beta_iH_\mathrm{u}\widetilde{L}_i + \text{H. c.}\right].
\end{aligned}
\tag{B.4}
$$

Here, i, j are indices for the generations, which run 1–3, and we omit the gauge indices. Note that the terms of the last line, ξ's terms and β term, also violate the B- or L-number, and are usually omitted by imposing the R-parity conservation.

We have the following $\not{\mathrm{SUSY}}$ parameters:

- M_i : gaugino masses.
- $a_\bullet$: trilinear scalar couplings.
- $m_\bullet^2$: scalar masses, which must be Hermitian.
- b : Higgs off-diagonal mass, which is assumed to be real.
- ξ : B- or L-violating trilinear scalar couplings.
- β : L-violating bilinear scalar couplings.

Here we also use the convention

$$
\xi_{ijk} = -\xi_{jik}, \qquad\qquad \xi''_{ijk} = -\xi''_{ikj}. \tag{B.5}
$$

Section B.2 Proton Decay and R-Parity

Now we have introduced the MSSM, but with the B- and L-violating terms left. As we discussed in Sec. 2.1, these terms induce the proton decay problem, and usually omitted by imposing the R-parity.

However, as is also already mentioned, we overlooked the fact that we have to consider not only 4-dimensional operators but also higher-dimensional operators in order to realize the current bounds of the proton lifetime. In this section we will discuss this matter, and introduce a better symmetry, the proton hexality.

B.2.1 Higher Dimensional Operators and Proton Decay

We considered the 4-dimensional operators in the MSSM in Sec. 2.1, and saw that we can avoid proton decay with imposing the R-parity conservation. However, we should be more careful. We must consider 5-dimensional operators too.

Let us assume that we had a 5-dimensional term which invoke proton decay in the Lagrangian. The term is suppressed by a huge mass, e.g., the scale of the grand unification theories (GUTs) $M_{\text{GUT}} = 10^{16}$GeV as

$$\mathcal{L} \supset \frac{k}{M_{\text{GUT}}} XXXX, \tag{B.6}$$

where k is the coupling constant. Then the rate of proton decay can be roughly estimated as

$$\Gamma \lesssim \frac{|k|^2}{M_{\text{GUT}}^2} m_{\text{proton}}^3 = \frac{|k|^2}{2.5\text{yr}}, \tag{B.7}$$

therefore still we have to constrain the coupling constant as $|k| \lesssim 10^{-15}$. This constraint is still not usual, thus we have to eliminate, or at least pay attention to, the effect of 5-dimensional operators.[*2]

In the MSSM scheme, there may be the following terms which lay down 5-dimensional operators:

$$\begin{matrix} Q Q \bar{U} \bar{D}, & Q Q Q L, & Q Q Q H_{\text{d}}, & \bar{U} \bar{U} \bar{D} \bar{E}, & Q \bar{U} L \bar{E}, \\ Q \bar{U} H_{\text{d}} \bar{E}, & L H_{\text{u}} H_{\text{u}} H_{\text{d}}, & L L H_{\text{u}} H_{\text{u}}, & H_{\text{u}} H_{\text{u}} H_{\text{d}} H_{\text{d}}, & \end{matrix} \tag{B.8}$$

in the superpotential, and the following in the Kähler potential:

$$\begin{matrix} \bar{D}^\dagger \bar{U} \bar{E}, & \bar{D}^\dagger Q Q, & L^\dagger Q \bar{U}, & H_{\text{d}}^\dagger Q \bar{U}, \\ H_{\text{u}}^\dagger Q \bar{D}, & H_{\text{u}}^\dagger L \bar{E}, & H_{\text{u}}^\dagger H_{\text{d}} \bar{E}, & L^\dagger H_{\text{d}}. \end{matrix} \tag{B.9}$$

(And their Hermitian conjugates, surely.)

They can be classified as follows:

- Both P_{B}- and P_{L}-violating: $QQQL$ and $\bar{U}\bar{U}\bar{D}\bar{E}$,
- only P_{B}-violating: $QQQH_{\text{d}}$ and $\bar{D}^\dagger QQ$,
- only P_{L}-violating: $Q\bar{U}H_{\text{d}}\bar{E}$, $LH_{\text{u}}H_{\text{u}}H_{\text{d}}$, $\bar{D}^\dagger\bar{U}\bar{E}$, $L^\dagger Q\bar{U}$, $H_{\text{u}}^\dagger H_{\text{d}}\bar{E}$ and $L^\dagger H_{\text{d}}$,
- and neither P_{B}- nor P_{L}-violating ones.

[*2] To use the Planck scale M_{pl} instead of M_{GUT} is not a remedy. Also we can see from this discussion that 6-dimensional operators, which is suppressed by M_{GUT}^2, is not so critical.

Here, $P_{\rm B} := (-1)^{3B}$ is the baryon parity, and $P_{\rm L} := (-1)^{L}$ is the lepton parity.

In particular, the four $P_{\rm B}$-violating terms are critical, because the proton decay owes to them, as we mentioned in Sec. 2.1. Now we would like to see how the above $P_{\rm B}$-violating operators invoke proton decay.

* * *

In the following estimation, we use as the mass scales

$$M_{\rm GUT} = 10^{16}{\rm GeV}, \qquad m_{\rm SUSY} = 10^{3}{\rm GeV}. \tag{B.10}$$

◆$QQQL$ and $\bar{U}\bar{U}\bar{D}\bar{E}$ terms

The terms $Q_iQ_jQ_kL_l$ and $\bar{U}_i\bar{U}_j\bar{D}_k\bar{E}_l$, where i, j, k and l are the generation indices, violate not only B ($P_{\rm B}$) but also L ($P_{\rm L}$). Note that these terms *do respect* the R-parity, and thus we cannot omit these terms by imposing the R-parity conservation.

Here, we cannot choose the generation indices arbitrarily for the asymmetry of the gauge indices. For $\bar{U}_i\bar{U}_j\bar{D}_k\bar{E}_l$, we have to choose them so that $i \neq j$. For $Q_iQ_jQ_kL_l$, if we select the SU(2)$_{\rm weak}$ indices (a and b, where $a \neq b$) as $Q_i^aQ_j^bQ_k^aL_l^b$, we have to satisfy $i \neq k$.

Ones of the main channels of the proton decay processes are described in Fig. B.1. The decay rate can be roughly estimated as

$$\Gamma_{QQQL} \sim \left|\frac{\alpha \cdot k/M_{\rm GUT}}{m_{\rm SUSY}}\right|^2 \cdot m_{\rm proton}^5 = \frac{|k|^2}{5.4\times 10^{10}{\rm yr}}, \tag{B.11}$$

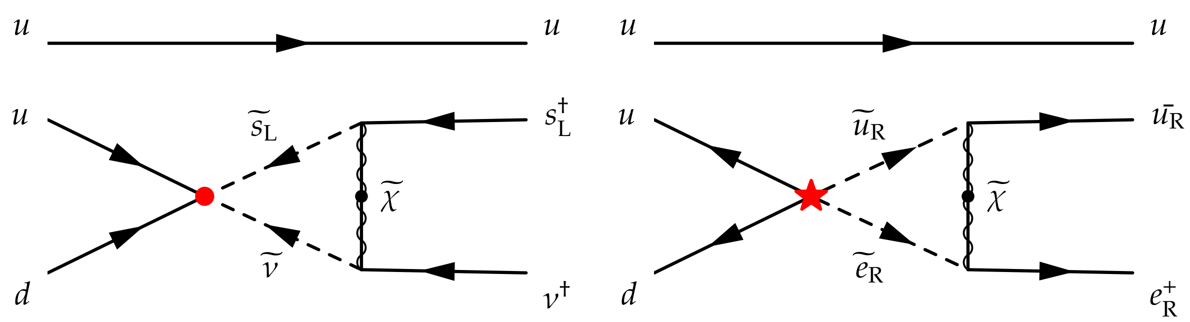

Fig. B.1 Channels of the proton decay induced by $QQQL$ (left) and $\bar{U}\bar{U}\bar{D}\bar{E}$ (right) terms. The star ★ in the $\bar{U}\bar{U}\bar{D}\bar{E}$ case denotes the CKM mixing, which we have to use to convert from the charm quark to the up quark. $\widetilde{\chi}$ denotes the neutralinos.

and the experimental bounds are [7]

$$\tau(p \to K^+\nu) > 6.7 \times 10^{32}\text{yr}. \tag{B.12}$$

Thus we need an unnatural constraint, $|k| \lesssim 10^{-11}$, for the $QQQL$ coupling. For the $\bar{U}\bar{U}\bar{D}\bar{E}$ coupling the constraint is a bit relaxed because of the CKM mixing, but still unnatural.

Therefore, it is favorable to introduce a symmetry to forbid these terms.

* * *

These 5-dimensional operators are, since they are naturally present in SU(5) GUTs, well studied. The decay rate for $QQQL$ is more precisely calculated [37, 38] as

$$\Gamma(p \to K^+\nu_i^\dagger) = \frac{(m_\text{proton}^2 - m_K^2)^2}{32\pi m_\text{proton}^3 f_\pi^2}\left|\beta C\left(1 + \frac{m_\text{proton}}{m_B}(D+F)\right)\right|^2. \tag{B.13}$$

f_π *in this equation* is the pion decay constant $\approx$ 139MeV. C is the coupling constant which is defined through the 6-dimensional effective operator $O(sud\nu)$ as

$$\mathcal{L} \supset C \cdot O(sud\nu). \tag{B.14}$$

β is a parameter related to the hadron matrix element $\left\langle 0 \middle| udd \middle| \text{proton}\right\rangle$ which ranges

$$\beta = (0.003\text{–}0.03)\text{GeV}^3. \tag{B.15}$$

The latter part in the big bracket of Eq. (B.13) expresses the effects of the Quantum Chromo Dynamics in the Standard Model, and in the references $D = 0.81$ and $F = 0.44$ are used as the values. m_B is typical light baryon mass $\approx$ 1150MeV.

Anyway, in our language the coupling constant is

$$C \sim \frac{k}{M_\text{GUT}}\frac{1}{m_\text{SUSY}}, \tag{B.16}$$

and therefore we obtain

$$\tau = \frac{1.9 \times 10^{13}\text{yr}}{k^2} \cdot \left(\frac{M_\text{GUT}}{10^{16}\text{GeV}}\frac{m_\text{SUSY}}{10^3\text{GeV}}\right)^2 \left(\frac{0.003\text{GeV}^3}{\beta}\right)^2, \tag{B.17}$$

which roughly meets our estimate and yields a constraint $|k| \lesssim 10^{-10}$.

◆$QQQH_\text{d}$ term

This term $QQQH_\text{d}$ violates only B, thus we need a source of L-violation. Here, we introduce the bilinear term $H_\text{u}L_i$ as an example:

$$W = (\mu\epsilon_i)H_\text{u}L_i + \frac{k}{M_\text{GUT}}QQQH_\text{d}. \tag{B.18}$$

As we will explain in Appendix B.i, when the superpotential includes the bilinear term, the sneutrino obtains a vacuum expectation value (VEV), and the value can be approximated as

$$\langle \widetilde{\nu_i} \rangle \simeq \frac{1}{\sqrt{2}} \epsilon_i v \cos\beta, \tag{B.19}$$

Therefore one of the main Feynman diagrams is as Fig. B.2, whose decay rate is

$$\begin{aligned} \Gamma &\approx \left| \frac{\alpha_2 k}{M_{\rm GUT}} \cdot \frac{m_s}{v\cos\beta/\sqrt{2}} \cdot \frac{\epsilon_i v \cos\beta}{\sqrt{2}} \cdot \frac{1}{m_{\rm SUSY}} \right|^2 m_{\rm proton}^3 \\ &= \left| \frac{\alpha_2 k}{M_{\rm GUT}} \cdot \epsilon_i m_s \cdot \frac{1}{m_{\rm SUSY}} \right|^2 m_{\rm proton}^3 \\ &= \frac{|k\epsilon_i|^2}{2.0\times 10^{11}\mathrm{yr}} \left(\frac{m_{\rm SUSY}}{10^3\mathrm{GeV}} \right)^{-2} \left(\frac{M_{\rm GUT}}{10^{16}\mathrm{GeV}} \right)^{-2}. \end{aligned} \tag{B.20}$$

Here m_s is the mass of the strange quark. We can see that the experimental constraint is roughly $|k\epsilon_i| \lesssim 10^{-11}$.

The constraint from this term is weaker than those of the previous two interactions because here we need an L-violating term.

◆$\bar{D}^\dagger QQ$ term

The term $\bar{D}^\dagger QQ$ is similar to the 4-dimensional $\bar{U}\bar{D}\bar{D}$ term. One of the processes is as Fig. B.3, where we use

$$\int \mathrm{d}\theta^2 \mathrm{d}\bar{\theta}^2\, \bar{D}^\dagger QQ \supset \int \mathrm{d}\theta^2 \mathrm{d}\bar{\theta}^2 \left(\sqrt{2}\bar{\theta}\bar{\psi}_{\bar{D}} \right) \left(\sqrt{2}\theta\psi_Q \right) \left(\mathrm{i}\theta\sigma^\mu\bar{\theta}\partial_\mu\phi_Q \right) \tag{B.21}$$

term, and a $LQ\bar{D}$ term as a source of L-violation. The decay rate is estimated as

$$\Gamma \approx \left| \frac{k\cdot m_{\rm proton}}{M_{\rm GUT}} \lambda' \right|^2 \frac{m_{\rm proton}^5}{m_{\rm SUSY}^4} = \frac{|k\lambda'|^2}{3.3\times 10^{12}\mathrm{yr}}, \tag{B.22}$$

while the experimental bounds are [7]

$$\tau(p \to K^+\nu) > 6.7\times 10^{32}\mathrm{yr}, \qquad \tau(p\to \pi^+\nu) > 2.5\times 10^{31}\mathrm{yr}. \tag{B.23}$$

Therefore the constraint is

$$|k\lambda'| \lesssim 10^{-10}. \tag{B.24}$$

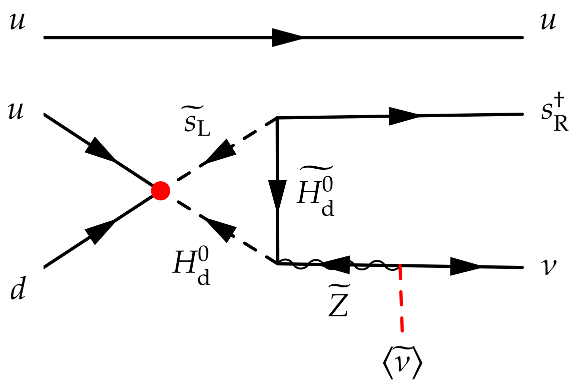

Fig. B.2 One of the main channels of the proton decay induced by $QQQH_{\mathrm{d}}$. Here we use the sneutrino VEV.

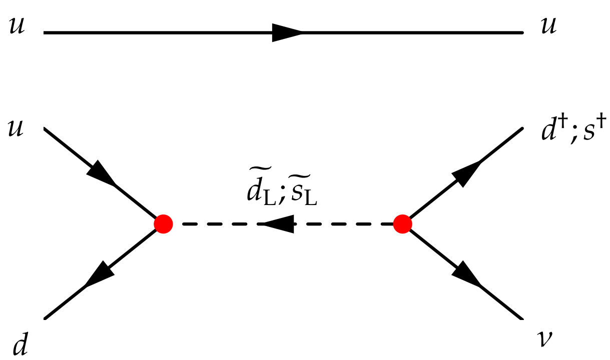

Fig. B.3 One of the main channels of the proton decay induced by $\bar{D}^{\dagger}QQ$. The intermediate particle in this diagram must be left-handed.

Section B.3 R-Parity Violation and Other Symmetries

So far, we saw that we have the following B-violating interactions, and they are very harmful to the proton decay problem:

$$W \supset \lambda'' \bar{U}\bar{U}\bar{D} + \frac{k_1}{M_{\mathrm{GUT}}} QQQL + \frac{k_2}{M_{\mathrm{GUT}}} \bar{U}\bar{U}\bar{D}\bar{E} + \frac{k_3}{M_{\mathrm{GUT}}} QQQH_{\mathrm{d}}, \tag{B.25}$$

$$K \supset \frac{k_4}{M_{\mathrm{GUT}}} \bar{D}^\dagger QQ. \tag{B.26}$$

To solve the proton decay problem, usually we impose the R-parity conservation on the MSSM. However, even if we do so, $QQQL$ and $\bar{U}\bar{U}\bar{D}\bar{E}$ terms cannot be omitted. Thus if we want to omit all these terms, we must find another symmetry.

B.3.1 Discrete Gauge Symmetry

Here, we should mention quantum gravitational effects on the symmetries. It is said that any *global* symmetries are violated by those effect, for example virtual blackhole exchange and wormhole tunneling. Thus all the global symmetries we impose on the MSSM (or other effective theories) must be a *remnant* of a *gauge* symmetry. [39]

Consider a U(1) gauge theory with two scalar fields η and ξ carrying charge Ne and e, respectively, and η is much heavier than ξ. Here, if η is condensed at some high energy scale, that is, acquires VEV, then its low-energy effective theory is only the theory with ξ, and it respects a *global* Z_N symmetry $\xi \mapsto \exp(2\pi\mathrm{i}/N)\xi$ as a consequence of the original gauge invariance. Discrete symmetries of this type are called "discrete gauge symmetries." Discrete gauge symmetries are protected from the quantum gravity effects.

However, gauge symmetries must satisfy anomaly cancellation conditions. We can regard a global symmetry as a discrete gauge symmetry if and only if there is an anomaly-free gauge symmetry from which the global symmetry is obtained.

Here note that the high energy theory might have another particle whose mass is very heavy and thus already integrated out. Therefore, the gauge symmetry need *not* be anomaly-free only with the low-energy particles. In other words, we can add some heavy particles so that the gauge symmetry should be anomaly-free.

For example, the R-parity *is anomalous* only with the MSSM particles (See: [40, Sec.22.4]), but adding the right-handed neutrino $\bar{N}$ makes it anomaly-free. An example of the anomaly-free charge assignments is as follows.

Q	L	$\bar{U}$	$\bar{D}$	$\bar{E}$	H_u	H_d	$\bar{N}$
1	−3	−5	3	7	4	−4	−1

Here, though we do not prove, we are free to shift all these values by kY, where k is an arbitral coefficient and Y is the hypercharge of the particle.

Ibáñez and Ross studied this "discrete gauge anomaly"[41]. They assume that all the massive fermions, which is added in order that the gauge symmetry be anomaly-free, have *integer* Z_N charges[*3], and under this assumption, they proved [42] that, among Z_2 and Z_3 symmetries, only two symmetries are anomaly-free. One is the standard Z_2 R-parity R_2, and the other is "baryon triality" B_3.

Dreiner, Luhn, and Thormeier extended this result to arbitrary Z_N symmetries, and propose a new symmetry "proton-hexality" P_6 [10], which is anomaly-free without including fractionally charged heavy particles. Also Luhn and Thormeier [43] studied about the symmetries which is suitable to the MSSM+$\bar{N}$ (right-handed neutrino) model and their GUT-compatibility, and proposed other several symmetries.

We write down the charge assignments of the symmetries at Tab. B.2, and allowed terms of the MSSM at Tab. B.3, as references.

B.3.2 Other Ways but R-Parity

Forget about the matters of high-energy theories, and concentrate on the low-energy effective theory. What can we say about the MSSM superpotential?

You can see that we have three ways. (Here, P_B and P_L are the baryon and the lepton parity as we have introduced.)

(i) R-parity conserving case The first way is to impose a symmetry which forbids *both* P_B- and P_L-violating terms, e.g., P_6. In this case the LSP is still stable, and the (renormalizable) superpotential is

$$W = W_\mathrm{RPC} := \mu H_\mathrm{u} H_\mathrm{d} + y_{uij} H_\mathrm{u} Q_i \bar{U}_j + y_{dij} H_\mathrm{d} Q_i \bar{D}_j + y_{eij} H_\mathrm{d} L_i \bar{E}_j. \tag{B.27}$$

(ii) R-parity violation in lepton sector The second way is to forbid only P_B-violating terms with, for example, the baryon triality B_3. Then the LSP cannot be a candidate of the dark

[*3] Adding fractionally charged heavy particles will generally relax the anomaly cancellation conditions.

matter, but proton would not decay. The superpotential is

$$W = W_{\mathrm{RPC}} + \kappa_i H_{\mathrm{u}} L_i + \frac{1}{2}\lambda_{ijk} L_i L_j \bar{E}_k + \lambda'_{ijk} L_i Q_j \bar{D}_k. \tag{B.28}$$

(iii) R-parity violation in baryon sector The last way is to use the lepton triality L_3 etc. to forbid only P_{L}-violating terms, with imposing another condition that the LSP is heavier than proton. The LSP cannot be a dark matter candidate, and the proton decay would not occur as we discussed in App. 2.1.3. The superpotential is

$$W = W_{\mathrm{RPC}} + \frac{1}{2}\lambda''_{ijk} \bar{U}_i \bar{D}_j \bar{D}_k. \tag{B.29}$$

We refer the first way as "R-parity conserving case" or "SUSY with R-parity," and the other two ways as "R-parity violating case" or "SUSY without R-parity," although the symmetry we do or do not impose is not the R-parity.

> The fact that the LSP is stable under P_{B}- and P_{L}-conservation needs some explanation.
>
> First consider the interactions induced from the superpotential. Note that the operators which induced by the same term in the superpotential have the same R-parity. Since the conservation of both P_{B} and P_{L} the superfields means the R-parity conservation in the superfields, the LSP would not decay via the superpotential interaction in this case.
>
> How about the gauge interactions? Supersymmetric gauge interactions are obtained by "supersymmetrizing" an even number of fields in a gauge interaction operator. Since this operation does not change the R-parity, the gauge interactions never violate the R-parity as long as the gauge boson is even in the R-parity. However, now P_{B} and P_{L} are conserved, and therefore gauge bosons must be R-even. Therefore the LSP would not decay via the gauge interactions in this case.

Table B.2 The charge assignments to the MSSM particles (and right-handed neutrino $\bar{N}$) of discrete gauge symmetries. Among these symmetries, the R-parity R_2, the baryon triality B_3 [42] and the proton hexality P_6 [10], are anomaly-free without adding fractionally charged heavy particles. The other symmetries are Luhn and Tohrmeier's work [43].

	type	Q	L	$\bar{U}$	$\bar{D}$	$\bar{E}$	H_u	H_d	$\bar{N}$
R_2	Z_2	1	1	1	1	1	0	0	1
B_3	Z_3	0	2	2	1	2	1	2	0
P_6	Z_6	0	4	1	5	1	5	1	3
B_2	Z_2	1	0	1	1	0	0	0	0
L_2	Z_2	0	1	0	0	1	0	0	1
L_3	Z_3	0	2	0	0	1	0	0	1
M_3	Z_3	0	1	2	1	0	1	2	1
R_3	Z_3	0	0	2	1	1	1	2	2
$Z_6(102)$	Z_6	0	4	5	1	3	1	5	1
$Z_6(302)$	Z_6	0	4	3	3	5	3	3	5

Table B.3 Allowed interaction of the MSSM with a discrete symmetry.

	disastrous	harmful		not good		L-violating		
	$\bar{U}\bar{D}\bar{D}$	$QQQL$	$\bar{U}\bar{U}\bar{D}\bar{E}$	$QQQH_d$	$\bar{D}^\dagger QQ$	LH_u	$LQ\bar{D}$	$LL\bar{E}$
R_2		✓	✓					
B_3						✓	✓	✓
P_6								
B_2						✓	✓	✓
L_2	✓			✓	✓			
L_3	✓			✓	✓			
M_3								
R_3		✓	✓					
$Z_6(102)$								
$Z_6(302)$								

Appendix B.i Higgs Mechanism under R-Parity Violation

If the R-parity is not conserved, the superpotential and the $\not{\text{SUSY}}$ terms of the MSSM are extended, and the Higgs Mechanism is modified. Especially the violation is in the lepton sector, we cannot distinguish the down-type Higgs from the leptons because we have no "lepton number." In this appendix we discuss these matters [11].

B.i.1 Higgs Potential

First, we put the Higgs superfield and the lepton superfields into a vector

$$L_\alpha = (L_0, L_i) := (H_\mathrm{d}, L_i). \tag{B.30}$$

(Greek letters to run 1–4, and Latin letters 1–3.) The superpotential is described as

$$W = y_{\mathrm{u}ij} H_\mathrm{u} Q_i \bar{U}_j + \mu_\alpha H_\mathrm{u} L_\alpha + \frac{1}{2} y_{\alpha\beta j} L_\alpha L_\beta \bar{E}_j + {y'}_{\alpha ij} L_\alpha Q_i \bar{D}_j, \tag{B.31}$$

where $y_{\alpha\beta j} = -y_{\beta\alpha j}$, and the $\not{\text{SUSY}}$ part is now

$$\begin{aligned}\mathcal{L}_{\not{\text{SUSY}}} = (\text{Gaugino mass term}) &- \left[(a_u)_{ij}\tilde{\bar{u}}_i\tilde{Q}_j H_\mathrm{u} - a_{\alpha ij}\tilde{\bar{d}}_i\tilde{Q}_j\tilde{L}_\alpha - a'_{\alpha ij}\tilde{\bar{e}}_i\tilde{L}_j\tilde{L}_\alpha + \text{H. c.}\right]\\ &- \left[\left(m_Q^2\right)_{ij}\tilde{Q}_i^*\tilde{Q}_j + \left(m_{\bar{U}}^2\right)_{ij}\tilde{\bar{u}}_i^*\tilde{\bar{u}}_j + \left(m_{\bar{D}}^2\right)_{ij}\tilde{\bar{d}}_i^*\tilde{\bar{d}}_j + \left(m_{\bar{E}}^2\right)_{ij}\tilde{\bar{e}}_i^*\tilde{\bar{e}}_j + m_{H_\mathrm{u}}^2 H_\mathrm{u}^* H_\mathrm{u}\right]\\ &- \left(m_L^2\right)_{\alpha\beta}\tilde{L}_\alpha^*\tilde{L}_\beta - \left[\beta_\alpha H_\mathrm{u}\tilde{L}_\alpha + \text{H. c.}\right].\end{aligned} \tag{B.32}$$

Here note especially that $\mu_\alpha = (\mu_0, \mu_i) = (\mu, \kappa_i)$.

The classical scalar potential for the Higgs bosons is now

$$\begin{aligned}V_\text{Higgs} = &\left(|\mu_\alpha|^2 + m_{H_\mathrm{u}}^2\right)\left(|H_\mathrm{u}^0|^2 + |H_\mathrm{u}^+|^2\right)\\ &+ \left(m_L^2\right)_{\alpha\beta}\left(\tilde{e}_\alpha^*\tilde{e}_\beta + \tilde{\nu}_\alpha^*\tilde{\nu}_\beta\right) + \left|\mu_\alpha\tilde{\nu}_\alpha\right|^2 + \left|\mu_\alpha\tilde{e}_\alpha\right|^2 + 4\sum_i\left|y_{\alpha\beta i}\tilde{\nu}_\alpha\tilde{e}_\beta\right|^2\\ &+ \left[\beta_\alpha\left(H_\mathrm{u}^+\tilde{e}_\alpha - H_\mathrm{u}^0\tilde{\nu}_\alpha\right) + \text{H. c.}\right]\\ &+ \frac{g_1^2 + g_2^2}{8}\left(\left|H_\mathrm{u}^+\right|^2 + \left|H_\mathrm{u}^0\right|^2 - \left|\tilde{\nu}_\alpha\right|^2 - \left|\tilde{e}_\alpha\right|^2\right)^2\\ &+ \frac{g_2^2}{8}\left[\left|H_\mathrm{u}^+\tilde{\nu}_\alpha^* + H_\mathrm{u}^0\tilde{e}_\alpha^*\right|^2 - \sum_{\alpha<\beta}\left|\tilde{\nu}_\alpha\tilde{e}_\beta - \tilde{e}_\alpha\tilde{\nu}_\beta\right|^2\right],\end{aligned} \tag{B.33}$$

which is very complicated. Here, as usual, not to break the electromagnetic symmetry, we redefine the up-type Higgs field so that $\langle H_\mathrm{u}^+\rangle = 0$. This is the rotation of $\mathrm{SU}(2)_\text{weak}$.

The VEV-condition

$$\left\langle \frac{\partial V}{\partial H_{\mathrm{u}}^{+}} \right\rangle \Big|_{\langle H_{\mathrm{u}}^{+}\rangle=0} = 0 \tag{B.34}$$

yields

$$\left\langle \left(\beta_\alpha + \frac{g_2}{8} \nu_\alpha^* H_{\mathrm{u}}^{0*} \right) \widetilde{e}_\alpha \right\rangle = 0, \qquad \text{i.e.,} \quad \langle \widetilde{e}_\alpha \rangle = 0, \tag{B.35}$$

from which we see that the electromagnetic symmetry does not break up. Thus we obtain

$$\begin{aligned} V_{\text{Higgs}} &= \left(|\mu_\alpha|^2 + m_{H_{\mathrm{u}}}^2 \right) |H_{\mathrm{u}}^0|^2 + \left(m_L^2 \right)_{\alpha\beta} \widetilde{\nu}_\alpha^* \widetilde{\nu}_\beta + \left| \mu_\alpha \widetilde{\nu}_\alpha \right|^2 \\ &\quad - \left[\beta_\alpha H_{\mathrm{u}}^0 \widetilde{\nu}_\alpha + \text{H. c.} \right] + \frac{g_1^2 + g_2^2}{8} \left(\left| H_{\mathrm{u}}^0 \right|^2 - \left| \widetilde{\nu}_\alpha \right|^2 \right)^2 . \end{aligned} \tag{B.36}$$

Here, as usual, we can set $\beta_\alpha > 0$ by redefining the phases of L_α.

B.I.2 Mass Matrices and Alignment

This discussion is along Ref. [44].

Now the fields which may have VEVs are H_{u}^0 and $\widetilde{\nu}_\alpha$. Define the VEVs of those fields as

$$\left\langle H_{\mathrm{u}}^0 \right\rangle =: v_u, \qquad \qquad \langle \widetilde{\nu}_\alpha \rangle =: v_\alpha. \tag{B.37}$$

Then the mass matrices of the neutralino and the chargino sector are

$$\mathcal{L} \supset \left[-\frac{1}{2} (\psi_0)^{\mathrm{T}} M_{\mathrm{N}} \psi_0 - (\psi_+)^{\mathrm{T}} M_{\mathrm{C}} \psi_- \right] + \text{H. c.} \tag{B.38}$$

$$M_{\mathrm{N}} = \begin{pmatrix} c^2 M_1 + s^2 M_2 & cs(M_2 - M_1) & 0 & 0 & 0 \\ cs(M_2 - M_1) & s^2 M_1 + c^2 M_2 & -\overline{g} v_u & \overline{g} v_0 & \overline{g} v_i \\ 0 & -\overline{g} v_u & 0 & -\mu & -\kappa_i \\ 0 & \overline{g} v_0 & -\mu & 0 & 0 \\ 0 & \overline{g} v_i & -\kappa_i & 0 & 0 \end{pmatrix}, \quad \psi_0 := \begin{pmatrix} \widetilde{\gamma} \\ \widetilde{Z} \\ \widetilde{H_{\mathrm{u}}^0} \\ \nu_0 \\ \nu_i \end{pmatrix}; \tag{B.39}$$

$$M_{\mathrm{C}} = \begin{pmatrix} M_2 & g v_0/\sqrt{2} & g v_i/\sqrt{2} \\ g v_u/\sqrt{2} & \mu & \kappa_i \\ 0 & (y_d)_{kj} v_k & -(y_d)_{ij} v_0 + \lambda_{ikj} v_k \end{pmatrix}, \qquad \psi_+ := \begin{pmatrix} \widetilde{W}^+ \\ \widetilde{H_{\mathrm{u}}^0} \\ e_{\mathrm{R}i}^{+} \end{pmatrix}, \quad \psi_- := \begin{pmatrix} \widetilde{W}^- \\ e_0 \\ e_j \end{pmatrix}, \tag{B.40}$$

where $\overline{g} := g_2/2\cos\theta_{\mathrm{W}}$, $c := \cos\theta_{\mathrm{W}}$ and $s := \sin\theta_{\mathrm{W}}$.

Here, two eigenvalues of M_{N} are zero, and five are non-zero. This means we have two massless "neutralinos" and five massive ones after the EWPT. Note that these five massive higgsinos contains one neutrino, which is nearly massless.

The product of the masses can be calculated as

$$\prod_{i=1...5} m_i^{\mathrm{N}} = \left(c^2 M_1 + s^2 M_2\right)\left(\overline{g}\left\|\boldsymbol{\mu}\right\| \|\boldsymbol{v}\| \sin\xi\right)^2 \tag{B.41}$$

where

$$\boldsymbol{\mu} := \mu_\alpha, \qquad \boldsymbol{v} := v_\alpha, \qquad \cos\xi := \frac{\boldsymbol{\mu}\cdot\boldsymbol{v}}{\left\|\boldsymbol{\mu}\right\| \|\boldsymbol{v}\|}. \tag{B.42}$$

Here, we can expect that $M_1 \sim M_2 \sim \left\|\boldsymbol{\mu}\right\| \sim 100\mathrm{GeV}$, and thus the mass of one massive neutrino can be approximated as

$$m_\nu \sim (100\mathrm{GeV})\cdot\sin^2\xi, \tag{B.43}$$

That is, we can expect that ξ is very small, or in other words, $\boldsymbol{\mu}$ and $\boldsymbol{v}$ are nearly *aligned*.

* * *

When we assume $\kappa_i \ll \mu$, we can obtain the following expressions of VEVs:

$$\left\langle H_{\mathrm{u}}^0\right\rangle = \frac{1}{\sqrt{2}}v\cdot\sin\beta, \qquad \left\langle H_{\mathrm{d}}^0\right\rangle \simeq \frac{1}{\sqrt{2}}v\cdot\cos\beta, \qquad \langle\widetilde{\nu}_i\rangle \simeq -\frac{\kappa_i}{\mu}\cdot\frac{1}{\sqrt{2}}v\cdot\cos\beta, \tag{B.44}$$

where $v = 246\mathrm{GeV}$ is the Standard Model Higgs VEV (See: Sec. A.2.2), and β, the well-known value of the MSSM, is defined as

$$\tan\beta = \frac{\left\langle H_{\mathrm{u}}^0\right\rangle}{\left\langle H_{\mathrm{d}}^0\right\rangle}. \tag{B.45}$$

We used this basis in the decay rate approximation of the $QQQH_{\mathrm{d}}$ proton decay in Sec. B.2.1.

B.I.3 Conditions for Alignment

As the end of this appendix, let us discuss the conditions on the parameters for the alignment. To this end, we take the basis where $\kappa_i = 0$. If the alignment is realized, v_i must vanish in this basis.

In this basis, the Higgs potential would be

$$\begin{aligned} V_{\mathrm{Higgs}} = {} & \left(|\mu|^2 + m_{H_{\mathrm{u}}}^2\right)|H_{\mathrm{u}}^0|^2 + \left(m_L^2\right)_{\alpha\beta}\widetilde{\nu}_\alpha^*\widetilde{\nu}_\beta + |\mu|^2\left|H_{\mathrm{d}}^0\right|^2 \\ & - \beta_\alpha\left[H_{\mathrm{u}}^0\widetilde{\nu}_\alpha + \mathrm{H.\,c.}\right] + \frac{g_1^2+g_2^2}{8}\left(\left|H_{\mathrm{u}}^0\right|^2 - \left|\widetilde{\nu}_\alpha\right|^2\right)^2. \end{aligned} \tag{B.46}$$

Here, as usual, we can rotate the fields so that $v_u > 0$ and $v_d > 0$, and express them as

$$v_u = v \sin\theta, \qquad v_d = v\cos\theta. \tag{B.47}$$

(We use θ instead of the usual β.) The VEV-conditions are expressed as

$$\left(|\mu|^2 + m_{H_\mathrm{u}}^2\right)\sin\theta - \beta_0\cos\theta - \frac{{g_1}^2 + {g_2}^2}{4}v^2\cos 2\theta\sin\theta = 0, \tag{B.48}$$

$$\left[(|\mu|^2 + \left(m_L^2\right)_{00}\right]\cos\theta - \beta_0\sin\theta + \frac{{g_1}^2 + {g_2}^2}{4}v^2\cos 2\theta\sin\theta = 0, \tag{B.49}$$

$$\left(m_L^2\right)_{i0}\cos\theta - \beta_i\sin\theta = 0. \tag{B.50}$$

Therefore, the conditions for the alignment are expressed as

$$2|\mu|^2 + m_{H_\mathrm{u}}^2 + \left(m_L^2\right)_{00} = \frac{2\beta_0}{\sin 2\theta}, \tag{B.51}$$

$$\frac{{g_1}^2 + {g_2}^2}{4}v^2 = \frac{\left[\left(m_L^2\right)_{00} + |\mu|^2\right]\cos^2\theta - \left[m_{H_\mathrm{u}}^2 + |\mu|^2\right]\sin^2\theta}{\cos 2\theta}, \tag{B.52}$$

$$\frac{\left(m_L^2\right)_{10}}{\beta_1} = \frac{\left(m_L^2\right)_{20}}{\beta_2} = \frac{\left(m_L^2\right)_{30}}{\beta_3} = \tan\theta \tag{B.53}$$

in this basis. Here, the first and the second conditions are the same ones of the R-parity conserving MSSM, and what is important is the last one.

Appendix C
Cosmology

In this thesis, we discussed cosmological constraints on the R-parity violating parameters. In the discussion, we have used the fact that this universe is expanding, where the expansion rate is given by the Hubble parameter. Now, for the sake of completeness, we will obtain the Hubble expansion rate in this appendix.

Section C.1 The Expanding Universe

C.1.1 Underlying Structure

We, the human being, live in this universe. We know that this universe is spatially homogeneous and isotropic. Or more precisely speaking, this universe is *macroscopically* homogeneous and isotropic as far as we know. This is so-called "Cosmological Principle."[*1]

Under this axiom, the spacetime metric g_{ij} of the universe is restricted as follows, the "Фридман (Friedmann)–Lemaître–Robertson–Walker (FLRW) metric:"

$$\begin{aligned} \mathrm{d}s^2 &:= g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu && \text{(C.1)} \\ &= \mathrm{d}t^2 - a(t)^2\left[\|\mathrm{d}\boldsymbol{x}\|^2 + \frac{K\,(\boldsymbol{x}\cdot\mathrm{d}\boldsymbol{x})^2}{1-K\,\|\boldsymbol{x}\|^2}\right] && \text{(C.2)} \\ &= \mathrm{d}t^2 - a(t)^2\left[\frac{\mathrm{d}r^2}{1-Kr^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\cdot\mathrm{d}\phi^2\right)\right], && \text{(C.3)} \end{aligned}$$

where t is the time coordinate, $\boldsymbol{x}$ and (r,θ,ϕ) are the Cartesian and the polar coordinates for the (3-dimensional) space, a is a time-dependent parameter, which is called the scale

[*1] Steven Weinberg mentioned [45] that this principle is valid only for "typical observers," those who move with the average velocity of typical galaxies in their respective neighborhoods. This is true surely.

factor, and K denotes the curvature of the space,

$$\begin{cases} K > 0 & \text{for a closed universe (+1 for spherical),} \\ K = 0 & \text{for a flat universe,} \\ K < 0 & \text{for an open universe } (-1 \text{ for hyperspherical).} \end{cases} \tag{C.4}$$

Under this metric, the Einstein equation is calculated as

$$\left(\frac{\dot a}{a}\right)^2 + \frac{K}{a^2} = \frac{8\pi G}{3} T_{00}, \qquad g_{ij}\left(\frac{\ddot a}{a} + \frac{\dot a^2}{a^2} + \frac{K}{a^2}\right) = 8\pi G T_{ij}. \tag{C.5}$$

The definition of the Einstein equation and the detail procedure of this calculation is given in Appendix C.i.

Recent observations proved that the universe is extremely flat. Therefore we set $K = 0$ from now.

C.1.2 Energy–Momentum Tensor

The axiom that the universe is spatially isotropic and homogeneous also leads us an approximation that the substance of the universe can be approximated as the perfect fluid. The perfect fluid is a fluid that has no viscosity and no heat conduction. Its energy–momentum tensor $T^\mu{}_\nu$ is

$$T^\mu{}_\nu = \mathrm{diag}(\rho, -p, -p, -p). \tag{C.6}$$

Thus the Einstein equation is now

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho(t), \qquad \frac{\ddot a}{a} + \frac{\dot a^2}{a^2} = -8\pi G \cdot p(t), \tag{C.7}$$

and therefore

$$\frac{\dot a}{a} = \sqrt{\frac{8\pi G}{3}\rho(t)}, \qquad \frac{\ddot a}{a} = -\frac{8\pi G}{3}\left(\rho + 3p\right). \tag{C.8}$$

The energy density $\rho(t)$ and the pressure $p(t)$ depend on the property of the substance. Let us calculate these values.

To discuss the energy density and the pressure, we first introduce the well-known momentum distributions, the Fermi–Dirac (FD), the Bose–Einstein (BE), and the Maxwell–Boltzmann (MB) ones:

$$f_{\mathrm{MB}}(\boldsymbol{k}) = \frac{1}{\mathrm{e}^{(E-\mu)/T}}, \qquad f_{\mathrm{BE}}(\boldsymbol{k}) = \frac{1}{\mathrm{e}^{(E-\mu)/T} - 1}, \qquad f_{\mathrm{FD}}(\boldsymbol{k}) = \frac{1}{\mathrm{e}^{(E-\mu)/T} + 1}, \tag{C.9}$$

where $E := \sqrt{\|\boldsymbol{k}\|^2 + m^2}$ is the energy of the particle. $\boldsymbol{k}$, μ and m denote the momentum, the chemical potential, and the mass of the particle, and T is the temperature of the universe.

When a particle is in a thermal bath, its energy density ρ, pressure p, and number density n, are given by

$$\rho = g\int \frac{\mathrm{d}^3k}{(2\pi)^3} f(\boldsymbol{k})\cdot E, \qquad p = g\int \frac{\mathrm{d}^3k}{(2\pi)^3} f(\boldsymbol{k})\cdot \frac{\|\boldsymbol{k}\|}{3E}, \qquad n = g\int \frac{\mathrm{d}^3k}{(2\pi)^3} f(\boldsymbol{k}), \tag{C.10}$$

where $f(\boldsymbol{k})$ depends on the statistics of the particle.

C.1.3 Hubble Parameter

◆Massless approximation

Now, almost all have been done. We know that the Hubble parameter is given only by the energy density ρ (C.7), and ρ can be calculated by the above expression. Thus, theoretically, we can obtain the Hubble parameter at any temperature.

However, actually, this calculation cannot be done analytically in general, and we cannot obtain the analytical expression of the Hubble parameter, which we want to use in the Boltzmann equations. Therefore, here we come down to do an approximation, $m \ll T$.

If we use the approximation $m \ll T$, which means that all the particles are nearly massless, or the temperature is extremely high, we can continue the calculation analytically. Especially, if we can approximate μ is small enough, i.e., $\mu \ll T$, we expand the expression to the 1st order of μ to obtain the following result:

$$\rho^{\mathrm{BE}} = gT^4\left[\frac{\pi^2}{30} + \frac{3\zeta(3)}{\pi^2}\bar{\mu}\right], \qquad p^{\mathrm{BE}} = \frac{1}{3}\rho^{\mathrm{BE}}, \qquad n^{\mathrm{BE}} = gT^3\left[\frac{\zeta(3)}{\pi^2} + \frac{1}{6}\bar{\mu}\right], \tag{C.11}$$

$$\rho^{\mathrm{FD}} = gT^4\left[\frac{7\pi^2}{240} + \frac{9\zeta(3)}{4\pi^2}\bar{\mu}\right], \qquad p^{\mathrm{FD}} = \frac{1}{3}\rho^{\mathrm{FD}}, \qquad n^{\mathrm{FD}} = gT^3\left[\frac{3\zeta(3)}{4\pi^2} + \frac{\pi^2}{12}\bar{\mu}\right], \tag{C.12}$$

where $\bar{\mu} := \mu/T$.

Now we can calculate the Hubble parameter, which is defined as

$$H := \frac{\dot{a}}{a} = \sqrt{\frac{8\pi G}{3}\rho(t)}. \tag{C.13}$$

The particle which we have and their degrees of freedom are presented in Tab. C.1.*2 As

*2 Also in the table the *mood* of the mass are presented. "Massive" denotes around 100GeV, and "heavy" denotes 300–1000GeV, which describe only the *mood*, or the tendency.

	Boson	Fermion	D.o.F g
Q	HEAVY	0	36
$\bar{U}$	HEAVY	0	18
$\bar{D}$	HEAVY	0	18
$L, \bar{E}$	massive	0	12 + 6
$H_\mathrm{u}, H_\mathrm{d}$	massive	0	4 + 4
g	0	HEAVY	16
W	0	massive	6
B	0	massive	2

Table C.1 The *mood* of mass, and the degree of freedom, of the MSSM particles.

you can see, the energy density and the pressure is calculated as

$$\rho(T) = \frac{\pi^2}{30}T^4\left(122 + \frac{7}{8}\cdot 122\right), \tag{C.14}$$

$$p(T) = \frac{\pi^2}{90}T^4\left(122 + \frac{7}{8}\cdot 122\right), \tag{C.15}$$

under this massless approximation.

Finally, we obtain the Hubble parameter

$$H = \xi\frac{T^2}{M_\mathrm{pl}} \qquad \text{where} \quad \xi := \sqrt{\frac{8\pi}{3}\frac{\pi^2}{30}\cdot 122\left(1 + \frac{7}{8}\right)} \approx 25.1. \tag{C.16}$$

(Note that the Planck mass is defined as $M_\mathrm{pl} := G^{-1/2}$.) We used this value, which is approximated to be constant for simplicity in calculation, in Chap. 4.

Also the second order differential of the scale factor is obtained as

$$\frac{\ddot{a}}{a} = -\xi'\frac{T^4}{M_\mathrm{pl}^2} \qquad \text{where} \quad \xi' := \frac{8\pi}{3}\frac{\pi^2}{15}\cdot 122\left(1 + \frac{7}{8}\right) \approx 1.26\times 10^3. \tag{C.17}$$

◆General result

Here, we will try to obtain the Hubble parameter without the approximation $m \ll T$. We present numerical results here.

As we can still use the approximation $\mu \ll T$, or $\bar{\mu} \ll 1$, we expand the values as, for example the energy density ρ of a boson,

$$\rho^\mathrm{BE}(\bar{m}, \bar{\mu}) = \rho_0^\mathrm{BE}(\bar{m}) + \bar{\mu}\rho_1^\mathrm{BE}(\bar{m}) + \mathrm{O}(\bar{\mu}^2), \tag{C.18}$$

where $\bar{m}$ is defined as: $\bar{m} := m/T$, as we did in $\bar{\mu}$. We put the numerical result of the energy density ρ of a massive boson, and fermion, in Fig. C.1 and Fig. C.2, respectively. Also the pressures in Fig. C.3, and the number densities in Fig. C.4. For the energy density, the approximation seems to be still good for $m \lesssim T$, and for the pressure and the number density, it is good for only $m \lesssim T/3$.

As we presented in Tab. C.1, we have

$$\begin{array}{lll} \text{massless:} & g_{\text{boson}} = 24, & g_{\text{fermion}} = 98 \\ \text{massive:} & g_{\text{boson}} = 26, & g_{\text{fermion}} = 8 \\ \text{heavy:} & g_{\text{boson}} = 72, & g_{\text{fermion}} = 16 \end{array}$$

particles. Then, assuming that the "massive" particles are all $m = 100\text{GeV}$, and "HEAVY" particles are $m = 600\text{GeV}$, and estimating the energy density of the massive particles from Figs. C.1 and C.2, we can approximate

$$\rho(T = 100\text{GeV}) \approx \sum_{\text{BE, FD}} \left[(g \cdot \rho)_{\text{massless}} + (g \cdot \rho)_{\text{massive}} + (g \cdot \rho)_{\text{heavy}} \right] \tag{C.19}$$

$$\approx \frac{\pi^2}{30} T^4 \left[24 + 26 \cdot 0.9 + 72 \cdot 0.1 \right] + \frac{7}{8} \frac{\pi^2}{30} T^4 \left[98 + 8 \cdot 0.9 + 16 \cdot 0.1 \right], \tag{C.20}$$

and finally,

$$H(100\text{GeV}) \simeq 20 \cdot \frac{T^2}{M_{\text{pl}}}. \tag{C.21}$$

Here we have ignored $\bar{\mu}$.

This result tells us that our massless approximation is not so bad even when $T = 100\text{GeV}$, our lowest temperature under consideration.

◆One more note

So far, we have not consider the complexity that the particle might get out of the thermal bath. When the temperature falls down so that the mass becomes not negligible, the creation processes become less frequent, and meanwhile, when the particle becomes very dilute due to the expansion of the universe, the pair annihilation processes also less frequent. These effects make the distribution of the particle different from the original (MB, BE or FD) one, and eventually the particles can travel freely.

If we would like to discuss the expansion (the value of $\dot{a}$) precisely, surely we had to include these effects. However in this thesis (the main part of thesis: Chap. 3 and Chap. 4), we have ignored these effects in the calculation of the Hubble parameter for simplicity.

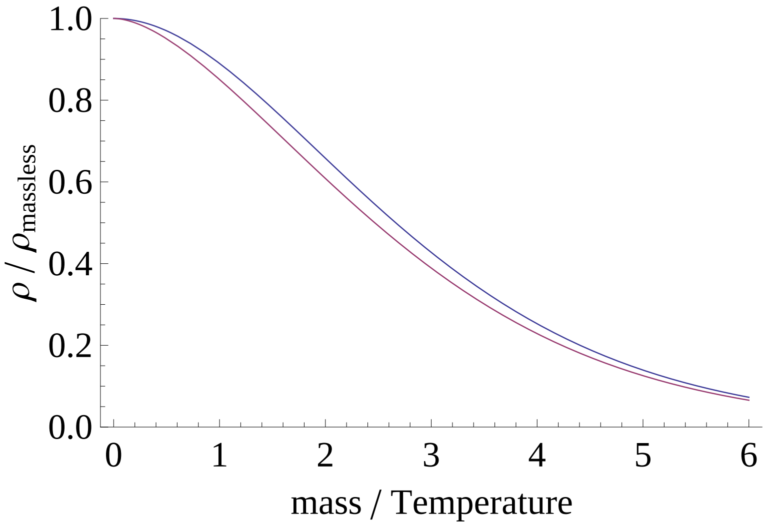

Fig. C.1 The functions $\rho_i^{\text{BE}}(\bar{m})$ normalized by the massless result $\rho_i(0)$, which describe the energy density of a massive *boson*. The blue line is $\rho_0^{\text{BE}}(\bar{m})/\rho_0^{\text{BE}}(0)$, and the red line is $\rho_1^{\text{BE}}(\bar{m})/\rho_1^{\text{BE}}(0)$. See Eq. (C.18) for the definition.

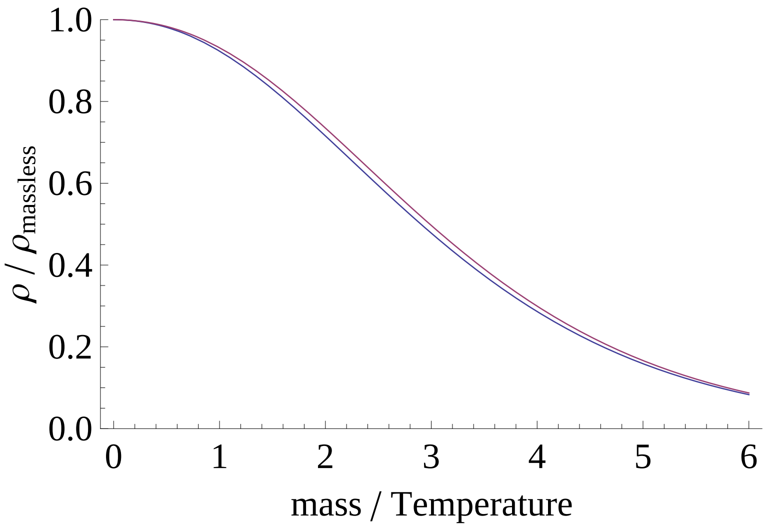

Fig. C.2 The same as Fig. C.1, but for a massive *fermion*.

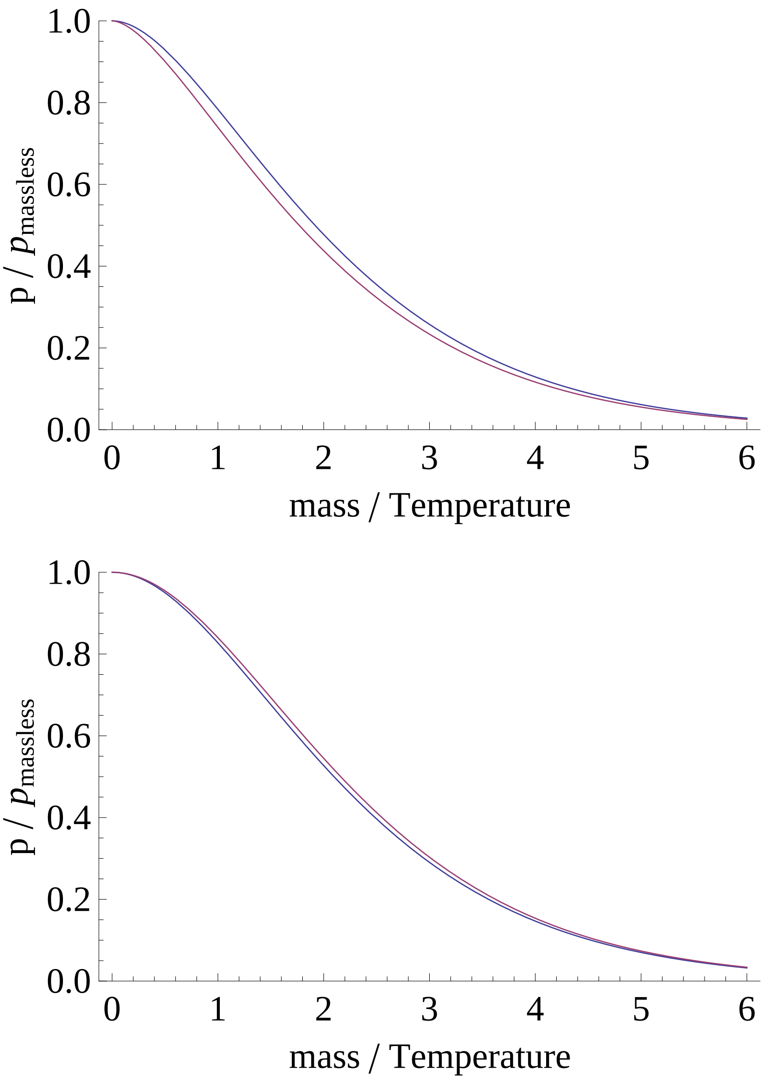

Fig. C.3 The pressures, the same as Fig. C.1 and Fig. C.2. The upper figure is for a massive boson, and the lower is for fermion. The blue lines for the 0th order (p_0), and the red lines for the 1st order (p_1).

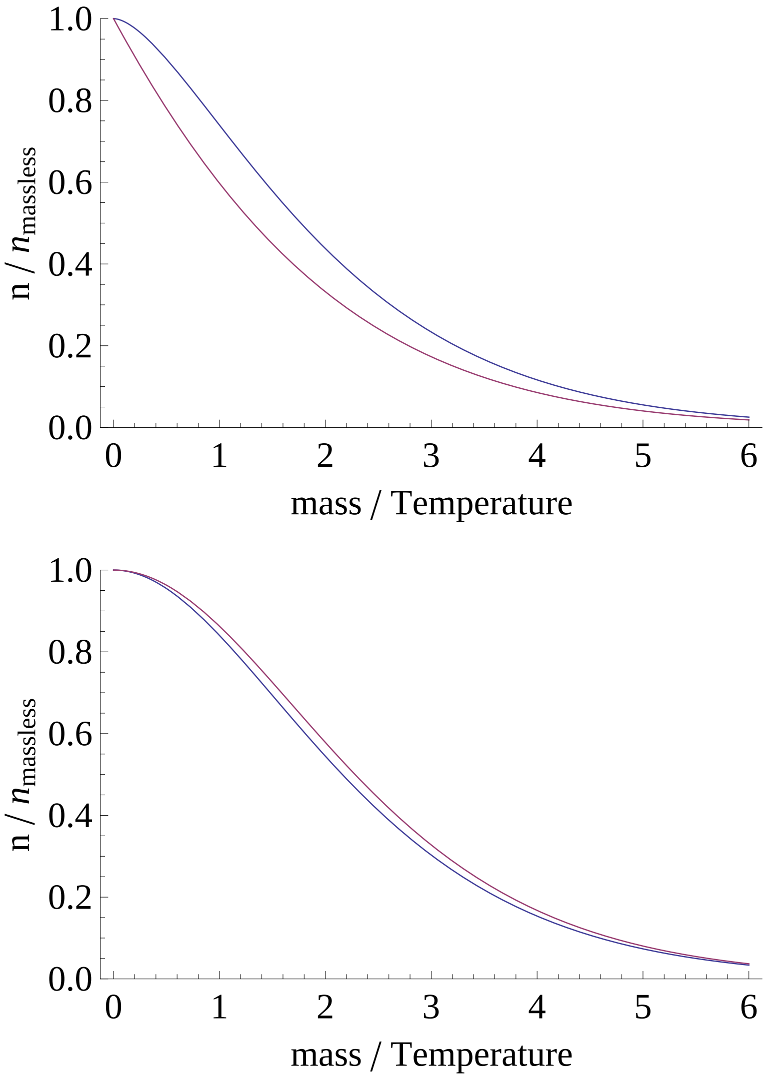

Fig. C.4 The number densities, the same as Fig. C.3.

Appendix C.i Metric and Einstein Equation

In this appendix, we introduce the valuables which we use to express the curvature of the space, calculate their values in under Фридман–Lemaître–Robertson–Walker metric, and introduce the Einstein equation.

C.i.1 The Valuables

When we want to express the curvature of the space, we usually use several valuables which are derived from metric. At first, we give the definitions of the valuables [45, 46, 47].[*3]

- Christoffel symbol (affine connection)

$$\Gamma^{\alpha}{}_{\mu\nu} := \frac{1}{2} g^{\alpha\beta} \left(\frac{\partial}{\partial x^{\nu}} g_{\beta\mu} + \frac{\partial}{\partial x^{\mu}} g_{\beta\nu} - \frac{\partial}{\partial x^{\beta}} g_{\mu\nu} \right). \tag{C.22}$$

- Riemann curvature tensor

$$R^{\alpha}{}_{\beta\mu\nu} := \frac{\partial}{\partial x^{\mu}} \Gamma^{\alpha}{}_{\beta\nu} - \frac{\partial}{\partial x^{\nu}} \Gamma^{\alpha}{}_{\beta\mu} + \Gamma^{\alpha}{}_{\sigma\mu} \Gamma^{\sigma}{}_{\beta\nu} - \Gamma^{\alpha}{}_{\sigma\nu} \Gamma^{\sigma}{}_{\beta\mu}. \tag{C.23}$$

- Ricci curvature tensor and Ricci curvature scalar

$$R_{\mu\nu} := R^{\alpha}{}_{\mu\alpha\nu}, \qquad R := g^{\mu\nu} R_{\mu\nu}. \tag{C.24}$$

- Einstein tensor

$$G_{\mu\nu} := R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}. \tag{C.25}$$

As the metric is symmetric, i.e., $g_{\mu\nu} = g_{\nu\mu}$, these valuables also have the following features related to the (anti-)symmetricity:

$$\Gamma^{\alpha}{}_{\mu\nu} = \Gamma^{\alpha}{}_{\nu\mu}, \qquad R_{\alpha\beta\mu\nu} = -R_{\beta\alpha\mu\nu} = -R_{\alpha\beta\nu\mu} = R_{\mu\nu\alpha\beta}, \qquad R_{\mu\nu} = R_{\nu\mu}. \tag{C.26}$$

[*3] Weinberg [45] uses different definitions. He use $R^{\text{his}}_{\mu\nu} = -R^{\text{ours}}_{\mu\nu}$ as the Ricci tensor, thus his Einstein equation is $R^{\text{his}}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta} R^{\text{his}}_{\alpha\beta} = -8\pi T_{\mu\nu}$.

C.I.2 Values under FLRW Metric

The FLRW metric is, under our time-respecting notation $\eta = \mathrm{diag}(1, -1, -1, -1)$, given by

$$g_{00} = 1, \qquad g_{0i} = g_{i0} = 0, \qquad g_{ij} = -a(t)^2\left(\delta_{ij} + \frac{Kx_ix_j}{1 - K\|\boldsymbol{x}\|^2}\right). \tag{C.27}$$

From this metric, we can obtain the Christoffel symbol, the Ricci curvature tensor, and the Ricci scalar as follows:

$$\Gamma^0{}_{ij} = -\frac{\dot{a}}{a}g_{ij}, \qquad \Gamma^i{}_{0j} = \frac{\dot{a}}{a}\delta^i_j, \qquad \Gamma^i{}_{jk} = -\frac{K}{a^2}x^i g_{jk}, \qquad \text{(Others)} = 0, \tag{C.28}$$

$$R_{00} = -\frac{3\ddot{a}}{a}, \qquad R_{0i} = R_{i0} = 0, \qquad R_{ij} = -\left(\frac{\ddot{a}}{a} + \frac{2\dot{a}^2}{a^2} + \frac{2K}{a^2}\right)g_{ij}, \tag{C.29}$$

$$R = -6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{K}{a^2}\right). \tag{C.30}$$

$$G_{00} = 3\left(\frac{\dot{a}^2}{a^2} + \frac{K}{a^2}\right), \qquad G_{0i} = G_{i0} = 0, \qquad G_{ij} = \left(\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{K}{a^2}\right)g_{ij}. \tag{C.31}$$

For your information, we give the values in the space-respecting notation $\eta = \mathrm{diag}(-1, 1, 1, 1)$. In this notation the FLRW metric is modified as

$$g_{00} = -1, \qquad g_{0i} = g_{i0} = 0, \qquad g_{ij} = a(t)^2\left(\delta_{ij} + \frac{Kx_ix_j}{1 - K\|\boldsymbol{x}\|^2}\right),$$

and the results are

$$\Gamma^0{}_{ij} = \frac{\dot{a}}{a}g_{ij}, \qquad \Gamma^i{}_{0j} = \frac{\dot{a}}{a}\delta^i_j, \qquad \Gamma^i{}_{jk} = \frac{K}{a^2}x^i g_{jk},$$

$$R_{00} = -\frac{3\ddot{a}}{a}, \qquad R_{ij} = \left(\frac{\ddot{a}}{a} + \frac{2\dot{a}^2}{a^2} + \frac{2K}{a^2}\right)g_{ij}, \qquad R = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{K}{a^2}\right).$$

C.I.3 EINSTEIN EQUATION

The Einstein equation is the equation which describes the gravitational interactions. It is expressed by the energy-momentum tensor $T_{\mu\nu}$ and the Einstein tensor $G_{\mu\nu}$ as

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{C.32}$$

where G is the gravitational constant.

Sometimes a term with the cosmological constant Λ is inserted to the Einstein equation, as

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}. \tag{C.33}$$

Then the result (C.8) which we have obtained under the perfect fluid approximation is modified as

$$\frac{\dot{a}}{a} = \sqrt{\frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{K}{a^2}}, \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}. \tag{C.34}$$

Meanwhile, if we add the “dark energy” as a substance which satisfy $\rho_{\mathrm{DE}}(t) = -p_{\mathrm{DE}}(t)$, the Einstein equation is modified as

$$\frac{\dot{a}}{a} = \sqrt{\frac{8\pi G}{3}(\rho + \rho_{\mathrm{DE}}) - \frac{K}{a^2}}, \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{8\pi G}{3}\rho_{\mathrm{DE}}. \tag{C.35}$$

Therefore inserting the cosmological constant is equivalent to introducing the dark energy.

Actually it is known that the universe is accelerated, or $\ddot{a} > 0$, therefore we *need* the dark energy, and it is known that 70% of the whole energy of the universe is the dark energy. [9]

Bibliography

[1] Motoi Endo, Koichi Hamaguchi, and Sho Iwamoto. *Lepton Flavor Violation and Cosmological Constraints on R-parity Violation*. JCAP, **1002**, 032 (2010) [hep-ph/0912.0585]. — pp. ii, 3, 34, and 36.

[2] Rudolf Haag, Jan T. Łopuszański, and Martin Sohnius. *All Possible Generators of Supersymmetries of the s Matrix*. Nucl. Phys., **B88**, 257 (1975). — p. 1.

[3] Stephen P. Martin. *A Supersymmetry Primer*. arXiv: hep-ph/9709356. — pp. 1 and 23.

[4] Pierre Fayet. *Supersymmetry and Weak, Electromagnetic and Strong Interactions*. Phys. Lett., **B64**, 159 (1976). — pp. 2 and 69.

[5] Pierre Fayet. *Spontaneously Broken Supersymmetric Theories of Weak, Electromagnetic and Strong Interactions*. Phys. Lett., **B69**, 489 (1977). — pp. 2 and 69.

[6] Glennys R. Farrar and Pierre Fayet. *Phenomenology of the Production, Decay, and Detection of New Hadronic States Associated with Supersymmetry*. Phys. Lett., **B76**, 575–579 (1978). — pp. 2, 6, and 69.

[7] C. Amsler, et al. *Review of particle physics*. Phys. Lett., **B667**, 1 (2008). and 2009 partial update at http://pdg.lbl.gov/. — pp. 2, 5, 11, 14, 28, 68, 74, and 75.

[8] E. Komatsu, et al. *Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation*. Astrophys. J. Suppl., **180**, 330–376 (2009) [astro-ph/0803.0547]. — p. 2.

[9] G. Hinshaw, et al. *Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Data Processing, Sky Maps, & Basic Results*. Astrophys. J. Suppl., **180**, 225–245 (2009) [astro-ph/0803.0732]. — pp. 2 and 95.

[10] Herbi K. Dreiner, Christoph Luhn, and Marc Thormeier. *What is the discrete gauge symmetry of the MSSM?* Phys. Rev., **D73**, 075007 (2006) [hep-ph/0512163]. — pp. 6, 78, and 80.

[11] R. Barbier, et al. *R-parity violating supersymmetry*. Phys. Rept., **420**, 1–202 (2005) [hep-ph/0406039]. — pp. 10, 12, 20, 60, and 81.

[12] Yee Kao and Tatsu Takeuchi. *Single-Coupling Bounds on R-parity violating Supersym-*

metry, an update. arXiv: hep-ph/0910.4980. — pp. 10, 11, 14, and 20.

[13] Yee Kao and Tatsu Takeuchi. *Constraints on R-parity violation from recent Belle/Babar data*. arXiv: hep-ph/0909.0042. — pp. 10, 12, 16, and 20.

[14] Vernon D. Barger, G. F. Giudice, and Tao Han. *Some New Aspects of Supersymmetry R-Parity Violating Interactions*. Phys. Rev., **D40**, 2987 (1989). — pp. 10 and 12.

[15] Margarete Herz. *Bounds on Leptoquark and Supersymmetric, R-parity violating Interactions from Meson Decays*. arXiv: hep-ph/0301079. Diplom thesis, in German. — p. 12.

[16] Herbert K. Dreiner, Giacomo Polesello, and Marc Thormeier. *Bounds on broken R-parity from leptonic meson decays*. Phys. Rev., **D65**, 115006 (2002) [hep-ph/0112228]. — pp. 12 and 21.

[17] M. E. Peskin and D. V. Schroeder. *An Introduction to Quantum Field Theory*. Westview Press, Jun. 1995. — pp. 21, 22, 49, and 61.

[18] N. S. Manton. *Topology in the Weinberg-Salam Theory*. Phys. Rev., **D28**, 2019 (1983). — p. 24.

[19] Frans R. Klinkhamer and N. S. Manton. *A Saddle Point Solution in the Weinberg-Salam Theory*. Phys. Rev., **D30**, 2212 (1984). — p. 24.

[20] Gerard 't Hooft. *Computation of the quantum effects due to a four-dimensional pseudoparticle*. Phys. Rev., **D14**, 3432–3450 (1976). — p. 24.

[21] V. A. Kuzmin, V. A. Rubakov, and M. E. Shaposhnikov. *On Anomalous Electroweak Baryon-Number Non-conservation in the Early Universe*. Phys. Lett., **B155**, 36 (1985). — pp. 25 and 35.

[22] A. Ringwald. *Rate of Anomalous Baryon and Lepton Number Violation at Finite Temperature in Standard Electroweak Theory*. Phys. Lett., **B201**, 510 (1988). — p. 25.

[23] A. Bouquet and P. Salati. *R Parity Breaking and Cosmological Consequences*. Nucl. Phys., **B284**, 557 (1987). — pp. 28 and 34.

[24] A. D. Sakharov. *Violation of CP Invariance, c Asymmetry, and Baryon Asymmetry of the Universe*. Pisma Zh. Eksp. Teor. Fiz., **5**, 32–35 (1967). — p. 33.

[25] Sacha Davidson. *Cosmological constraints on B−L violation*. arXiv: hep-ph/9808427. — pp. 34, 35, and 37.

[26] Bruce A. Campbell, Sacha Davidson, John R. Ellis, et al. *On B and L violation in the laboratory in the light of cosmological and astrophysical constraints*. Astropart. Phys., **1**, 77–98 (1992). — p. 35.

[27] Bruce A. Campbell, Sacha Davidson, John R. Ellis, et al. *Cosmological baryon asym-*

metry constraints on extensions of the standard model. Phys. Lett., **B256**, 484–490 (1991). — p. 35.

[28] W. Fischler, G. F. Giudice, R. G. Leigh, et al. *Constraints on the baryogenesis scale from neutrino masses*. Phys. Lett., **B258**, 45–48 (1991). — p. 35.

[29] James M. Cline, Kimmo Kainulainen, and Keith A. Olive. *Protecting the primordial baryon asymmetry from erasure by sphalerons*. Phys. Rev., **D49**, 6394–6409 (1994) [hep-ph/9401208]. — p. 35.

[30] Tomoyuki Inui, Tomoyasu Ichihara, Yukihiro Mimura, et al. *Cosmological baryon asymmetry in supersymmetric Standard Models and heavy particle effects*. Phys. Lett., **B325**, 392–400 (1994) [hep-ph/9310268]. — p. 35.

[31] Bruce A. Campbell, Sacha Davidson, John R. Ellis, et al. *On the baryon, lepton flavor and right-handed electron asymmetries of the universe*. Phys. Lett., **B297**, 118–124 (1992) [hep-ph/9302221]. — p. 35.

[32] Herbert K. Dreiner and Graham G. Ross. *Sphaleron erasure of primordial baryogenesis*. Nucl. Phys., **B410**, 188–216 (1993) [hep-ph/9207221]. — p. 35.

[33] J. Hisano, T. Moroi, K. Tobe, et al. *Lepton-Flavor Violation via Right-Handed Neutrino Yukawa Couplings in Supersymmetric Standard Model*. Phys. Rev., **D53**, 2442–2459 (1996) [hep-ph/9510309]. — p. 41.

[34] Tsutomu Yanagida. *Horizontal Symmetry and Masses of Neutrinos*. Prog. Theor. Phys., **64**, 1103–1105 (Sept. 1980). — p. 41.

[35] W. Buchmuller, R. D. Peccei, and T. Yanagida. *Leptogenesis as the origin of matter*. Ann. Rev. Nucl. Part. Sci., **55**, 311–355 (2005) [hep-ph/0502169]. — p. 41.

[36] Julius Wess and Jonathan Bagger. *Supersymmetry and Supergravity*. Princeton University Press, 2nd edition, 1992. — p. 70.

[37] S. Chadha and M. Daniel. *Chiral Lagrangian Calculation of Nucleon Decay Modes Induced by $d = 5$ supersymmetric operators*. Nucl. Phys., **B229**, 105 (1983). — p. 74.

[38] J. Hisano, H. Murayama, and T. Yanagida. *Nucleon decay in the minimal supersymmetric SU(5) grand unification*. Nucl. Phys., **B402**, 46–84 (1993) [hep-ph/9207279]. — p. 74.

[39] Lawrence M. Krauss and Frank Wilczek. *Discrete Gauge Symmetry in Continuum Theories*. Phys. Rev. Lett., **62**, 1221 (1989). — p. 77.

[40] Steven Weinberg. *The Quantum Theory of Fields*, Vol. 2; Modern Applications. Cambridge University Press, May 2005. — p. 78.

[41] Luis E. Ibáñez and Graham G. Ross. *Discrete gauge symmetry anomalies*. Phys. Lett., **B260**, 291–295 (1991). — p. 78.

[42] Luis E. Ibáñez and Graham G. Ross. *Discrete gauge symmetries and the origin of baryon and lepton number conservation in supersymmetric versions of the standard model*. Nucl. Phys., **B368**, 3–37 (1992). — pp. 78 and 80.

[43] Christoph Luhn and Marc Thormeier. *Dirac neutrinos and anomaly-free discrete gauge symmetries*. Phys. Rev., **D77**, 056002 (2008) `[hep-ph/0711.0756]`. — pp. 78 and 80.

[44] Tom Banks, Yuval Grossman, Enrico Nardi, et al. *Supersymmetry without R-parity and without lepton number*. Phys. Rev., **D52**, 5319–5325 (1995) `[hep-ph/9505248]`. — p. 82.

[45] Steven Weinberg. *Cosmology*. Oxford University Press, Apr. 2008. — pp. 85 and 93.

[46] Bernard F. Schutz. *A first course in general relativity*. Cambridge University Press, 1985. — p. 93.

[47] Edward W. Kolb and Michael Turner. *The Early Universe*. Westview Press, 1994. — p. 93.