



# ILC capability of measuring SUSY-contrib. to $(g-2)_\mu$

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[BURI 2014](#) @ [University of Toyama](#)

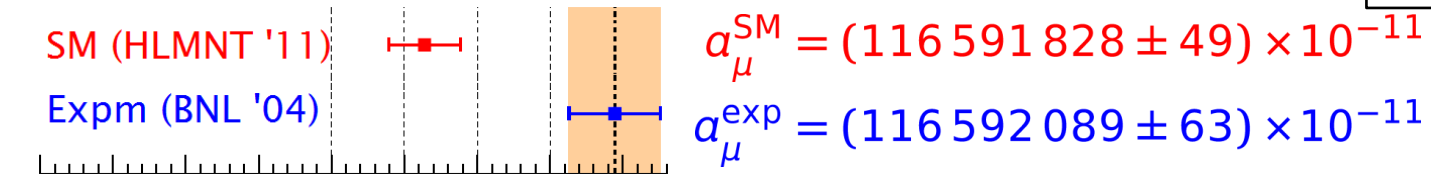
Reference)

M. Endo, K. Hamaguchi, S. I., T. Kitahara, T. Moroi [[1310.4496](#)].

# Muon $g-2$ Problem

$$\left( a_\mu := \frac{g_\mu - 2}{2} \right)$$

Muon  $g-2$  (anomalous magnetic moment)



Hagiwara, Liao, Martin, Nomura, Teubner [[1105.3149](#)]

**3.3 $\sigma$  discrepancy**

**New Physics?**

can be explained with **SUSY**.

Lopez, Nanopoulos, Wang [[ph/9308336](#)]  
Chattopadhyay, Nath [[ph/9507386](#)]  
Moroi [[ph/9512396](#)]

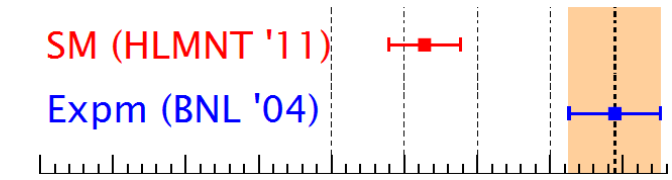
## SUSY!

- ✓ Dark matter problem,
- ✓ Hierarchy problem,
- ✓ Muon  $g-2$  problem,
- ✓ Grand unification,
- ✓ **will be discovered at LHC.**

# Muon $g-2$ Problem

$$\left( a_\mu := \frac{g_\mu - 2}{2} \right)$$

Muon  $g-2$  (anomalous magnetic moment)



$$a_\mu^{\text{SM}} = (116\,591\,828 \pm 49) \times 10^{-11}$$

$$a_\mu^{\text{exp}} = (116\,592\,089 \pm 63) \times 10^{-11}$$

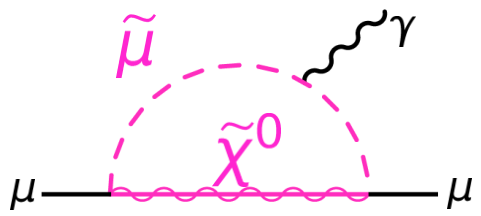
Hagiwara, Liao, Martin, Nomura, Teubner [[1105.3149](#)]

**3.3 $\sigma$  discrepancy**

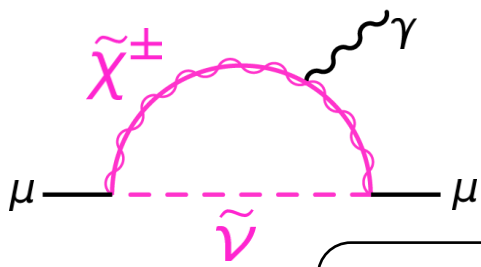
**New Physics?**  
 can be explained with **SUSY**.

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 Moroi [[ph/9512396](#)]

$$\left[ \rightsquigarrow (\tilde{\chi}^0, \tilde{\mu}) \text{ or } (\tilde{\chi}^\pm, \tilde{\nu}) = \mathcal{O}(100)\text{GeV} \right]$$



$$a_\mu^{\text{SUSY}}(\tilde{\chi}^0, \tilde{\mu}) \approx \frac{g_Y^2}{(4\pi)^2} \frac{m_\mu^2}{m_{\text{soft}}^2} \text{sgn}(\mu) \tan \beta + \dots,$$



$$a_\mu^{\text{SUSY}}(\tilde{\chi}^\pm, \tilde{\nu}) \approx \frac{g_2^2}{(4\pi)^2} \frac{m_\mu^2}{m_{\text{soft}}^2} \text{sgn}(\mu) \tan \beta.$$

$W \ni \mu H_u H_d$  (Higgsino mass term),  $\tan \beta = \frac{\langle H_u \rangle}{\langle H_d \rangle}$ ,  
 $m_{\text{soft}}$  : SUSY-particle mass-scale,  $g_i$  : Gauge couplings.

# Muon $g-2$ Problem

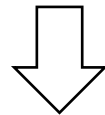
Muon  $g-2$  (a)

SM (HLMNT)

Expm (BNL)

Hagiwara, Liac

201X : SUSY discovery @ LHC



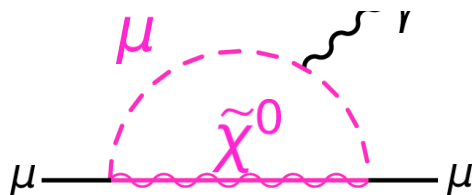
202X : SUSY measurement @ ILC

$$a_\mu^{\text{SUSY}} = \frac{g_\mu - 2}{2}$$

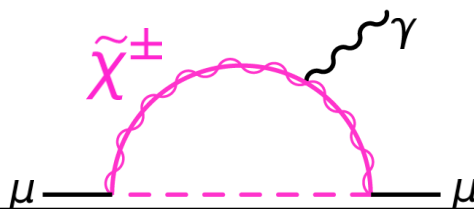
discrepancy

Can we measure these contribution?

[08336](#)  
[07386](#)  
[12396](#)



$$a_\mu^{\text{SUSY}}(\tilde{\chi}^0, \tilde{\nu}) \approx \frac{g_Y^2}{(4\pi)^2} \frac{m_\mu^2}{m_{\text{soft}}^2} \text{sgn}(\mu) \tan \beta + \dots,$$



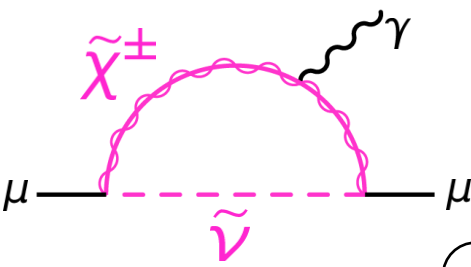
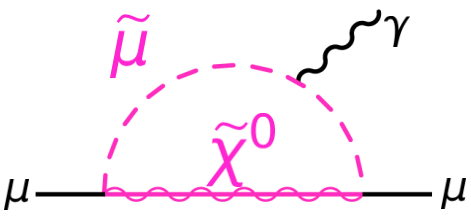
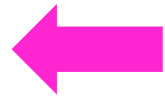
$$a_\mu^{\text{SUSY}}(\tilde{\chi}^\pm, \tilde{\nu}) \approx \frac{g_2^2}{(4\pi)^2} \frac{m_\mu^2}{m_{\text{soft}}^2} \text{sgn}(\mu) \tan \beta.$$

Note: We consider MSSM;  $R$ -parity conserved.  
 (Minimal SUSY Standard Model)

$m_{\text{soft}}$  : SUSY-particle mass-scale,  $g_i$  : Gauge couplings.

$$\left( a_\mu := \frac{g_\mu - 2}{2} \right)$$

reconstruct

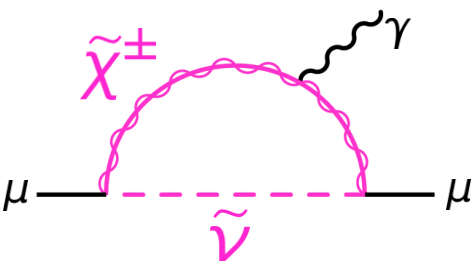
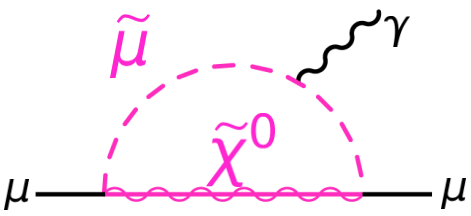


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 $m_{\text{soft}}$  : SUSY-particle mass-scale,  $g_i$  : Gauge couplings.

$$\left( a_\mu := \frac{g_\mu - 2}{2} \right)$$



$$a_\mu^{\text{SUSY}}(\tilde{\chi}^0, \tilde{\mu}) = f_N \left( m_{\tilde{\mu}_a}, m_{\tilde{\chi}_i^0}, \frac{g_Y N_{1i}^* + g_2 N_{2i}^*}{\sqrt{2}} E_{1a}^* - Y_\mu N_{3i}^* E_{2a}^*, -Y_\mu N_{3i} E_{1a}^* - \sqrt{2} g_Y N_{1i} E_{2a}^* \right)$$

$$a_\mu^{\text{SUSY}}(\tilde{\chi}^\pm, \tilde{\nu}) = f_C \left( m_{\tilde{\nu}_\mu}, m_{\tilde{\chi}_i^\pm}, -g_2 D_{1i}, Y_\mu C_{2i} \right)$$

where  $f_N(M, M_{\tilde{\chi}}, g_L, g_R) = \frac{1}{16\pi^2} \left[ -\frac{|g_L|^2 + |g_R|^2}{6} \frac{m_\mu^2}{M^2} N_1 \left( \frac{M_{\tilde{\chi}}^2}{M^2} \right) - \text{Re}(g_L^* g_R) \frac{m_\mu M_{\tilde{\chi}}}{M^2} N_2 \left( \frac{M_{\tilde{\chi}}^2}{M^2} \right) \right]$

$f_C(M, M_{\tilde{\chi}}, g_L, g_R) = \frac{1}{16\pi^2} \left[ +\frac{|g_L|^2 + |g_R|^2}{6} \frac{m_\mu^2}{M^2} C_1 \left( \frac{M_{\tilde{\chi}}^2}{M^2} \right) - \text{Re}(g_L^* g_R) \frac{m_\mu M_{\tilde{\chi}}}{M^2} C_2 \left( \frac{M_{\tilde{\chi}}^2}{M^2} \right) \right]$

$N_1(x) := \frac{1 - 6x + 3x^2 + 2x^3 - 6x^2 \log x}{(1-x)^4}, \quad N_2(x) := \frac{1 - x^2 + 2x \log x}{(1-x)^3}$

$C_1(x) := \frac{2 + 3x - 6x^2 + x^3 + 6x \log x}{(1-x)^4}, \quad C_2(x) := \frac{3 - 4x + x^2 + 2 \log x}{(1-x)^3}$

$$\left( a_\mu := \frac{g_\mu - 2}{2} \right)$$

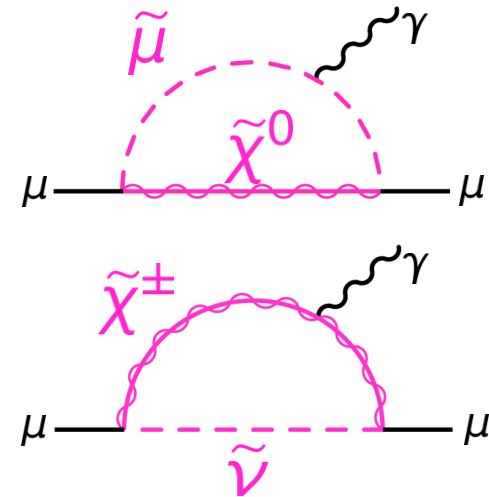
Why so complex?

$$\therefore \tilde{\chi}^0 = (\tilde{B}, \tilde{W}^0, \tilde{H}_u^0, \tilde{H}_d^0),$$

$$\tilde{\chi}^\pm = (\tilde{W}^\pm, \tilde{H}^\pm),$$

$$\tilde{\mu} = (\tilde{l}_L, \tilde{l}_R).$$

→ We consider  $\mu \gg 100 \text{ GeV}$ ,  
 $M_2 \gg 100 \text{ GeV}$ ,  
 $M_{\text{colored}} \gg 100 \text{ GeV}$ .



$$a_\mu^{\text{SUSY}}(\tilde{\chi}^0, \tilde{\mu}) = f_N \left( m_{\tilde{\mu}_a}, m_{\tilde{\chi}_i^0}, \frac{g_Y N_{1i}^* + g_2 N_{2i}^*}{\sqrt{2}} E_{1a}^* - Y_\mu N_{3i}^* E_{2a}^*, \right. \\ \left. - Y_\mu N_{3i} E_{1a}^* - \sqrt{2} g_Y N_{1i} E_{2a}^* \right)$$

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$$\left( a_\mu := \frac{g_\mu - 2}{2} \right)$$

Why so complex?

decoupled

$$\therefore \tilde{\chi}^0 = (\tilde{B}, \tilde{W}^0, \tilde{H}_u^0, \tilde{H}_d^0),$$

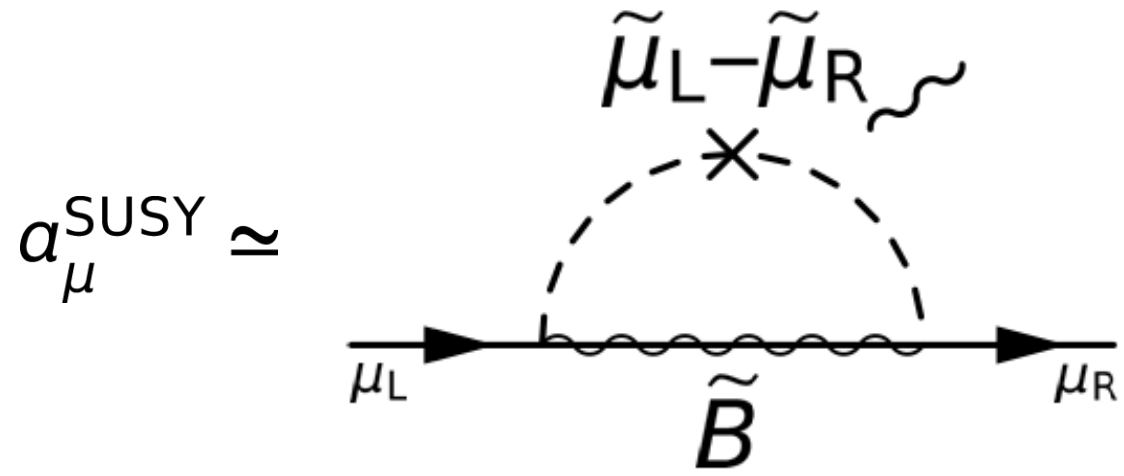
$$\tilde{\chi}^\pm = (\tilde{W}^\pm, \tilde{H}^\pm),$$

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→ We consider  $\mu \gg 100 \text{ GeV},$

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$M_{\text{colored}} \gg 100 \text{ GeV}.$





$$\left( a_\mu := \frac{g_\mu - 2}{2} \right)$$

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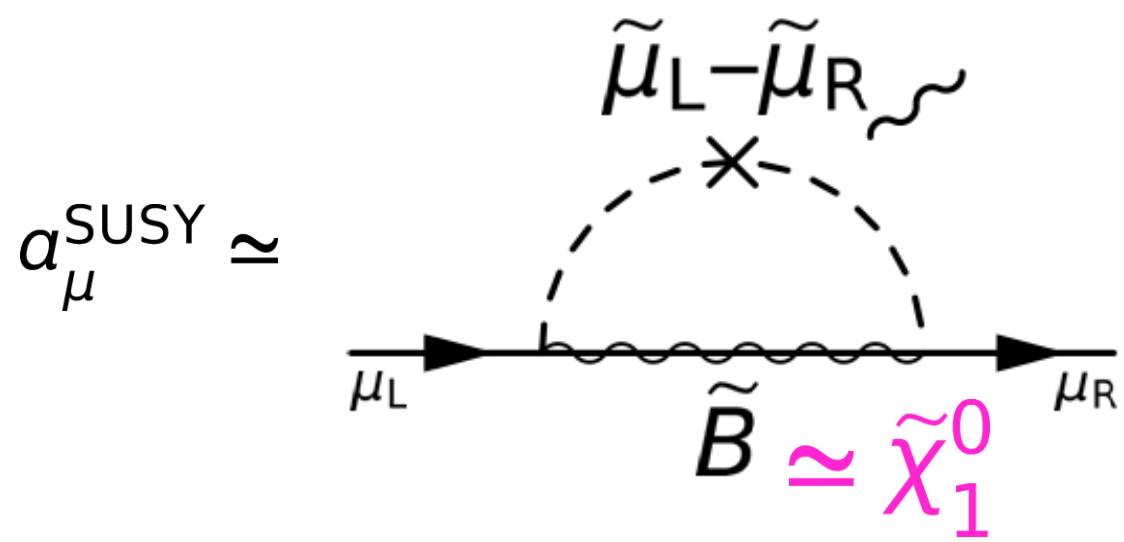
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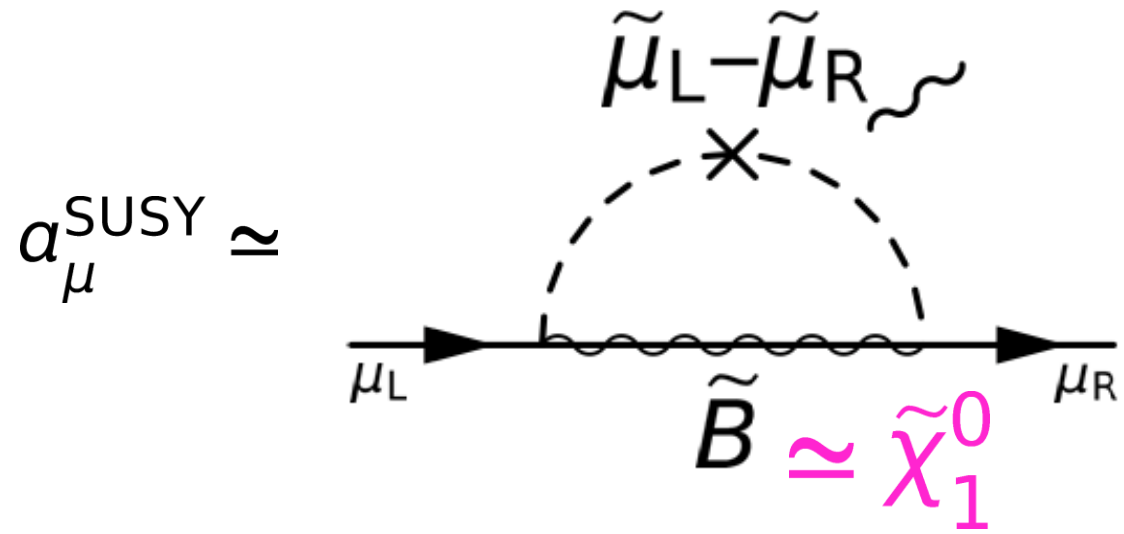
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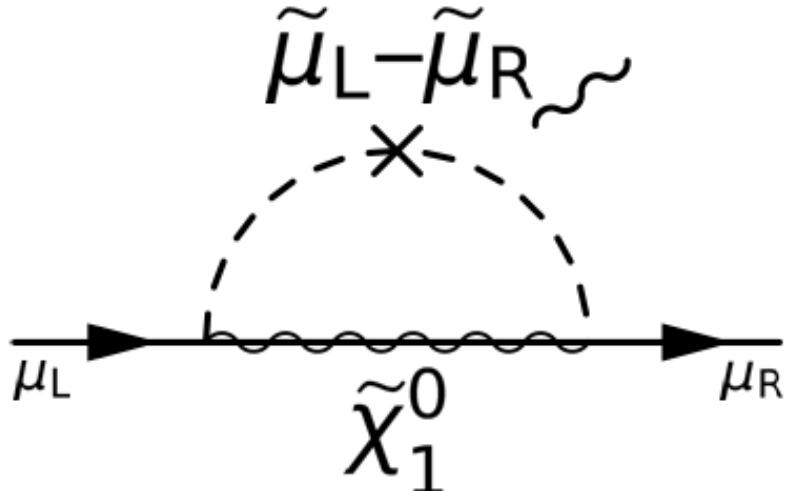
→ We consider  $\mu \gg 100 \text{ GeV}$ ,  
 $M_2 \gg 100 \text{ GeV}$ ,  
 $M_{\text{colored}} \gg 100 \text{ GeV}$ .



## What keys should we collect?

- Let's measure  $a_\mu^{\text{SUSY}}$ !

## What should be measured?

$$a_\mu^{\text{SUSY}} \approx$$


$\tilde{\mu}_L - \tilde{\mu}_R$

$\tilde{\chi}_1^0$

$\mu_L$   $\mu_R$

$$\left( \propto \frac{m_\mu \cdot M_{LR}^2 \cdot \tilde{g}_L \tilde{g}_R}{(\text{mass})^3} \right)$$

## What keys should we collect?

- Let's measure  $\alpha_{\mu}^{\text{SUSY}}$ !

## What should be measured?

➤ Mass of  $\tilde{\mu}_1, \tilde{\mu}_2, \tilde{\chi}_0^1$

➤ Mixing  $M_{LR}^2$

➤ Coupling  $\tilde{g}_L, \tilde{g}_R$

$\neq g_Y$  because

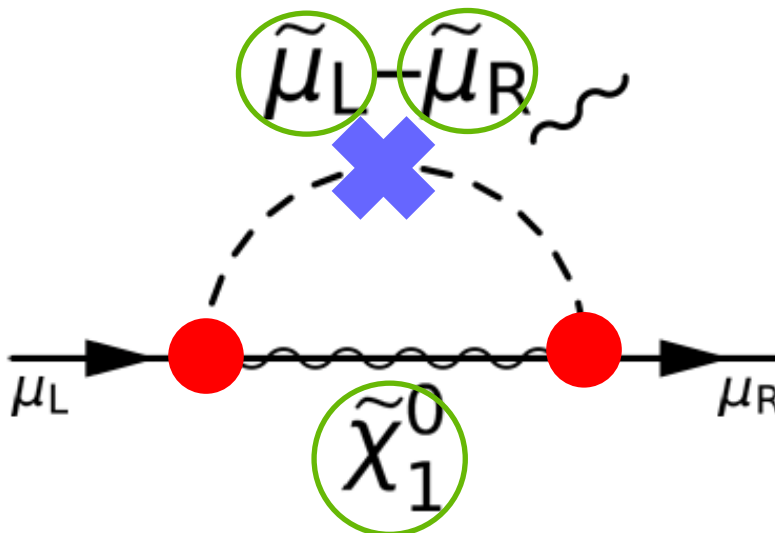
- SUSY effect.
- $\tilde{\chi}_1^0 \neq$  "pure"  $\tilde{B}$ .

$$\mathcal{M}_{\tilde{\mu}}^2 = \begin{pmatrix} m_L^2 & M_{LR}^2 \\ M_{LR}^2 & m_R^2 \end{pmatrix}$$

$$(M_{LR}^2 \simeq m_{\mu} \mu \tan \beta)$$

$$M_{LR}^2 = -\frac{1}{2} (m_2^2 - m_1^2) \sin 2\theta$$

$\alpha_{\mu}^{\text{SUSY}} \simeq$



$$\left( \propto \frac{m_{\mu} \cdot M_{LR}^2 \cdot \tilde{g}_L \tilde{g}_R}{(\text{mass})^3} \right)$$

## What keys should we collect?

- Let's measure  $a_{\mu}^{\text{SUSY}}$ !

## What should be measured?

➤ Mass of  $\tilde{\mu}_1, \tilde{\mu}_2, \tilde{\chi}_0^1$

➤ Mixing  $M_{LR}^2$

➤ Coupling  $\tilde{g}_L, \tilde{g}_R$

$\left( \neq g_Y \text{ because } \begin{array}{l} \bullet \text{ SUSY effect.} \\ \bullet \tilde{\chi}_1^0 \neq \text{"pure"} \tilde{B}. \end{array} \right)$

$$\mathcal{M}_{\tilde{\mu}}^2 = \begin{pmatrix} m_L^2 & M_{LR}^2 \\ M_{LR}^2 & m_R^2 \end{pmatrix}$$

$$(M_{LR}^2 \simeq m_{\mu} \mu \tan \beta)$$

$$M_{LR}^2 = -\frac{1}{2} (m_2^2 - m_1^2) \sin 2\theta$$

## What we will see:

$\tilde{e}_1, \tilde{e}_2, \tilde{\mu}_1, \tilde{\mu}_2, \tilde{\tau}_1, \tilde{\tau}_2$  (&  $\tilde{B}$ -LSP) observed @ ILC

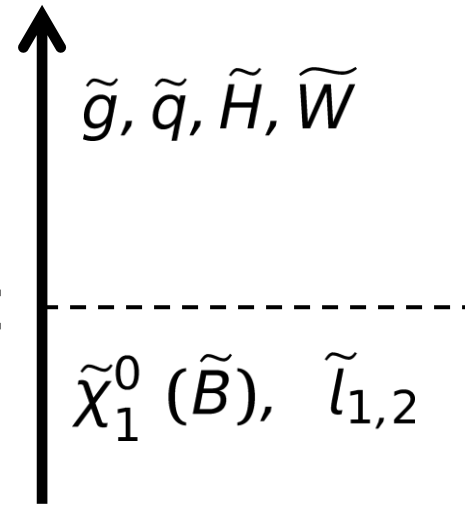
→ Mass, mixing, couplings can be measured.

→   $(\simeq a_{\mu}^{\text{SUSY}})$  can be reconstructed.

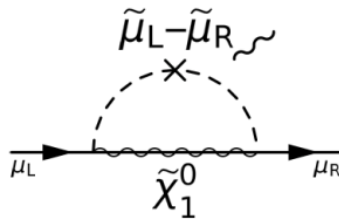
$\tilde{e}_1, \tilde{\mu}_1$	$\tilde{e}_2, \tilde{\mu}_2$	$\tilde{\tau}_1$	$\tilde{\tau}_2$	$\tilde{\chi}_1^0$	$\mu \tan \beta$
126	200	108	210	90	$6.1 \times 10^3$

[in GeV]

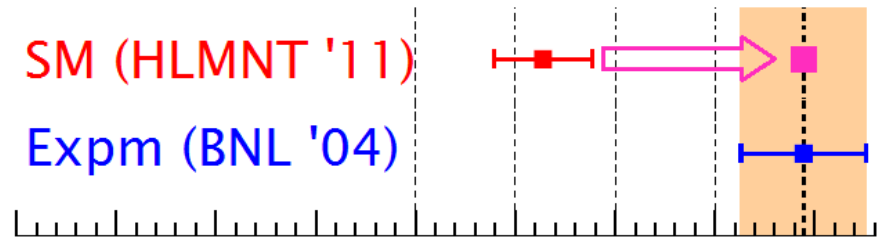
ILC



$\rightsquigarrow \sin \theta_{\tilde{\mu}} = 0.027, \sin \theta_{\tilde{\tau}} = 0.36,$



$\approx 2.6 \times 10^{-9}$



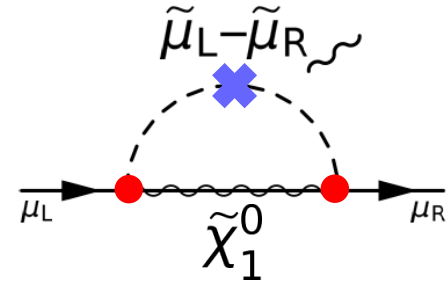
- Satisfies LEP/LHC constraints.
- Close to SPS1a(')

→ We can consult Previous works! Don't call us lazy :)

## How can we measure

- Mass of  $\tilde{\mu}_1, \tilde{\mu}_2, \tilde{\chi}_0^1$
- Mixing  $M_{LR}^2$
- Coupling  $\tilde{g}_L, \tilde{g}_R$  ?

and How accurately?



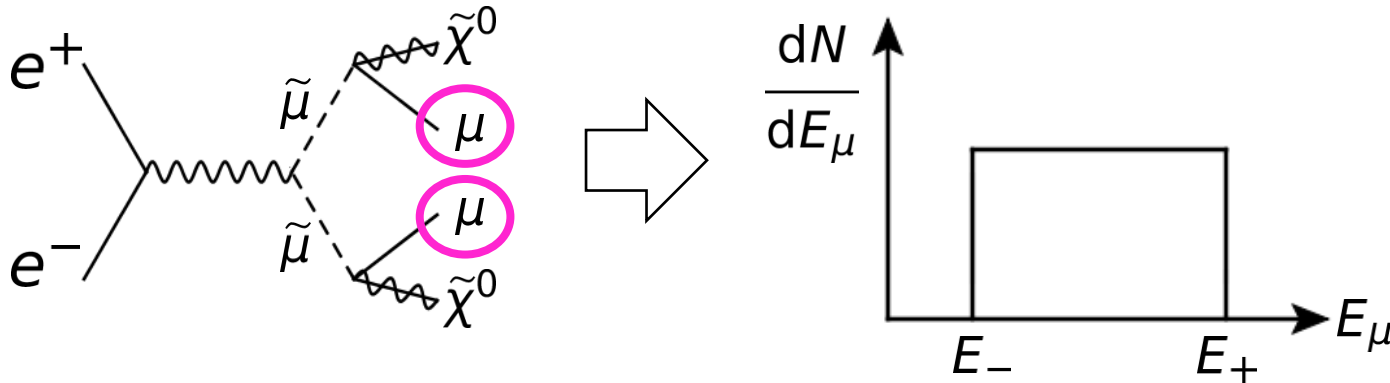
# How can we measure

## ➤ Mass of $\tilde{\mu}_1, \tilde{\mu}_2, \tilde{\chi}_0^1$

End-point analysis  $\rightarrow \Delta m_{\tilde{\mu}}, \Delta m_{\text{LSP}} \sim 100\text{--}200 \text{ MeV}$   
 (dominated by **stat.** unc.) ( $\sim 0.1\%$ )

@  $\sqrt{s} = 500 \text{ GeV}$ ,  $\int \mathcal{L} = 500 \text{ fb}^{-1}$

[ILC-TDR Vol.2 Sec.7.5.4]



$$E_{\pm} = \frac{\sqrt{S}}{4} (1 \pm \beta) \left( 1 - \frac{m^2}{M^2} \right); \quad \beta = \sqrt{1 - \frac{4M^2}{S}}.$$



$$\sin \theta_{\tilde{\mu}} = 0.027, \quad M_{LR}^2 = -\frac{1}{2} (m_2^2 - m_1^2) \sin 2\theta$$

$$\sin \theta_{\tilde{\tau}} = 0.36,$$

## How can we measure

➤ Mixing  $M_{LR}^2$

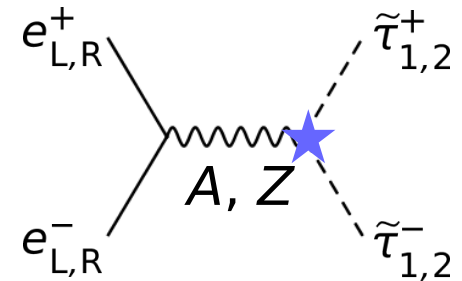
$$\mathcal{M}_{\tilde{\mu}}^2 = \begin{pmatrix} m_L^2 & M_{LR}^2 \\ M_{LR}^2 & m_R^2 \end{pmatrix}$$

$$(M_{LR}^2 \simeq m_\mu \mu \tan \beta)$$

$$\sigma(e^+ e^- \rightarrow \tilde{\tau}_A^+ \tilde{\tau}_B^-)$$

⇒  $\tilde{\tau}$  mixing  $M_{LR}^2(\tilde{\tau})$  measured.

$$\Rightarrow M_{LR}^2 = \frac{m_\mu}{m_\tau} M_{LR}^2(\tilde{\tau}) \quad (\text{as long as } A_{\tilde{\tau}} \simeq 0.)$$



$$\sigma(e^+ e^- \rightarrow \tilde{\tau}_i \tilde{\tau}_j) = \frac{8\pi\alpha^2}{3s} v^3 \left[ c_{ij}^2 \frac{\Delta_Z^2}{\sin^4 2\theta_W} (\mathcal{P}_{-+} L^2 + \mathcal{P}_{+-} R^2) \right. \\ \left. + \delta_{ij} \frac{1}{16} (\mathcal{P}_{-+} + \mathcal{P}_{+-}) + \delta_{ij} c_{ij} \frac{\Delta_Z}{2 \sin^2 2\theta_W} (\mathcal{P}_{-+} L + \mathcal{P}_{+-} R) \right];$$

$$v^2 = [1 - (m_{\tilde{\tau}_i} + m_{\tilde{\tau}_j})^2/s][1 - (m_{\tilde{\tau}_i} - m_{\tilde{\tau}_j})^2/s], \quad \Delta_Z = s/(s - m_Z^2),$$

$$c_{11/22} = \frac{1}{2} [L + R \pm (L - R) \cos 2\theta_{\tilde{\tau}}],$$

$$c_{12} = c_{21} = \frac{1}{2} (L - R) \sin 2\theta_{\tilde{\tau}},$$

$$L = -\frac{1}{2} + \sin^2 \theta_W,$$

$$R = \sin^2 \theta_W.$$

$$\sin \theta_{\tilde{\mu}} = 0.027, \quad M_{LR}^2 = -\frac{1}{2} (m_2^2 - m_1^2) \sin 2\theta$$

$$\sin \theta_{\tilde{\tau}} = 0.36,$$

## How can we measure

➤ Mixing  $M_{LR}^2$

$$\sigma(e^+ e^- \rightarrow \tilde{\tau}_A^+ \tilde{\tau}_B^-)$$

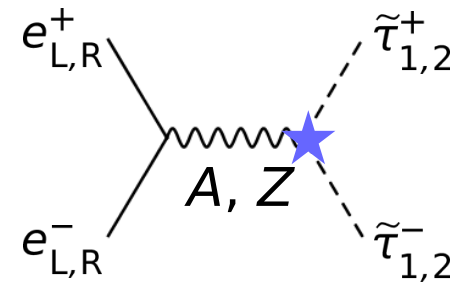
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$$\sigma(\tilde{\tau}_A \tilde{\tau}_B) = \begin{cases} (\dots) + (\dots) \cos 2\theta + (\dots) \cos^2 2\theta & (A = B) \\ (\dots) \sin^2 2\theta & (A \neq B) \end{cases}$$

$$M_{\tilde{\mu}}^2 = \begin{pmatrix} m_L^2 & M_{LR}^2 \\ M_{LR}^2 & m_R^2 \end{pmatrix}$$

$$(M_{LR}^2 \simeq m_\mu \mu \tan \beta)$$



# How can we measure

➤ Mixing  $M_{LR}^2$

$$\begin{aligned} \sin \theta_{\tilde{\mu}} &= 0.027, & M_{LR}^2 &= -\frac{1}{2} (m_2^2 - m_1^2) \sin 2\theta \\ \sin \theta_{\tilde{\tau}} &= 0.36, \end{aligned}$$

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$$\sigma(e^+ e^- \rightarrow \tilde{\tau}_A^+ \tilde{\tau}_B^-)$$

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$$\Rightarrow M_{LR}^2 = \frac{m_\mu}{m_\tau} M_{LR}^2(\tilde{\tau}) \quad (\text{as long as } A_{\tilde{\tau}} \simeq 0.)$$

$$\Delta M_{LR}^2 = \underbrace{\Delta m_{\tilde{\tau}1}}_{\sim 0.1\%} \oplus \underbrace{\Delta m_{\tilde{\tau}2}}_{\sim 3\%} \oplus \Delta\sigma \left( \tilde{\tau}_A^+ \tilde{\tau}_B^- \right)$$

$\left[ \begin{array}{l} @500 \text{ GeV}, 500 \text{ fb}^{-1}; \\ (P_+, P_-) = (-0.3, +0.8) \end{array} \right]$

ILC-TDR Vol.2 Sec.7.5.4 (and Bechtle, Berggren, et al. [0908.0876])

# How can we measure

➤ Mixing  $M_{LR}^2$

$$\begin{aligned} \sin \theta_{\tilde{\mu}} &= 0.027, & M_{LR}^2 &= -\frac{1}{2} (m_2^2 - m_1^2) \sin 2\theta \\ \sin \theta_{\tilde{\tau}} &= 0.36, \end{aligned}$$

$$\sigma(\tilde{\tau}_A \tilde{\tau}_B) = \begin{cases} (\dots) + (\dots) \cos 2\theta + (\dots) \cos^2 2\theta & (A = B) \\ (\dots) \sin^2 2\theta & (A \neq B) \end{cases}$$

$$\Delta\sigma(\tilde{\tau}_1 \tilde{\tau}_1) = \frac{1.8 \text{ fb}}{54 \text{ fb}} = 3.4\% \quad \Rightarrow \quad \Delta M_{LR}^2 = 12\%$$

(stat. dominated)

Not precise...

$$\Delta M_{LR}^2 = \Delta m_{\tilde{\tau}_1} \oplus \Delta m_{\tilde{\tau}_2} \oplus \Delta\sigma(\tilde{\tau}_A^+ \tilde{\tau}_B^-)$$

$\downarrow$   
 $\sim 0.1\%$

$\downarrow$   
 $\sim 3\%$

$\left[ \begin{array}{l} @500 \text{ GeV}, 500 \text{ fb}^{-1}; \\ (P_+, P_-) = (-0.3, +0.8) \end{array} \right]$

ILC-TDR Vol.2 Sec.7.5.4 (and Bechtle, Berggren, et al. [0908.0876])

[All values are for the sample mass spectrum.] **20**/<sub>26</sub>

$$\begin{aligned} \sin \theta_{\tilde{\mu}} &= 0.027, & M_{LR}^2 &= -\frac{1}{2} (m_2^2 - m_1^2) \sin 2\theta \\ \sin \theta_{\tilde{\tau}} &= 0.36, & & \end{aligned}$$

## How can we measure

➤ Mixing  $M_{LR}^2$

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(stat. dominated)

Not precise...

$$\Delta\sigma(\tilde{\tau}_1 \tilde{\tau}_2) = \frac{??? \text{ fb}}{2.7 \text{ fb}} = \dots \quad \rightarrow \text{ should be studied!}$$

$$\Delta M_{LR}^2 = \Delta m_{\tilde{\tau}_1} \oplus \Delta m_{\tilde{\tau}_2} \oplus \Delta\sigma(\tilde{\tau}_A^+ \tilde{\tau}_B^-)$$

$\downarrow$   
 $\sim 0.1\%$

$\downarrow$   
 $\sim 3\%$

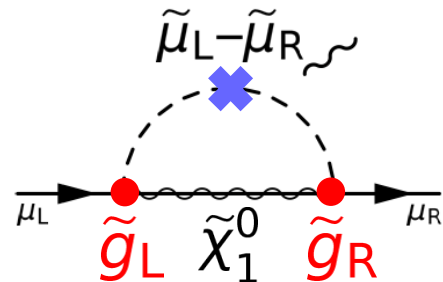
$\left[ \begin{array}{l} @500 \text{ GeV}, 500 \text{ fb}^{-1}; \\ (P_+, P_-) = (-0.3, +0.8) \end{array} \right]$

ILC-TDR Vol.2 Sec.7.5.4 (and Bechtle, Berggren, et al. [0908.0876])

[All values are for the sample mass spectrum.] **21** / 26

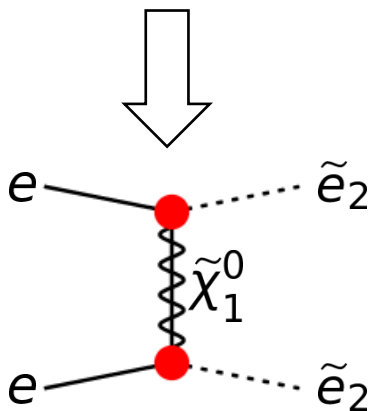
# How can we measure

➤ Coupling  $\tilde{g}_L, \tilde{g}_R$



$$\mu_R \text{---} \tilde{g}_R \text{---} \tilde{\chi}_1^0 \text{---} \tilde{\mu}_R = e_R \text{---} \tilde{g}_R^{(e)} \text{---} \tilde{\chi}_1^0 \text{---} \tilde{e}_R (\approx \tilde{e}_2) + \tilde{H}^0\text{-contribution} (\propto Y_\mu)$$

measured via



< 0.4% contrib.  
for  $\tilde{H} > 500 \text{ GeV}$

$$\Delta\sigma \sim \frac{4.7 \text{ fb}}{316 \text{ fb}} = 1.5\% \rightsquigarrow \Delta\tilde{g}_R^{(e)} \sim \underline{0.4\%}$$

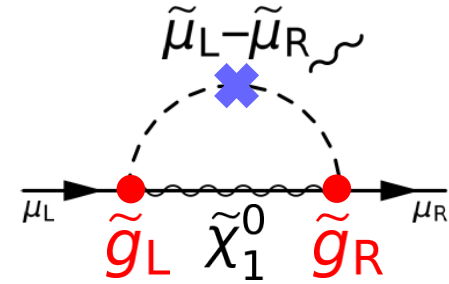
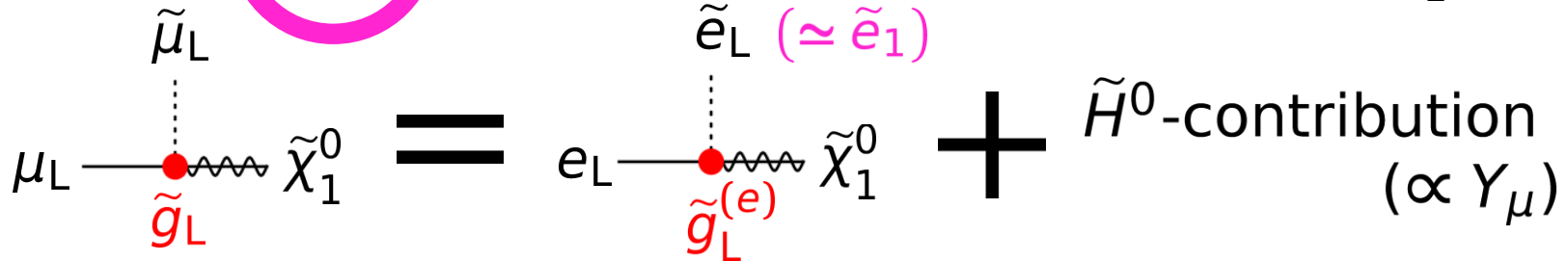
$$\therefore \Delta\tilde{g}_R \lesssim 1\%$$

Freitas, Kalinowski, et al. [[ph/0211108](#)]  
 Freitas, Manteuffel, Zerwas [[ph/0310382](#)]  
 Kilian, Zerwas [[ph/0601217](#)]

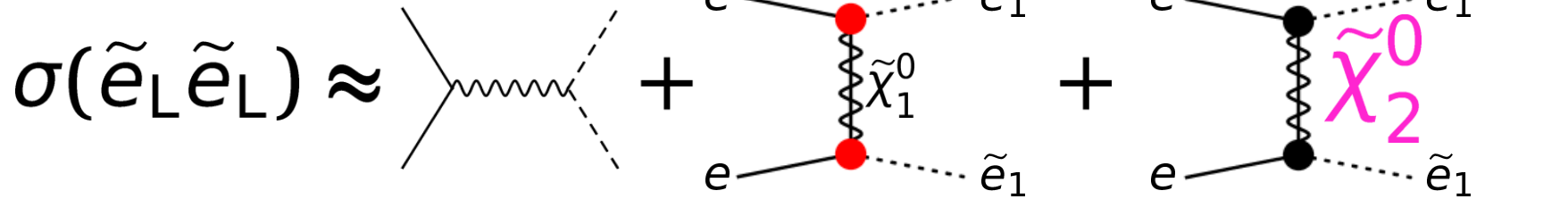
(@500 GeV, 500 fb<sup>-1</sup>;  
 (P<sub>+</sub>, P<sub>-</sub>) = (-0.3, +0.8))

# How can we measure

➤ Coupling  $\tilde{g}_L, \tilde{g}_R$



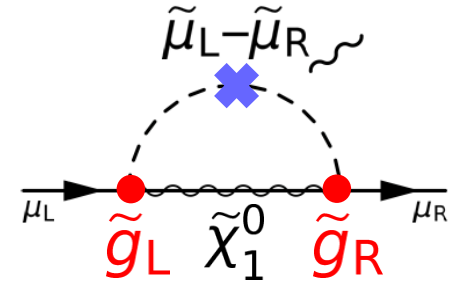
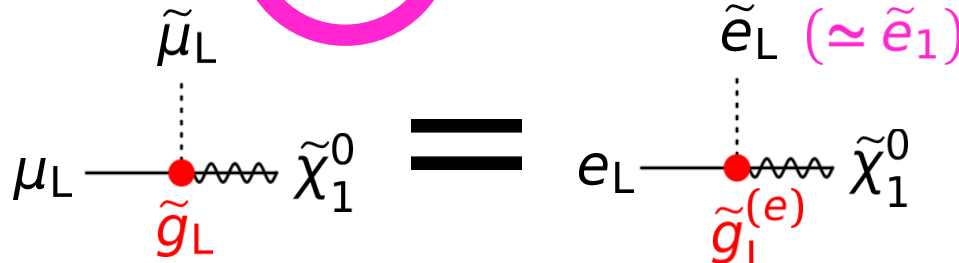
However



We use  $\sigma(\tilde{e}_L \tilde{e}_R)$ . ← cannot be neglected.

# How can we measure

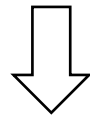
➤ Coupling  $\tilde{g}_L, \tilde{g}_R$



$\tilde{H}^0$ -contribution  
( $\propto Y_\mu$ )

< 0.9% contrib.  
for  $\tilde{H}, \tilde{W} > 500$  GeV

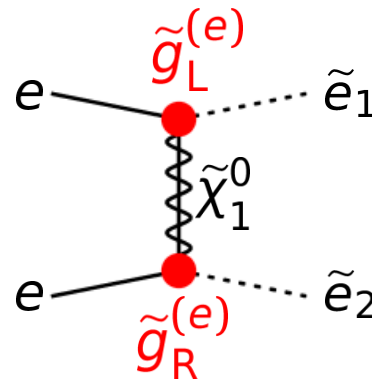
$$\sigma(\tilde{e}_L \tilde{e}_R) \approx$$



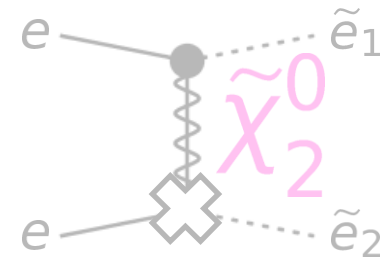
$$\Delta\sigma \sim \text{???}\% \quad (\sigma = 5.5 \text{ fb})$$

should be studied...

Here we use  $\Delta\tilde{g}_L^{(e)} \sim \underline{\text{a few}\%}$



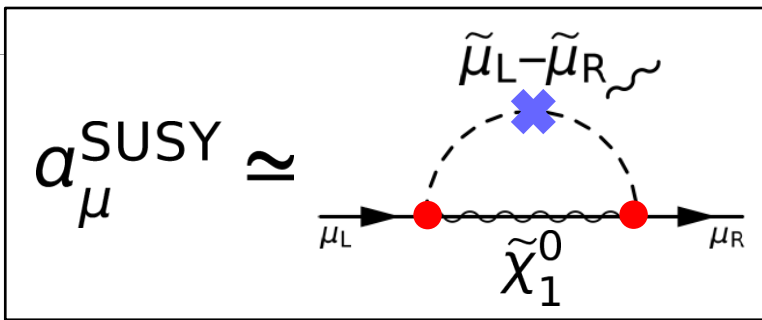
+



$\therefore \Delta\tilde{g}_L \sim 1 + \text{a few}\%$



Summary



$\therefore \Delta a_\mu^{\text{SUSY}} = 13\%$

Mass of  $\tilde{\mu}_1, \tilde{\mu}_2, \tilde{\chi}_1^1$

Mixing  $M_{LR}^2$

coupling  $\tilde{g}_L, \tilde{g}_R$

$e^+e^- \rightarrow \tilde{\mu}^+\tilde{\mu}^-$   
end-point

$\sigma(ee \rightarrow \tilde{\tau}_1\tilde{\tau}_1)$

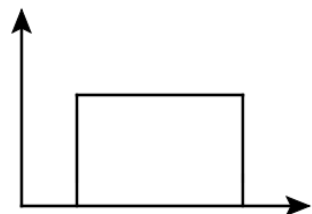
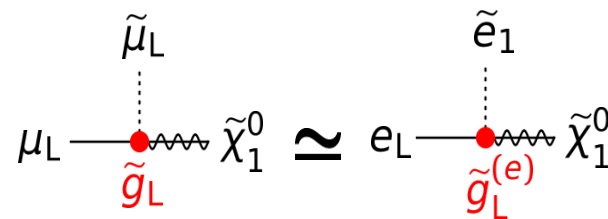
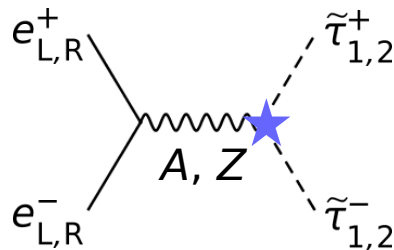
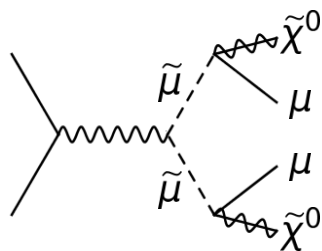
$\sigma(ee \rightarrow \tilde{e}_R\tilde{e}_R),$   
 $\sigma(ee \rightarrow \tilde{e}_L\tilde{e}_R)$

$\rightarrow \sim 0.1\%$

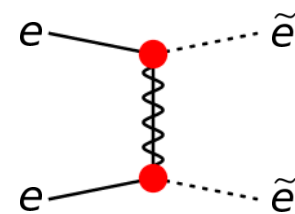
$\rightarrow \sim 12\%$

$\rightarrow R: \sim 1\%$

$L: (\text{a few} + 1)\%$

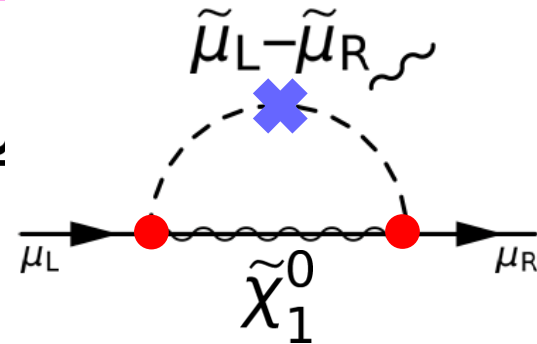


$M_{LR}^2 \simeq m_\mu \mu \tan \beta$   
 $\simeq \frac{m_\mu}{m_\tau} M_{LR}^2(\tilde{\tau})$



## For the scenario

- $\tilde{g}, \tilde{q}, \tilde{H}, \tilde{W} \gg 100 \text{ GeV}$ ,
- $\tilde{e}, \tilde{\mu}, \tilde{\tau} < \text{ILC reach}$ ,



$a_{\mu}^{\text{SUSY}}$  can reconstructed via

Mass of  $\tilde{\mu}_1, \tilde{\mu}_2, \tilde{\chi}_0^1$

$e^+e^- \rightarrow \tilde{\mu}^+\tilde{\mu}^-$   
end-point

Mixing  $M_{LR}^2$

$\sigma(ee \rightarrow \tilde{\tau}_1\tilde{\tau}_1)$

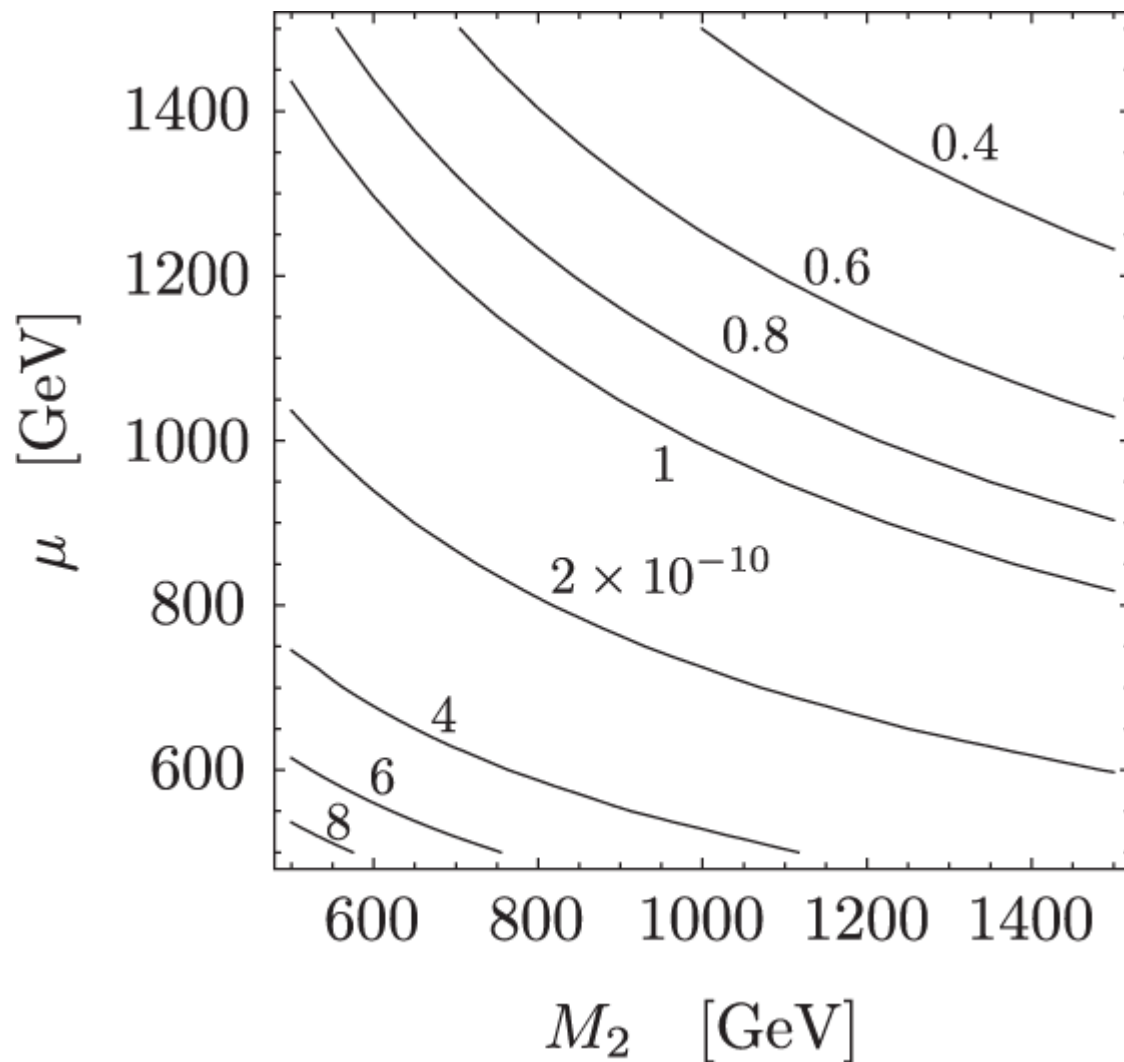
coupling  $\tilde{g}_L, \tilde{g}_R$

$\sigma(ee \rightarrow \tilde{e}_R\tilde{e}_R)$ ,  
 $\sigma(ee \rightarrow \tilde{e}_L\tilde{e}_R)$

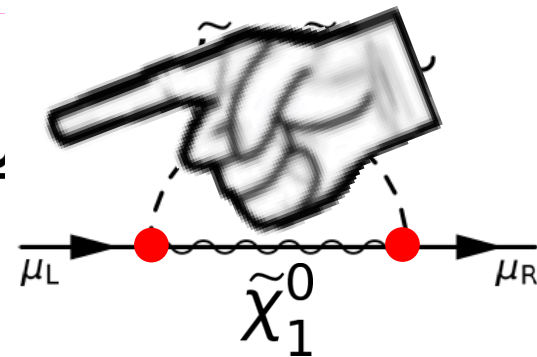
with the precision **13%** (at our sample point).

can be improved if we use  
 $\sigma(ee \rightarrow \tilde{\tau}_1\tilde{\tau}_2)$ .

Largely depends  
on **mixing**.



$$\alpha_{\mu}^{\text{SUSY}} \approx$$



$$= 2.6 \times 10^{-9}$$